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SEMI-ACTIVE REDUNDANT FAULT TOLERANT CONTROL FOR AN OMNIDIRECTIONAL REHABILITATIVE TRAINING ROBOT WITH CENTER OF GRAVITY SHIFT

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Abstract. *A complex control program for an omnidirectional rehabilitative training robot (ORTR) with actuator fault and center of gravity shift was proposed in this paper. Firstly, a stochastic nonlinear model was constructed for the ORTR with structure parameter uncertainty based on stochastic theory. And then, by combining the merits of passive fault tolerant control (PFTC), active fault tolerant control (AFTC), and redundant fault tolerant control (RFTC), the semi-active redundant fault tolerant control (SRFTC) was proposed and applied to the ORTR. Finally, simulations were conducted to show the designed controller can synchronously resolve the problems of trajectory tracking, center of gravity shift, and actuator fault.*

Keywords: Semi-active fault tolerant control, Center of gravity shift, Trajectory tracking

1. **Introduction.** It is well known that there are many random disturbances and random parameter uncertainties in robot systems, which would evidently affect the motion performance of robots. In recent years, with the improvement and development of modern stochastic control theory, robot systems with random uncertainties and disturbances have turned into a hot topic of scholars' attention. [1] proposed an adaptive control method for a flexible joint robot with random noises. [2] investigated a feedback path tracking control technique for a random Lagrangian system. [3] presented a method of model construction for stochastic Lagrangian control system and brought up an adaptive tracking controller. It is worth noting that the above-mentioned stochastic models just take the random noise of input channel into consideration. Nevertheless, the internal structure of robots still includes more random parameters such as the shift in gravity center of ORTR [4].

In addition, mobile robots inevitably have faults during their operation, which will cause the performance of robot system to degrade or even become unstable. Recently, research results on fault tolerant control for robot have made great progress, such as passive fault tolerant control (PFTC), active fault tolerant control (AFTC), and redundant fault tolerant control (RFTC). However, all of them have the advantages and disadvantages. For the PFTC, the fault information is considered as a disturbance, but this approach requires partial knowledge of fault, such as the upper bound of disturbance, which limits its applications [5]. AFTC is hardly applied to mobile robot systems, because it requires accurate fault information obtained from fault diagnosis, and the sensor is hardly installed

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on the motor drive of mobile robot. RFTC is a very effective fault-tolerant approach by switching a damaged drive to a normal one, but in the process of drive switching, the states of system would be changed suddenly, which is not allowed in precise robots, such as ORTR [6]. As for the reasons above, we invent a semi-active redundant fault tolerant control (SRFTC), which is quite fit to apply into the precision mechanical system. The SRFTC, as well as AFTC and RFTC, can eliminate the impact of actuator failures more effectively than PFTC. Moreover, SRFTC can maintain the states of system to be changed steady, but does not require accurate fault value from fault diagnosis sensor, to insure the safety of training process. The basic condition of SRFTC system includes: 1) the mechanical system has redundant drives; 2) the fault of actuator can be discovered, but the fault information need not be diagnosed. The SRFTC system will use the disturbance rejection technology to the redundant normal actuator, and mechanical system can continue to work without the interruption in fault actuator.

In this paper, we will investigate an ORTR and the main tasks are described as the following. 1) By converting the random structural parameters into random disturbances, we construct an appropriate stochastic model to describe the movement of ORTR with the center of gravity shift. 2) In order to design an SRFTC, we first detected which actuator is broken, and separated the broken actuator from the control vector. Then, in order to adapt the broken actuator, we design a corresponding adaptive law. 3) With the application of SRFTC to the ORTR, the trajectory tracking error system is exponentially practically stable in mean absolute. Simulation results show the efficiency of the controller.

2. **Stochastic Model Construction for the ORTR with Center of Gravity Shift.** Structure of ORTR is shown in Figure 1, and Figure 2 presents reference frame of ORTR [4].

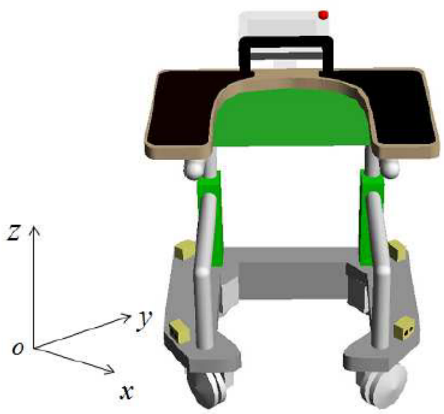

FIGURE 1. Structure of ORTR

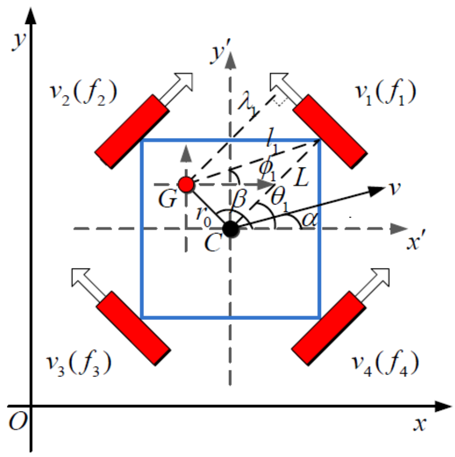

FIGURE 2. Reference frame of ORTR

In Figure 2, $\Sigma(x, O, y)$ is the world-coordinate; $\Sigma(x', C, y')$ is the local-coordinate; v is the speed of ORTR; v_i $(i = 1, 2, 3, 4)$ are the speeds of omniwheels; f_i are the forces on each omniwheels; G is the center of gravity of ORTR; r_0 is the distance between gravity center G and geometric center C with a load; α is the included angle between x'-axis and v; β is the included angle between x'-axis and r_0; L is the distance between geometric center of ORTR and apiece omniwheels; l_i are the distances between gravity center and apiece omniwheels; θ_i are the angles between x'-axis and center points of apiece omniwheels; ϕ_i are the angles between x'-axis and l_i.

The dynamic model of ORTR is presented below [4],

$$M_0K\ddot{X}(t) + M_0\dot{K}\dot{X}(t) = B(\theta)u(t) \tag{1}$$

where

$$M_0 = diag\left\{M+m, M+m, I_0+mr_0^2\right\}, X = [x(t), y(t), \theta(t)]^T, u(t) = [f_1, f_2, f_3, f_4]^T,$$

$$K = \begin{bmatrix} 1 & 0 & p \\ 0 & 1 & q \\ 0 & 0 & 1 \end{bmatrix}, \ B(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ \lambda_1 & -\lambda_2 & -\lambda_3 & \lambda_4 \end{bmatrix},$$

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} [(\lambda_1-\lambda_3)\sin\theta + (\lambda_2-\lambda_4)\cos\theta]/2 \\ [(\lambda_2-\lambda_4)\sin\theta - (\lambda_1-\lambda_3)\cos\theta]/2 \end{bmatrix}, \ \lambda_i = l_i\cos(\theta_i-\phi_i), \ (i=1,2,3,4).$$

Here, M is the mass of ORTR; m is the user's equivalent mass; I_0 is the inertia mass of ORTR. mr_0^2 is the user's inertia mass. f_1, f_2, f_3, and f_4 are the control input; r_0 and λ_1, λ_2, λ_3, λ_4 are the random parameters caused by center of gravity shift. θ is the angle between the center point of the 1st omniwheel and x'-axis, $\theta = \theta_1$, then we have $\theta_2 = \theta + \pi/2$, $\theta_3 = \theta + \pi$, and $\theta_4 = \theta + 3\pi/2$. In the consideration of $\lambda_1 + \lambda_3 = 2L$ and $\lambda_2 + \lambda_4 = 2L$, we extract the random parameters r_0, λ_1, λ_2, λ_3, λ_4 from (1) to get the stochastic model.

$$\ddot{X}(t) = M_1^{-1}B^*(\theta)u(t) + M_1^{-1}N\xi(t) \tag{2}$$

where

$$M_1 = diag\left\{M+m, M+m, I_0\right\}, B^*(\theta) = \begin{bmatrix} -\sin\theta_1 & \sin\theta_2 & \sin\theta_3 & -\sin\theta_4 \\ \cos\theta_1 & -\cos\theta_2 & \cos\theta_3 & \cos\theta_4 \\ L & L & L & L \end{bmatrix},$$

$$N = \begin{bmatrix} -(M+m)\sin\theta & \dot{\theta}^2(M+m)\sin\theta & -\dot{\theta}^2(M+m)\cos\theta & -(M+m)\cos\theta & 0 \\ (M+m)\cos\theta & -\dot{\theta}^2(M+m)\cos\theta & -\dot{\theta}^2(M+m)\sin\theta & -(M+m)\sin\theta & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\begin{aligned} \xi = \Big[&\ddot{\theta}(\lambda_1-L), (\lambda_2-L), (\lambda_1-L), \ddot{\theta}(\lambda_2-L), \\ &\Big((\lambda_1-L)(f_1+f_3) - (\lambda_2-L)(f_2+f_4) - \ddot{\theta}mr_0^2\Big)\Big]^T. \end{aligned}$$

In order to design the fault tolerant control that accounts for the actuator failure existing in input channels, a uniform actuator fault model [6] is introduced which would be used for separating the fault actuator.

$$u(t) = (I-\rho^*)u^*(t) = [u_i^*(t) \quad (1-\rho_i)\Delta u_i^*(t)]^T, \quad i = 1,2,3,4 \tag{3}$$

and $u^*(t)$, $B^*(\theta)$ can be broken down into,

$$u^*(t) = [u_i^*(t) \quad \Delta u_i^*(t)]^T \tag{4}$$

$$B^*(\theta) = [B_1^*(\theta) \quad B_2^*(\theta)] \tag{5}$$

where ρ^* can be described by $\rho^* = diag[\rho_1, \rho_2, \rho_3, \rho_4]$; $0 \le \rho_i \le 1$ is an unknown constant; the index i denotes the ith fault mode. $u_i^*(t)$ are well-functioning actuators control inputs and $\Delta u_i^*(t)$ is fault actuator input force. $B_1^*(\theta)$ is the homologous coefficient matrix of well-functioning input $u_i^*(t)$, and $B_2^*(\theta)$ is the homologous coefficient matrix of fault input $\Delta u_i^*(t)$. Then we separate the fault input $\Delta u_i^*(t)$ $(\rho_i \neq 0)$ from (2).

$$\begin{aligned} \ddot{X}(t) &= M_1^{-1}B^*(\theta)u(t) + M_1^{-1}N\xi(t) \\ &= M_1^{-1}B_1^*(\theta)u_1^*(t) + M_1^{-1}B_2^*(\theta)(1-\rho_i)\Delta u_1^*(t) + M_1^{-1}N\xi(t) \\ &= M_1^{-1}B_1^*(\theta)u_1^*(t) + M_1^{-1}\left[(1-\rho_i)B_2^*(\theta)\vdots N\right]\left[\Delta u_1^{*T}(t)\vdots\xi^T(t)\right]^T \end{aligned} \tag{6}$$

Assumption 2.1. *The random parameter uncertainties and the random fault input force* $\left[\Delta u_1^{*T}(t)\vdots\xi^T(t)\right]^T$ *are white noises.*

Assumption 2.2. *As M_1^{-1} is bounded, there are nonnegative parameters h_1 and h_2, such that*

$$0 \leq h_1 I_{3\times 3} \leq M_1^{-1} \leq h_2 I_{3\times 3} \tag{7}$$

Assumption 2.3. *Consider angular velocity $\dot{\theta}$ is bounded and the definition of the Frobenius norm, there exist unknown nonnegative constants h_3 and h_4, and smooth positive functions ψ_1, ψ_2 such that,*

$$\begin{aligned} & [(1-\rho_i)B_2^*(\theta_d(t)) \vdots N(\theta_d(t))]_F^2 \\ = \ & Tr\left\{(1-\rho_i)B_2^*(\theta_d(t))(1-\rho)B_2^*(\theta_d(t))^T\right\} + Tr\left\{N(\theta_d(t))N(\theta_d(t))^T\right\} \\ = \ & (1+L^2)(1-\rho_i)^2 + \left[2\dot{\theta}_d^4(t)(M+m)^2 + 2(M+m)^2 + 1\right] \\ \leq \ & \left(1+L^2\right)(1-\rho_i)^2 + h_3\psi_1 \end{aligned} \tag{8}$$

$$\left\|[(1-\rho_i)B_2^*(\theta) \vdots N(\theta)] - [(1-\rho_i)B_2^*(\theta_d(t)) \vdots N(\theta_d(t))]\right\|_F^2 \leq h_4\psi_2\left(e_2^T e_2\right)^2 \tag{9}$$

In view of Assumption 2.1, by replacing $\left[\Delta u_1^{*^T}(t) \vdots \xi^T(t)\right]^T$ with "dB/dt", the Stratonovich stochastic differential equation of (6) is achieved,

$$d\dot{X}(t) = M_1^{-1}B_1^*(\theta)u_1^*(t)dt + M_1^{-1}[(1-\rho_i)B_2^*(\theta) \vdots N] \circ dB \tag{10}$$

where B is a 6-dimensional independent Wiener process. We denote $[(1-\rho_i)B_2^*(\theta) \vdots N] = [\alpha_{ij}]_{i\times j}$ $(i=1,2,3;\ j=1,2,\dots,6)$. The Wong-Zakai correction term equals,

$$\frac{1}{2}\begin{bmatrix} \sum_{j=1}^{6}\left(\alpha_{1j}\cdot\frac{\partial\alpha_{1j}}{\partial\dot{x}(t)} + \alpha_{2j}\cdot\frac{\partial\alpha_{1j}}{\partial\dot{y}(t)} + \alpha_{3j}\cdot\frac{\partial\alpha_{1j}}{\partial\dot{\theta}(t)}\right) \\ \sum_{j=1}^{6}\left(\alpha_{1j}\cdot\frac{\partial\alpha_{2j}}{\partial\dot{x}(t)} + \alpha_{2j}\cdot\frac{\partial\alpha_{2j}}{\partial\dot{y}(t)} + \alpha_{3j}\cdot\frac{\partial\alpha_{2j}}{\partial\dot{\theta}(t)}\right) \\ \sum_{j=1}^{6}\left(\alpha_{1j}\cdot\frac{\partial\alpha_{3j}}{\partial\dot{x}(t)} + \alpha_{2j}\cdot\frac{\partial\alpha_{3j}}{\partial\dot{y}(t)} + \alpha_{3j}\cdot\frac{\partial\alpha_{3j}}{\partial\dot{\theta}(t)}\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

So, Itô stochastic differential equation of the ORTR with uniform input fault is obtained,

$$d\dot{X}(t) = M_1^{-1}B_1^*(\theta)u_1^*(t)dt + M_1^{-1}\left[(1-\rho_i)B_2^*(\theta) \vdots N\right]dB \tag{12}$$

The power spectral density of white noise $\left[\Delta u_1^{*^T}(t) \vdots \xi^T(t)\right]^T$ is supposed to be $\Sigma/2\pi$, i.e., the fact $dB = \Sigma dw$ holds. Then, stochastic model with random shift in the center of gravity and random fault input is obtained as

$$d\dot{X}(t) = M_1^{-1}B_1^*(\theta)u_1^*(t)dt + M_1^{-1}\left[(1-\rho_i)B_2^*(\theta) \vdots N\right]\Sigma dw \tag{13}$$

3. **Semi-Active Redundant Fault Tolerant Control.** In this section, we aim to design an SRFTC method that can track a designed trajectory when one wheel actuator is in fault. $X_d(t)$ is the desired trajectory, and $X(t)$ is the actual trajectory of ORTR; therefore, tracking errors are defined.

$$e_1 = X(t) - X_d(t) = [x(t) - x_d(t), y(t) - y_d(t), \theta(t) - \theta_d(t)]^T \tag{14}$$

$$e_2 = \dot{e}_1 + c_1 e_1 = \dot{X}(t) - \dot{X}_d(t) + c_1 e_1 \tag{15}$$

where $c_1 > 0$ is a design parameter.

Combining (14) and (15) with (13), we arrive the following tracking error system,

$$de_1 = (-c_1 e_1 + e_2)dt, \tag{16}$$

$$de_2 = \left[M_1^{-1} B_1^* u_1^*(t) - \ddot{X}_d(t) - c_1^2 e_1 + c_1 e_2 \right] dt + M_1^{-1} [(1-\rho_i) B_2^*(\theta) \vdots N] \Sigma dw \tag{17}$$

Suppose that $\hat{\Delta}$ is the estimate of $\Delta = 1/h_1$ with the estimate error $\tilde{\Delta} = \hat{\Delta} - \Delta$ and $\hat{h}$ is the estimate of h with the estimate error $\tilde{h} = \hat{h} - h$, where h will be designed later; $\hat{\rho}_i$ is the estimate of ρ_i with the estimate error $\tilde{\rho}_i = \hat{\rho}_i - \rho_i$.

Define the Lyapunov function as,

$$V = \sum_{i=1}^{2} \left(e_i^T e_i\right)^2/4 + h_1 \tilde{\Delta}^2/(2\gamma_1) + \tilde{h}^2/(2\gamma_2) + \tilde{\rho}_i^2/(2\gamma_3) \tag{18}$$

where $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 > 0$. The infinitesimal generator of $V(x,t)$ equals,

$$\begin{aligned}
& LV(x,t) \\
&= V_t(x,t) + V_x(x,t) f + Tr\left\{g^T V_{xx}(x,t) g\right\}/2 \\
&= -c_1 e_1^T e_1 e_1^T e_1 + e_1^T e_1 e_1^T e_2 + e_2^T e_2 e_2^T \left(M_1^{-1} B_1^{*^{-1}} u_1^*(t)\right) \\
&\quad - c_1^2 e_2^T e_2 e_2^T e_1 + c_1 e_2^T e_2 e_2^T e_2 - e_2^T e_2 e_2^T \ddot{X}_d \\
&\quad + Tr\left\{\Sigma^T \left[(1-\rho_i) B_2^*(\theta) \vdots N\right]^T M_1^{-1} \left(2 e_2 e_2^T + e_2^T e_2 I\right) M_1^{-1} \left[(1-\rho_i) B_2^*(\theta) \vdots N\right] \Sigma\right\}/2 \\
&\quad + h_1 \tilde{\Delta} \dot{\hat{\Delta}}/\gamma_1 + \tilde{h} \dot{\hat{h}}/\gamma_2 + \tilde{\rho}_i \dot{\hat{\rho}}_i \gamma_3
\end{aligned} \tag{19}$$

Using Young's inequality in the right-hand terms of (19), we have,

$$e_1^T e_1 e_1^T e_2 \le c_1 \left(e_1^T e_1\right)^2/4 + 27 \left(e_2^T e_2\right)^2 / \left(4 c_1^3\right) \tag{20}$$

$$-c_1^2 e_2^T e_2 e_2^T e_1 \le c_1 \left(e_1^T e_1\right)^2/4 + 3 c_1^{7/3} \left(e_2^T e_2\right)^2/4 \tag{21}$$

$$-e_2^T e_2 e_2^T \ddot{X}_r \le 3 \left(e_2^T e_2\right)^2 / \left(4 \varepsilon^{1/3}\right) + \varepsilon \ddot{X}_d^4/4 \tag{22}$$

where $\varepsilon > 0$ is a design parameter. Then, we design the well-functioning control $u_1^*(t)$ as,

$$u_1^*(t) = -B_1^{*-1} e_2 \hat{\Delta} \overline{u} \tag{23}$$

where $\overline{u} > 0$, one has $B_1^* u_1^*(t) = -e_2 \hat{\Delta} \overline{u}$. According to Equation (19), it leads to,

$$\begin{aligned}
& e_2^T e_2 e_2^T \left(M_1^{-1} B_1^* u_1^*(t)\right) \\
={}& -e_2^T e_2 e_2^T M_1^{-1} e_2 \hat{\Delta} \overline{u} \\
\le{}& -h_1 \left(e_2^T e_2\right)^2 \hat{\Delta} \overline{u} \\
={}& -\left(e_2^T e_2\right)^2 \overline{u} - h_1 \tilde{\Delta} \left(e_2^T e_2\right)^2 \overline{u}
\end{aligned} \tag{24}$$

Utilising the mean value inequality, from Assumption 2.3, one obtains,

$$\begin{aligned}
& [(1-\rho_i) B_2^*(\theta) \vdots N(\theta)]_F^2 \\
\le{}& 2 \left\| \left[(1-\rho_i) B_2^*(\theta) \vdots N(\theta)\right] - \left[(1-\rho_i) B_2^*(\theta_d(t)) \vdots N(\theta_d(t))\right] \right\|_F^2 \\
& + 2 \left[(1-\rho_i) B_2^*(\theta_d(t)) \vdots N(\theta_d(t))\right]_F^2 \\
={}& 2\left(1 + L^2\right)(1-\rho_i)^2 + 2 h_3 \psi_1 + 2 h_4 \psi_2 \left(e_2^T e_2\right)^2
\end{aligned} \tag{25}$$

which together with (10) leads to,

$$\begin{aligned}
& \left\| M_1^{-1} \left[(1-\rho_i) B_2^*(\theta) \vdots N(\theta)\right] \Sigma \right\|_F^2 \\
\le{}& h_2^2 \left[2(1+L^2)(1-\rho_i)^2 + 2 h_3 \psi_1 + 2 h_4 \psi_2 \left(e_2^T e_2\right)\right] \|\Sigma\|_F^2 \\
\le{}& h_2^4 \|\Sigma\|_F^4/2 + 2(1+L^2)^2 (1-\rho_i)^4 + 2 h_2^2 h_3 \|\Sigma\|_F^2 \psi_1 + 2 h_2^2 h_4 \|\Sigma\|_F^2 \psi_2 \left(e_2^T e_2\right)
\end{aligned} \tag{26}$$

Further, we have $(1-\rho_i)^4 \le (1-\rho_i) \le 1$ for $0 \le \rho_i \le 1$. According to Young's inequality, the definition of the Frobenius norm, (26) and (19), the following inequality is obtained,

$$\begin{aligned}
& Tr\Big\{\Sigma^T[(1-\rho_i)B_2^*(\theta)\vdots N(\theta)]^T M_1^{-1}\left(2e_2e_2^T + e_2^Te_2 I\right) M_1^{-1}[(1-\rho_i)B_2^*(\theta)\vdots N(\theta)]\Sigma\Big\}\Big/2 \\
\le\; & 3h_2^4\left\|\Sigma\right\|_F^4\left(e_2^Te_2\right)/4 + 3\left(1+L^2\right)^2(1-\rho_i)^4\left(e_2^Te_2\right) \\
& + 3h_2^2h_3\left\|\Sigma\right\|_F^2\psi_1\left(e_2^Te_2\right) + 3h_2^2h_4\left\|\Sigma\right\|_F^2\psi_2\left(e_2^Te_2\right)^2 \\
\le\; & (e_2^Te_2)^2\psi h + 33/8 + 9\left(1+L^2\right)^4(1-\rho_i)\left(e_2^Te_2\right)^2/8
\end{aligned} \tag{27}$$

where $h = \max\left\{3h_2^2h_4\left\|\Sigma\right\|_F^2, 9h_2^8\left\|\Sigma\right\|_F^8/8, 9h_2^4h_3^2\left\|\Sigma\right\|_F^4/8\right\}$ and $\psi = \psi_2 + \psi_1^2 + 1 > 0$. Substituting (20)-(27) into (19), one leads to,

$$\begin{aligned}
& LV(x,t) \\
\le\; & -\frac{c_1}{2}\left(e_1^Te_1\right)^2 + \left(e_2^Te_2\right)^2\left(\frac{27}{4c_1^3} - \overline{u} + \frac{3c_1^{7/3}}{4} + c_1 + \frac{3}{4\varepsilon^{1/3}} + \frac{9}{8}\left(1+L^2\right)^4(1-\hat{\rho}_i) + \psi\hat{h}\right) \\
& + \frac{1}{\gamma_1}h_1\tilde{\Delta}\left(-\gamma_1\left(e_2^Te_2\right)^2\overline{u} + \dot{\hat{\Delta}}\right) + \tilde{h}\left(-\gamma_2\psi\left(e_2^Te_2\right)^2 + \dot{\hat{h}}\right)\Big/\gamma_2 \\
& + \tilde{\rho}_i\left(\dot{\hat{\rho}}_i + \frac{9}{8}\gamma_3\left(1+L^2\right)^4\left(e_2^Te_2\right)^2\right)\Big/\gamma_3 + \varepsilon\ddot{X}_d^4(t)/4 + 33/8
\end{aligned} \tag{28}$$

Defining the function $\overline{u} = c_2/4 + 27/(4c_1^3) + 3c_1^{7/3}/4 + c_1 + 3/\left(4\varepsilon^{1/3}\right) + 9\left(1+L^2\right)^4(1-\hat{\rho}_i)/8$ $+\psi\hat{h}$ with $c_2 > 0$, $c = \min(2c_1, c_2)$, $d = \varepsilon\ddot{X}_d^4(t)/4 + 33/8$. The adaptive laws are designed as,

$$\dot{\hat{\Delta}} = \gamma_1\bar{u}\left(e_2^Te_2\right)^2 \tag{29}$$

$$\dot{\hat{h}} = \gamma_1\psi\left(e_2^Te_2\right)^2 \tag{30}$$

$$\dot{\hat{\rho}}_i = -9\gamma_3\left(1+L^2\right)^4\left(e_2^Te_2\right)^2/8 \tag{31}$$

Substituting (18) and (29)-(31) into (28), one leads to,

$$LV(x,t) \le -cV(x,t) + d \tag{32}$$

4. **Stability Analysis.**

Theorem 4.1. *For the stochastic model of ORTR with center of gravity shift and random fault input, the SRFTC and adaptive laws are designed such that, the closed-loop trajectory tracking error system is exponentially practically mean absolute stable for initial values* $e_1(t_0) \in R^n$, $e_2(t_0) \in R^n$. *The tracking errors* $e_1(t)$ *and* $\dot{e}_1(t)$ *satisfy,*

$$\lim_{t\to\infty} E\left|e_1(t)\right| \le (4d/c)^{1/4} \tag{33}$$

$$\lim_{t\to\infty} E\left|\dot{e}_1(t)\right| \le 2\sqrt{(1+c_1^2)}\,(4d/c)^{1/4} \tag{34}$$

In addition, if the design parameters are chosen appropriately, the right side of (33) and (34) could be small enough.

Proof: As the matrix M_1^{-1}, $B_1^*(\theta)$, and vector $u_1^*(t)$ are symmetric and positive definite, which is satisfied with local Lipschitz condition the trajectory tracking error system (16) and (17) also satisfies the local Lipschitz condition. Based on (18) and (32), and Lemma 1 in [3], the trajectory tracking error system has a unique strong solution on $[t_0, \infty)$ for initial values $e_1(t_0) \in R^3$, $e_2(t_0) \in R^3$; ulteriorly, the exponentially practical stability in mean square of the trajectory tracking error system is obtained.

What is more, multiplying Equation (31) with $e^{ct} > 0$, we have,

$$L\left(e^{ct}V(x,t)\right) = e^{ct}\left(LV(x,t) + cV(x,t)\right) \le e^{ct}d \tag{35}$$

Hence, integrating (34) from t_0 to t, it results in

$$E\left(e^{ct}V(x,t)\right) \le e^{ct_0}V(x_0,t_0) + E\int_{t_0}^{t} e^{cs}d\cdot ds \qquad \forall t \ge t_0 \tag{36}$$

From (32), one can deduce that,

$$\begin{aligned} E\left|e_1\right| \le\ & 2e^{c(t_0-t)/2}\left(\sum_{i=1}^{2}\left(e_i^Te_i\right)^2/4 + h_1\tilde{\Delta}^2/(2\gamma_1) + \tilde{h}^2/(2\gamma_2) + \tilde{\rho}_i^2/(2\gamma_3)\right)^{1/4} \\ & + (4d/c)^{1/4} \end{aligned} \tag{37}$$

$$\begin{aligned} E\left|e_2\right| \le\ & 2e^{c(t_0-t)/2}\left(\sum_{i=1}^{2}\left(e_i^Te_i\right)^2/4 + h_1\tilde{\Delta}^2/(2\gamma_1) + \tilde{h}^2/(2\gamma_2) + \tilde{\rho}_i^2/(2\gamma_3)\right)^{1/4} \\ & + (4d/c)^{1/4} \end{aligned} \tag{38}$$

Utilizing (37) and (38), in view of $|\dot{e}_1|^2 = (|e_2| + c_1|e_1|)^2 \le \left(|e_2|^2 + |e_1|^2\right)$, it follows that (33) and (34) hold.

5. **Simulation Results.** In this section, the proposed SRFTC algorithm is verified by the ORTR with center of gravity shift and fault input. For any t, that only one actuator fails is supposed. Without loss of generality, we consider the following possible cases: 1) four actuators are normal; 2) the fourth actuator cannot work at all and other actuators are normal; 3) the fourth actuator has lost effectiveness and other actuators are normal. The reference trajectory is a linear path $X_d(t)$, which is described by $x_d(t) = 20(1-e^{-0.2t})$, $y_d(t) = 20(1-e^{-0.2t})$, $\theta_d = \pi/4$. The physical parameters of the ORTR are $M = 58$ kg, $L = 0.4$ m, and $I_0 = 27.7$ kg.m^2. The random parameters are assumed to $r_0 = 0.1(1+\sin t)$ m, $\lambda_1 = L - r_0\sin t$ m, $\lambda_2 = L + r_0\cos t$ m, $\lambda_3 = L + r_0\sin t$ m, and $\lambda_4 = L - r_0\cos t$ m, $m = 80(1+\sin t)$ kg.

1) As four actuators are normal, the initial values are set as $\hat{\Delta}(0) = 0.001$, $\hat{h}(0) = 0.001$, and $\hat{\rho}_4(0) = 0.05$ and the design parameters are $c_1 = 2.3$, $c_2 = 0.5$, $\varepsilon = 1$, $\gamma_1 = 3$, $\gamma_2 = 3$, and $\gamma_3 = 0.5$. The simulation results are shown in the following figures.

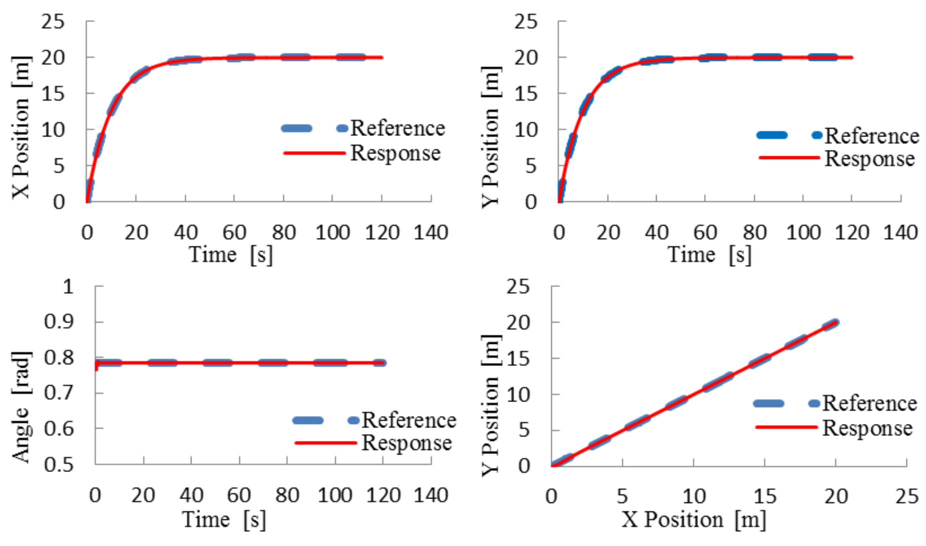

Figure 3. Trajectory tracking

Figure 3 shows the trajectory tracking on X-position, Y-position, the orientation angle, and path tracking of line, respectively. The ORTR can finish the trajectory tracking when all of the actuators are normal.

2) For the simulation of the fourth actuator outage fault, the initial values are $\hat{\Delta}(0) = 0.001$, $\hat{h}(0) = 0.001$, and $\hat{\rho}_i(0) = 0.05$, and design parameters are $c_1 = 2.3$, $c_2 = 0.5$, $\varepsilon = 1$, $\gamma_1 = 3$, $\gamma_2 = 3$, and $\gamma_3 = 0.5$. The simulation results are presented in the following figures.

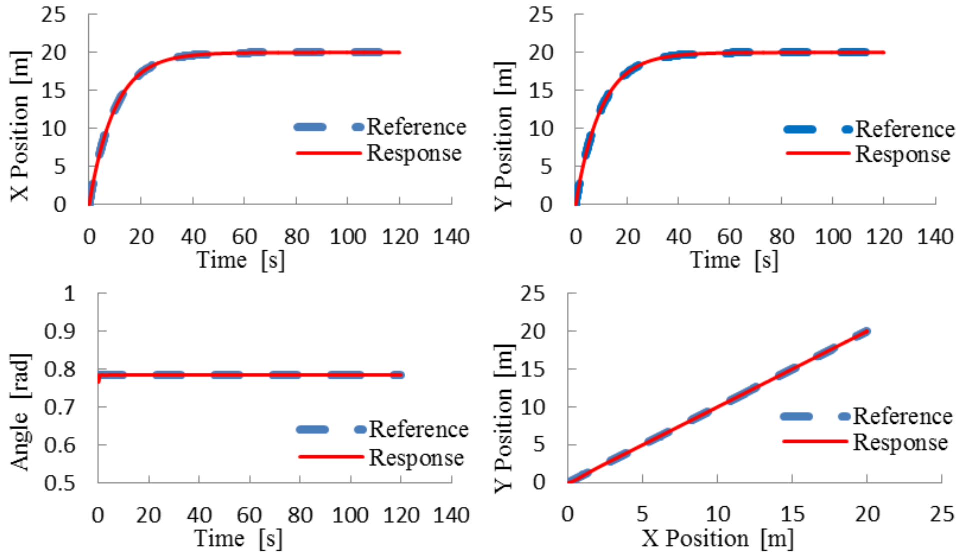

FIGURE 4. Trajectory tracking

Figure 4 shows the tracking performance of the ORTR. These simulation results show that the SRFTC is effective when one actuator cannot generate any force at all. Thus, three non-fault functioning actuators can support the ORTR to achieve trajectory tracking.

To verify the effectiveness of the proposed method, we conduct comparative simulations with [7], in which the problem of guaranteed cost non-fragile tracking control on the ORTR without the center of gravity shift is investigated. However, now, the center of gravity shifts is $r_0 = 0.1(1+\sin t)$ m, $\beta = 0.275\pi(1+\sin t)$. Simulation results are given as follows.

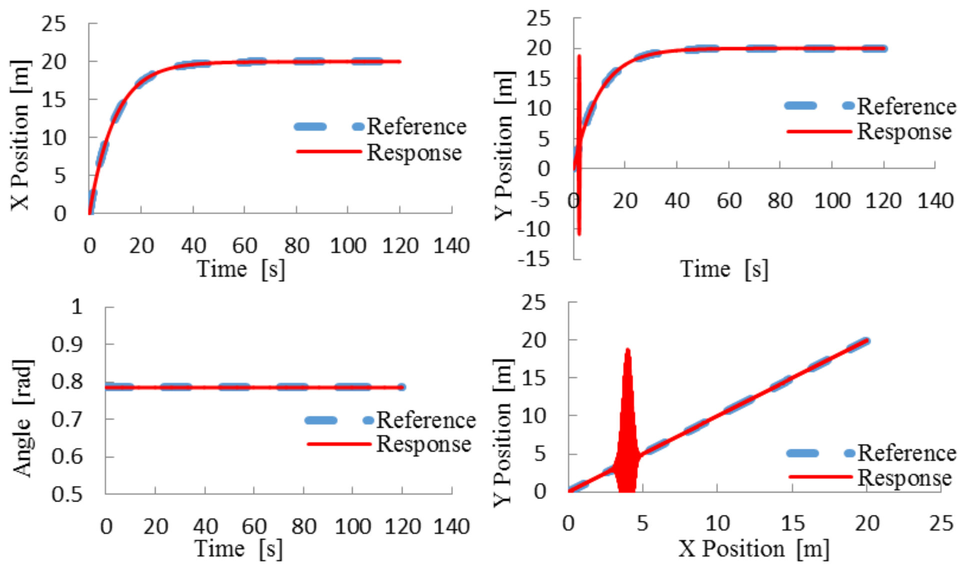

FIGURE 5. Trajectory tracking

Figure 5 plots the trajectory tracking of ORTR. It is evident the ORTR goes into vibration which shows the necessity of consideration on the actuator fault and center of gravity shift.

3) The fourth actuator is loss of effectiveness. We choose the initial values $\hat{\Delta}(0) = 0.001$, $\hat{h}(0) = 0.001$, and $\hat{\rho}_4(0) = 0.05$, and the design parameters are $c_1 = 2.3$, $c_2 = 3$, $\varepsilon = 1$, $\gamma_1 = 3$, $\gamma_2 = 2$, and $\gamma_3 = 0.5$. The simulation results are presented in the following figures.

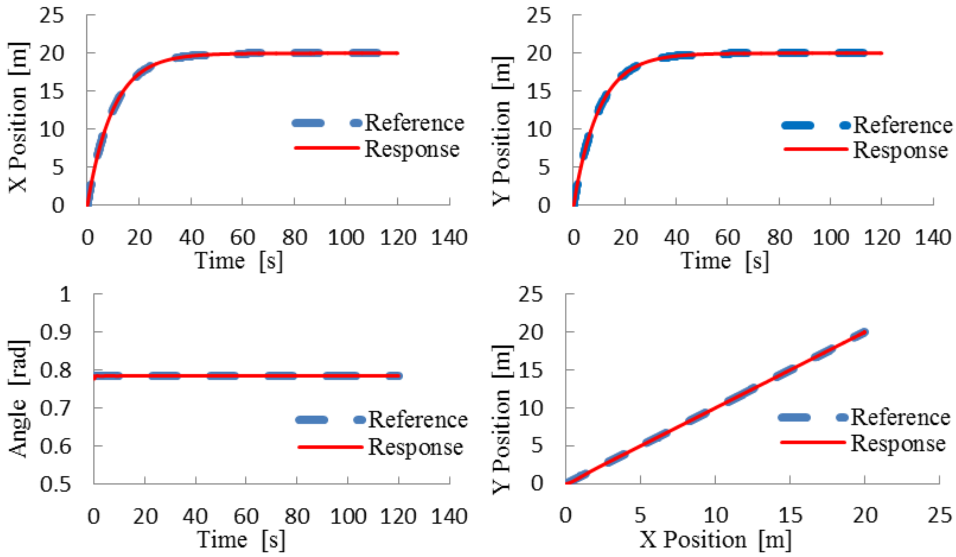

FIGURE 6. Trajectory tracking

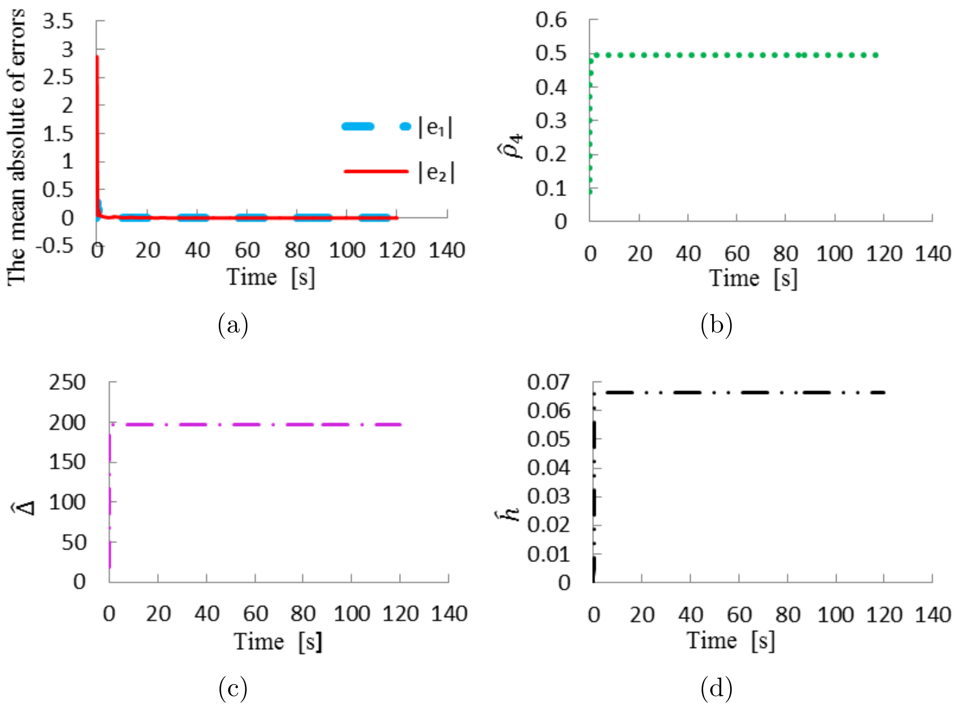

FIGURE 7. Adaptive laws and the mean absolute of errors

As shown in Figure 6, the ORTR can track the trajectory $X_d(t)$. The SRFTC method can guarantee the walker's continuous motion when the fourth actuator lost 50% of force. The ORTR relies on the three well-functioning actuators to maintain the training process. Figure 7(a) shows that tracking error system can realize asymptotic stability and the mean absolute of the tracking errors can be made small enough. Figures 7(b), 7(c) and 7(d) imply $0 \leq \rho_i \leq 1$, $\hat{\Delta}(t) > 0$ and $\hat{h}(t) > 0$, which satisfy the conditions of adaptive laws. The simulation results show that the redundant input stochastic model of ORTR and SRFTC can deal with center of gravity shift and fault input of ORTR.

6. **Conclusions.** A stochastic ORTR model with center of gravity shift and fault input is considered in this paper. Based on the Lyapunov stability theory, a new fault tolerant controller (SRFTC) is designed such that the mean absolute of the tracking error can be made small enough by choosing appropriate design parameters in controller. Meanwhile, the controller can render the closed-loop system exponential mean absolute stable. In the simulation section, the effectiveness of SRFTC has been fully specified and the ORTR with center of gravity shift can provide safe sequential motion which is verified when one wheel actuator is in fault. Therefore, in addition to the ORTR, our method could also be applied to other precision mechanical systems.

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