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A VERIFICATION FOR INTRINSIC ROBUSTNESS PROPERTY OF FINITE MEMORY STRUCTURE FILTER

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Abstract. *In this paper, in order to verify intrinsic robustness property, the finite memory structure (FMS) filter is applied for the state-space model with a couple of temporary uncertainties, model uncertainty and unknown input. Extensive computer simulations are performed for both nominal system and temporarily uncertain system. It is shown that there can exist the trade-off between the estimation error and the tracking ability in terms of the window length.*

Keywords: Infinite memory structure filter, Finite memory structure filter, Temporary uncertain system, Nominal system

1. **Introduction.** There have been many kinds of filters for diverse engineering problems [1,2]. Among them, the state estimation filter is to extract dynamic system's state values from full or partial measurements with noises. Real-time control systems rely on reliable state estimates in order to provide accurate and safe control of various dynamic systems.

Although the dynamic system is represented in state-space model accurately on a long time scale, it may undergo unpredictable changes, such as jumps in frequency, phase, and velocity. Because these effects typically occur over a short time horizon, they are called temporary uncertainties [3]. In estimation filtering for dynamic systems, the estimation filter should be robust to diminish the effects of temporary uncertainties. In contrast to the infinite memory structure (IMS) filter such as well-known Kalman filter [4,5], the finite memory structure (FMS) filter using only finite measurements on the most recent window has been known inherently to be bounded input/bounded output stable and more robust against temporary uncertainties [6-8]. This robustness property has been known as one of good intrinsic properties of the FMS filter. If this good intrinsic property is verified by a real application, this might be very informative for engineers and researchers in control and estimation communities, which is a main motivation of this paper.

Therefore, the FMS filter is applied for the state-space model with consideration of a couple of temporary uncertainties in order to verify intrinsic robustness property, which is a main contribution of this paper. Firstly, a model uncertainty can be considered. The state-space approach is commonly used when real physical systems and processes can be approximated with a reasonable number of states. The approximation implies model uncertainty that may cause an estimator to be biased and/or diverge. In other words, due to concerns for model misspecification, there can be a model uncertainty. Secondly, an unknown input can be considered. The unknown input has been used in many areas such as fault detection and diagnosis for various systems and maneuver detection and target tracking of flying objects.

Extensive computer simulations are performed for both nominal system and temporarily uncertain system. It is shown that the FMS filter can be more robust than the IMS filter

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when applied to the state-space model with a model uncertainty, although the FMS filter is designed with no consideration of robustness. When there is an unknown input in the state-space model, the FMS filter can be better than the IMS filter for estimation error. Moreover, it is shown that the noise suppression of the FMS filter might be closely related to the window length of past measurements. It is also shown that there can exist the trade-off between the estimation error and the tracking ability in terms of the window length.

2. **Finite Memory Structure Filtering for Temporary Uncertainties.**

The state-space model for the dynamic system can be represented by

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i + Gw_i, \\ z_i &= Cx_i + v_i, \end{aligned} \tag{1}$$

where x_i is the state variable, z_i is the measurement variable, and u_i is the control input variable. In addition, w_i is the system noise and v_i is the measurement noise, and their covariances are Q and R, respectively. A, B, G, and C are matrices for state-space model. Because noises w_i and v_i cause the system error, the desire system output should be corrected. Therefore, the state estimation filtering has been applied to estimating the state variable without noise interference. The IMS filtering, such as well-known Kalman filtering, has been successfully applied [3,4]. In contrast to the IMS filter, the FMS filter using only finite measurements on the most recent window has been known inherently to be bounded input/bounded output stable and more robust against temporary uncertainties, which means that the FMS filter has an intrinsic robustness property. Thus, in this paper, the FMS filter is applied for the state-space model with consideration of a couple of temporary uncertainties to verify this good intrinsic property.

2.1. **Model uncertainty.**

The state-space approach is commonly used when real physical systems and processes can be approximated with a reasonable number of states. The approximation implies model uncertainty. That is, due to concerns for model misspecification, there can be model uncertainty, which may cause an estimator to be biased and/or diverge. The state-space model with a model uncertainty can be represented by

$$\begin{aligned} x_{i+1} &= \left(A + \Delta A_i\right) x_i + Bu_i + Gw_i, \\ z_i &= \left(C + \Delta C_i\right) x_i + v_i. \end{aligned} \tag{2}$$

Although the estimation filtering is computed by the nominal discrete-time state-space model (1), actual measurements for the estimation filtering are obtained from the system with a model uncertainty (2).

Thus, as an alternative to the IMS filter, the FMS filter has been developed by the iterative form as well as the matrix form using only the most recent finite measurements on the window $[i - M, i]$. The window initial time $i - M$ will be denoted by i_M hereafter for simplicity. The FMS filter with the iterative form was developed from the well-known Kalman filter with the moving window strategy. The FMS filter with the iterative form provides an optimal state estimate $\hat{x}_i$ for the system state x_i as follows [5,6]:

$$\hat{x}_i = \Omega_M^{-1} \hat{\eta}_i, \tag{3}$$

where

$$\begin{aligned} \hat{\eta}_{i_M+j+1} = & \left[I + A^{-T}\left(\Omega_j + C^T R^{-1} C\right) A^{-1} G Q G^T\right]^{-1} A^{-T} \Big[\hat{\eta}_{i_M+j} + C^T R^{-1} z_{i_M+j} \\ & + \left(\Omega_j + C^T R^{-1} C\right) A^{-1} B u_{i_M+j}\Big], \ \hat{\eta}_{i_M} = 0, \\ \Omega_{j+1} = & \left[I + A^{-T}\left(\Omega_j + C^T R^{-1} C\right) A^{-1} G Q G^T\right]^{-1} A^{-T}\left(\Omega_j + C^T R^{-1} C\right) A^{-1}, \\ & \Omega_0 = 0, \ 0 \le j \le M. \end{aligned}$$

2.2. **Unknown input.** The state-space model with an unknown input can be represented by

$$x_{i+1} = Ax_i + Bu_i + Ep_i + Gw_i, \quad z_i = Cx_i + v_i. \tag{4}$$

The unknown input vector $p_i \in \Re^q$ in the system under consideration is to be represented by random-walk processes as $p_{i+1} = p_i + \delta_i$, where the unknown input $p_i \equiv \left[\begin{array}{cccc} p_i^1 & p_i^2 & \cdots & p_i^s \end{array}\right]^T$ and the unknown input noise $\delta_i \in \Re^s$ is a zero-mean white Gaussian random process with covariance Q_δ. It is noted that the random-walk process provides a general and useful tool for the analysis of unknown time-varying parameters and has been widely used in the detection and identification area.

The unknown input can be treated as auxiliary states and then the state-space model (4) can be rewritten as an augmented state-space model as

$$\begin{aligned} \left[\begin{array}{c} x_{i+1} \\ p_{i+1} \end{array}\right] &= A_a \left[\begin{array}{c} x_i \\ p_i \end{array}\right] + B_a u_i + G_a \left[\begin{array}{c} w_i \\ \delta_i \end{array}\right], \\ z_i &= Cx_i + v_i, \end{aligned} \tag{5}$$

where

$$A_a = \left[\begin{array}{cc} A & E \\ 0 & I \end{array}\right], \; B_a = \left[\begin{array}{c} B \\ 0 \end{array}\right], \; G_a = \left[\begin{array}{cc} G & 0 \\ 0 & I \end{array}\right],$$

and the system noise and the unknown input noise term $\left[\begin{array}{cc} w_i^T & \delta_i^T \end{array}\right]^T$ is a zero-mean white Gaussian random process with covariance $Q_a = diag\left(\left[\begin{array}{cc} Q & Q_\delta \end{array}\right]\right)$.

Although the estimation filtering is computed by the augmented discrete-time state-space model (5), actual measurements for the estimation filtering are obtained from the actual system with an unknown input (4). The FMS filter with the iterative form provides an optimal state estimate $\left[\begin{array}{cc} \hat{x}_i^T & \hat{p}_i^T \end{array}\right]^T$ for the augmented system state $\left[\begin{array}{cc} x_i^T & p_i^T \end{array}\right]^T$ as follows

$$\left[\begin{array}{c} \hat{x}_i \\ \hat{p}_i \end{array}\right] = \Sigma_M^{-1} \hat{\theta}_i,$$

where

$$\begin{aligned} \hat{\theta}_{i_M+j+1} &= \left[I + A_a^{-T}\left(\Sigma_j + C^T R^{-1} C\right) A_a^{-1} G_a Q_a G_a^T\right]^{-1} A_a^{-T} \left[\hat{\theta}_{i_M+j} + C^T R^{-1} z_{i_M+j} \right. \\ &\quad \left. + \left(\Sigma_j + C^T R^{-1} C\right) A_a^{-1} B_a u_{i_M+j}\right], \; \hat{\theta}_{i_M} = 0, \\ \Sigma_{i+1} &= \left[I + A_a^{-T}\left(\Sigma_j + C^T R^{-1} C\right) A_a^{-1} G_a Q_a G_a^T\right]^{-1} A_a^{-T}\left(\Sigma_j + C^T R^{-1} C\right) A_a^{-1}, \\ &\quad \Sigma_0 = 0, \; 0 \le j \le M. \end{aligned}$$

3. **Extensive Computer Simulations.** Using the direct current (DC) motor system [9-12], extensive computer simulations are performed for two kinds of temporary uncertainties. The following discrete-time state-space model for DC motor system is considered

$$\begin{aligned} A &= \left[\begin{array}{cc} 0.8178 & -0.0011 \\ 0.0563 & 0.3678 \end{array}\right], \; B = \left[\begin{array}{c} 0.1813 \\ 0.0069 \end{array}\right], \\ G &= \left[\begin{array}{cc} 0.0006 & 0 \\ 0 & 0.0057 \end{array}\right], \; C = \left[\begin{array}{cc} 1 & 0 \end{array}\right], \end{aligned} \tag{6}$$

where the motor is operated without any payload torque and the armature current is chosen as the output.

3.1. **Simulation for model uncertainty.** A model uncertainty as temporary uncertainty for DC motor system is set by

$$\Delta A_i = \begin{bmatrix} \delta_i & 0 \\ 0 & \delta_i \end{bmatrix}, \ \Delta C_i = \begin{bmatrix} 0.2 * \delta_i & 0.2 * \delta_i \end{bmatrix}, \ \delta_i = 0.1, \ 150 \leq i \leq 300. \tag{7}$$

System and measurement noise covariances are taken by $Q = diag\left(\begin{bmatrix} 0.1^2 & 0.1^2 \end{bmatrix}\right)$ and $R = 0.05^2$, respectively. Three kinds of estimation filters, the FMS filter with $M = 10$, and the FMS filter with $M = 20$, the IMS filter are compared. To make a clearer comparison of estimation performances, simulations of 20 runs are performed using different system and measurement noises, and each single simulation run lasts 500 samples.

Although the IMS filter and two FMS filters are computed by the nominal discrete-time state-space model (6) for DC motor system, actual measurements for these three filters are obtained from the system with the model uncertainty (7). The first figure of Figure 1 shows RMS estimation errors of the rotational speed for 20 simulations. The second figure of Figure 1 shows estimation error of the rotational speed for one of 20 simulations. As shown in Figure 1 both FMS filters can be better than the IMS filter in terms of error magnitude and error convergence. The estimation error of FMS filters is smaller than that of the IMS filter on the interval where the model uncertainty exists. In addition, the convergence of estimation error is faster than that of the IMS filter after the model uncertainty disappears. Therefore, FMS filters can be more robust than the IMS filter when applied to DC motor system with the model uncertainty, although FMS filters are designed with no consideration of robustness. In addition, the FMS filter can be comparable to the IMS filter after the effect of the model uncertainty completely disappears.

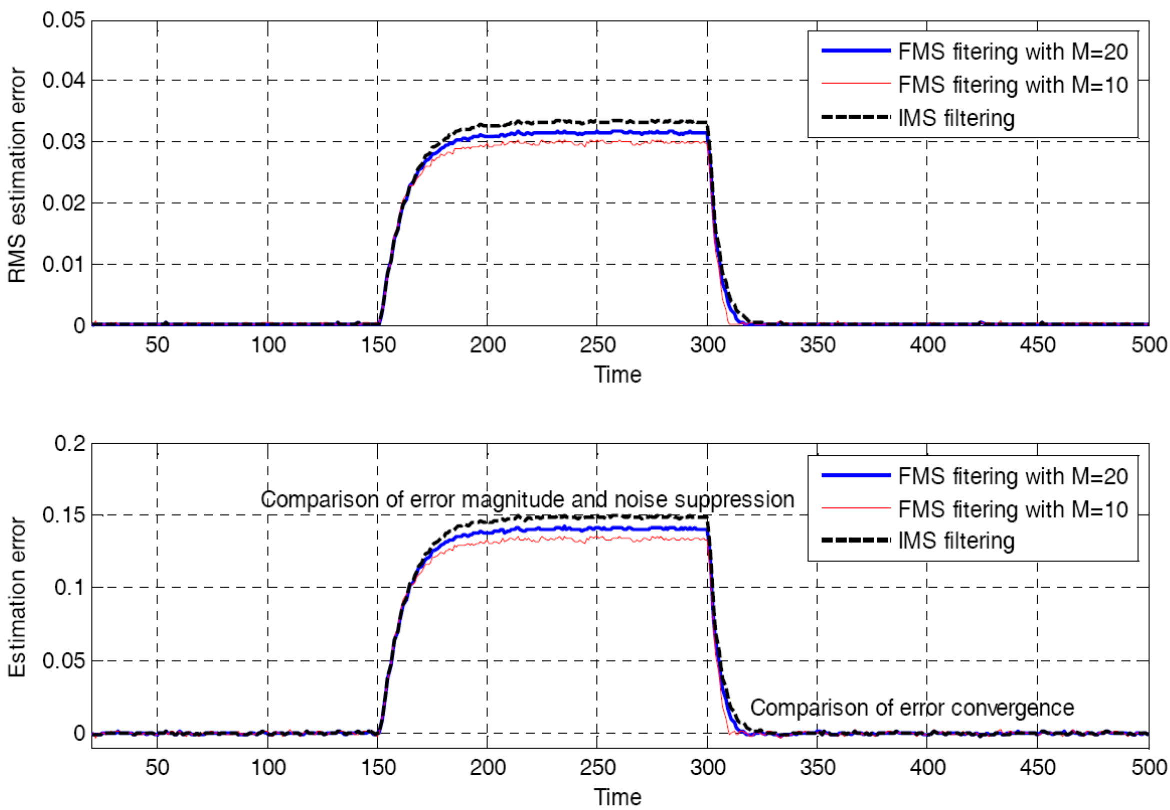

Figure 1. Simulation results for model uncertainty

3.2. **Simulation for unknown input.** The unknown input is emulated by the step-type load torque as follows:

$$p_i = 0.5, \quad 100 \le i \le 300, \tag{8}$$

and its matrix $E = \begin{bmatrix} -0.0069 & 6.3210 \end{bmatrix}^T$. System and measurement noise covariances are taken by $Q = diag\left(\begin{bmatrix} 0.01^2 & 0.01^2 \end{bmatrix}\right)$ and $R = 0.05^2$, respectively. The unknown input noise covariance is taken by $Q_\delta = 0.05^2$. Three kinds of filters, the FMS filter with $M = 10$, and the FMS filter with $M = 20$, the IMS filter are compared. To make a clearer comparison of estimation performances, simulations of 20 runs are performed and each single simulation run lasts 500 samples.

Although the IMS filter and two FMS filters are computed by the augmented discrete-time state-space model (5) for DC motor system, actual measurements for these estimation filters are obtained from the actual system with an unknown input (4). Figure 2 shows RMS estimation errors of the rotational speed for 20 simulations and estimation error of the rotational speed for one of 20 simulations. When there is an unknown input, FMS filters can be better than the IMS filter in terms of estimation error of the rotational speed. Especially, the FMS filter with $M = 20$ can be optimal in terms of noise suppression, estimation error and tracking speed. In addition, the FMS filter can be comparable to the IMS filter after the effect of the unknown input completely disappears.

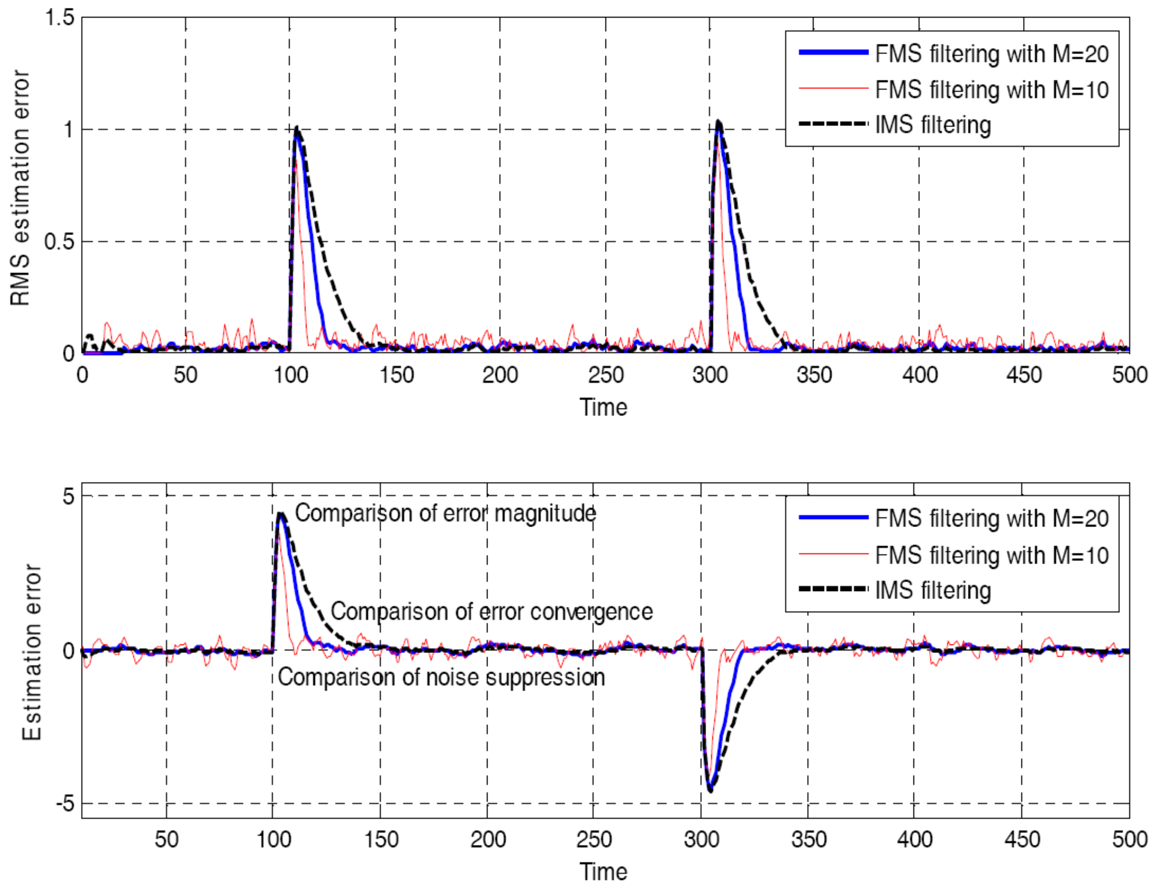

Figure 2. Simulation results for unknown input

3.3. **Effects of window length on simulation results.** As shown in simulations results with temporary uncertainties (7) and (8), the noise suppression of the FMS filtering might be closely related to the window length M. The FMS filtering can have greater noise suppression for both state estimate and unknown input estimate as the window length M increases. However, long M may yield the long convergence time of estimation error. As shown in Figure 3 which compares estimation errors of the rotational speed

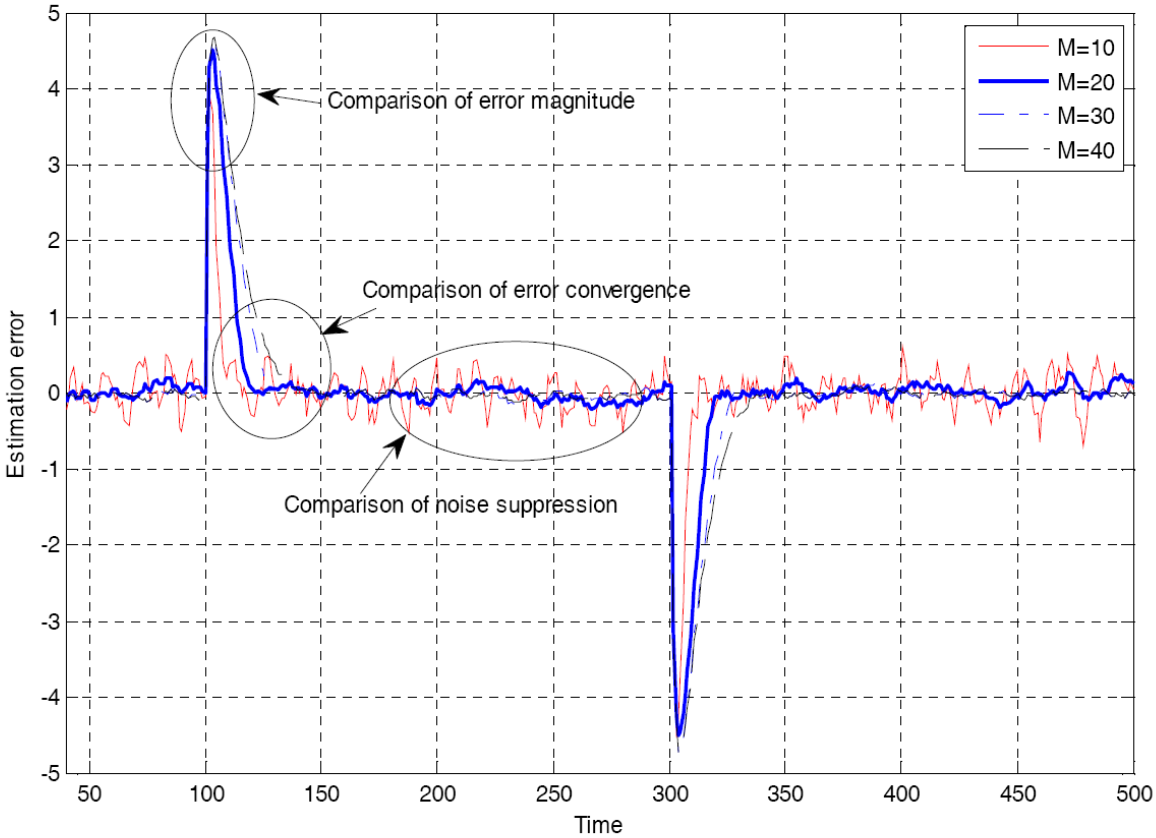

FIGURE 3. Simulation results for unknown input according to diverse window lengths

according to diverse window length, the window length M can make the tradeoff between the noise suppression and the tracking speed of the state estimation.

4. **Conclusions.** This paper has dealt with the FMS filter for the state-space model with two kinds of temporary uncertainties such as model uncertainty and unknown input to verify intrinsic robustness property of the FMS filter. To compare the FMS filter and the IMS filter for both nominal system and temporarily uncertain system, extensive computer simulations have been performed. It has been shown that the FMS filter can outperform the IMS filter for temporary uncertainties. On the other hand, the FMS filter can be comparable to the IMS filter after the effect of temporary uncertainties completely disappears. It has been shown that the noise suppression of the FMS filter might be closely related to the window length of past measurements. Meanwhile, in terms of the window length, it has been shown that there can exist the trade-off between the estimation error and the tracking ability.

The choice of the window length could still be somewhat nonsystematic although a guideline has been given through computer simulations. Hence, a more systematic approach of determining the window length should be researched as a future work.

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