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INTERVAL-VALUED FINANCIAL TIME SERIES PREDICTION BASED ON IMPROVED ELMAN NEURAL NETWORK

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Abstract. *In this paper, an improved Elman neural network is proposed for predicting interval-valued time series. Firstly, adaptive gradient descent method is used for training the parameters of interval-valued Elman neural network. And the initial parameters are optimized by the genetic algorithm. Then, the output value of the Elman model is corrected by an adaptive expectation compensation method to improve the forecasting accuracy. Finally, some numerical examples are provided to demonstrate the effectiveness of the proposed algorithm.*
Keywords: Elman neural network, Interval-valued time series, Adaptive expectation compensation model

1. **Introduction.** Interval-valued time series, which consists of a series of temporal interval-valued data, is widely applied in many fields such as meteorological forecasting, stock trading, and medical diagnosis. Especially in financial field, interval-valued financial times series, which contains the lowest and highest price of daily stock trading, is affected by many complex random factors and highly fluctuant. The challenges of predicting interval-valued financial time series consist in the processing way of interval-data and the modeling strategy of forecasting accuracy improvement.

In recent years, various artificial neural networks have been utilized in predicting time series. Among these neural networks, Elman neural network [1] which can capture the nonlinear characteristics of time series has gained much more attention by scholars. In [2], Elman neural network combined with stochastic time effective function is considered to predict the fluctuation of crude oil price. Elman-SVM model is proposed for predicting continuous time series data in [3]. In [4], a hybrid quantized Elman neural network is developed to investigate the short-term load forecasting problem, and the genetic algorithm (GA) is introduced for improving the accuracy of prediction. In [5], a decomposition method is introduced for improving the predictive ability of Elman neural network. In [6], a new hybrid model combined with improved wavelet transform and Elman neural network is designed to raise the wind speed prediction accuracy. In [7], Elman neural network optimized by the ant colony algorithm is used to forecast photovoltaic power, and the Markov method is used to revise the preliminary prediction value. In [8], a

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new learning rate scheme is proposed to improve convergence speed and generalization performance of Elman back-propagation algorithm.

From existing results, we find that above Elman networks are suitable for single-valued time series. However, how to design a novel Elman neural network for predicting interval-valued time series needs to be investigated. It is worth mentioning that for some financial time series, single-valued neural network could not comprehensively capture the dynamic feature of data. Besides, some compensation technologies can contribute to improving the forecasting accuracy, such as the adaptive expectation method (AEM) [9,10] which has been used as correction strategy for strengthening predicting capabilities especially of stock market data. Motivated by these facts, in this paper, based on Elman neural network and adaptive expectation compensation method, a combination model called AEM-Elman neural network, is proposed for interval-valued time series. The proposed AEM-Elman model takes advantage of Elman network and the adaptive expectation method together. And the adaptive expectation compensation process is refined further in this paper, which can favorably affect the results compared with some compensation models in the literature. For this model, genetic algorithm and adaptive gradient descent method are adopted to optimize parameters.

The paper is structured as follows. Section 2 introduces some basic concepts of interval-valued Elman neural network. In Section 3, AEM-Elman neural network model is proposed for predicting interval-valued time series. Section 4 demonstrates the effectiveness of the proposed model for financial data. Some conclusions are summarized in Section 5.

2. **Preliminaries of Interval-Valued Elman Neural Network.** In this section, we will introduce some basic concepts and mathematical representations of interval-valued Elman neural network.

Firstly, we will review some concepts of Elman neural network.

Elman neural network is a typical local recurrent network. A context layer is added in this neural network as a one-step delay operator. This kind of dynamic neural network can reflect the dynamic changes of the objective system and can be applied to predicting time series data.

The basic structure of Elman neural network is shown in Figure 1. It has four layers, including the input layer, the hidden layer, the context layer and the output layer. The context layer, which has the same number of neurons as the hidden layer, is used to save the output value of the hidden layer.

Assume that the numbers of input neurons and hidden neurons are m and n. $x(t) = [x_1(t), x_2(t), \ldots, x_m(t)]^T$ is the input vector. The outputs of hidden layer and context layer are represented by

$$h(t) = [h_1(t), h_2(t), \ldots, h_n(t)]^T, \quad c(t) = [c_1(t), c_2(t), \ldots, c_n(t)]^T = h(t-1).$$

The final output $x_{m+1}(t)$ for the network can be computed by the following equation:

$$x_{m+1}(t) = \sum_{j=1}^{n} w_{1j}^3 h_j(t) + b^o,$$

where

$$h_j(t) = f\left(\sum_{i=1}^{m} w_{ji}^1 x_i(t) + \sum_{p=1}^{n} w_{jp}^2 c_p(t) + b_j^h\right), \quad j = 1, 2, \ldots, n,$$

$$f(u) = \frac{1}{(1+\exp(-u))},$$

$b_j^h \in R^{1\times n}$ and $b^o \in R$ are the biases in hidden layer and output layer respectively. $w_{ji}^1 \in R^{n\times m}$, $w_{jp}^2 \in R^{n\times n}$ and $w_{1j}^3 \in R^{1\times n}$ are weight matrices of input layer, context layer and output layer respectively.

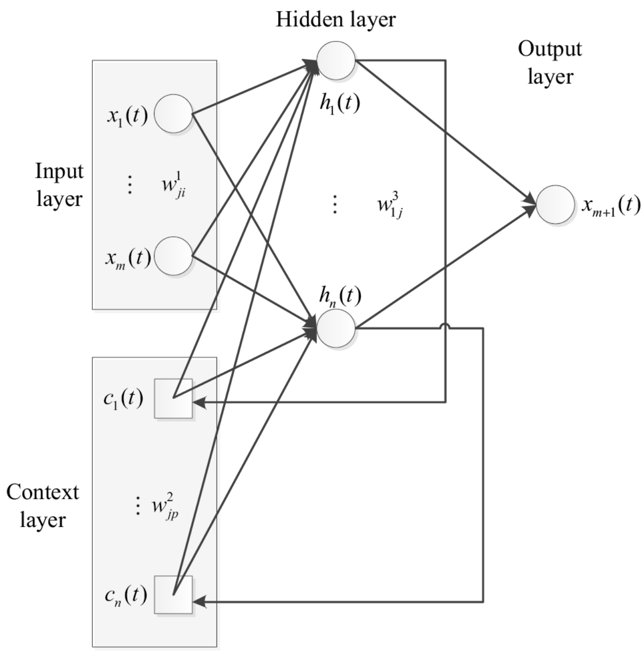

Figure 1. Elman neural network

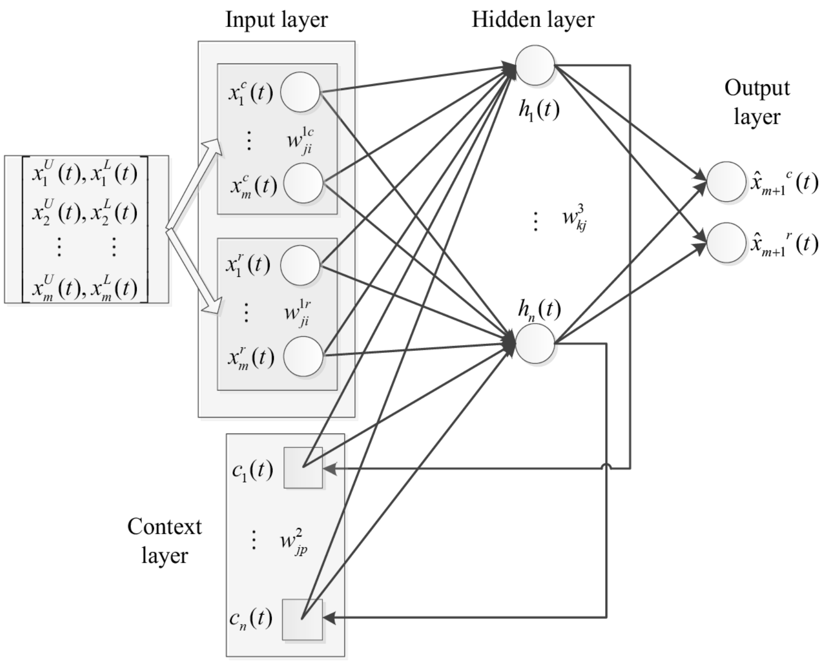

Figure 2. Interval-valued Elman neural network

Then, we will give the expressions of interval-valued Elman neural network.

An interval-valued time series $\left[x^L(t), x^U(t)\right]^T$ is a chronological sequence of interval-valued variables. $x^L(t)$ and $x^U(t)$ are the lower bound and upper bound at each time t which can represent the lowest and highest price of daily stock trading. The interval time series can also be converted into two numerical time series, i.e., the midpoint sequence $x^c(t)$ and the interval radius sequence $x^r(t)$, where

$$x^c(t) = \frac{x^U(t) + x^L(t)}{2}, \quad x^r(t) = \frac{x^U(t) - x^L(t)}{2}. \tag{1}$$

An interval-valued Elman network is shown in Figure 2. Compared to conventional Elman neural network, it is a system receiving interval-valued inputs and offering interval-valued outputs. Technically, on the one hand the original input data need to be transferred from lower and upper bounds to midpoint and interval radius sequence. On the other hand, the relevance between midpoint and radius should be considered in the excitation functions in the hidden layer and the output layer.

Suppose that the input vector is

$$x(t) = [x_1^c(t), x_2^c(t), \ldots, x_m^c(t), x_1^r(t), x_2^r(t), \ldots, x_m^r(t)]^T,$$

where $x_i^c(t) = x^c(t+i-1)$, $x_i^r(t) = x^r(t+i-1)$, $i = 1, 2, \ldots, m$. In the interval-valued Elman network, the outputs of context layer and hidden layer are represented by

$$h_j(t) = f\left(\sum_{i=1}^{m} w_{ji}^{1c} x_i^c(t) + \sum_{i=1}^{m} w_{ji}^{1r} x_i^r(t) + \sum_{p=1}^{n} w_{jp}^2 c_p(t) + b_j^h\right), \quad j = 1, 2, \ldots, n,$$

$$\hat{x}_{m+1}^c(t) = \sum_{j=1}^{n} w_{1j}^3 h_j(t) + b_1^o, \quad \hat{x}_{m+1}^r(t) = \sum_{j=1}^{n} w_{2j}^3 h_j(t) + b_2^o,$$

where $b_j^h \in R^{1\times n}$, b_1^o, $b_2^o \in R^{1\times 2}$, $w_{ji}^{1c} \in R^{n\times m}$, $w_{ji}^{1r} \in R^{n\times m}$, $w_{jp}^2 \in R^{n\times n}$ and w_{1j}^3, $w_{2j}^3 \in R^{2\times n}$ are biases and weight matrices respectively.

Accordingly, we can obtain the interval-valued output $\left[\hat{x}_{m+1}^L(t), \hat{x}_{m+1}^U(t)\right]$, where $\hat{x}_{m+1}^L(t) = \hat{x}_{m+1}^c(t) - \hat{x}_{m+1}^r(t)$ and $\hat{x}_{m+1}^U(t) = \hat{x}_{m+1}^c(t) + \hat{x}_{m+1}^r(t)$.

Interval-valued Elman neural network can handle the prediction of interval-valued time series. Considering the significant volatility for financial time series, for improving predicting accuracy, an improved adaptive expectation method is presented to compensate the predict error of Elman neural network subsequently.

3. **Interval-Valued AEM-Elman Forecasting Model.** In this section, we will design an AEM-Elman neural network for interval-valued financial time series.

An interval-valued Elman model with $2m$ input nodes, n hidden nodes, n context nodes and 2 output nodes is shown in Figure 2. Here, we utilize vector model to represent the interval-valued model, which means that an interval-valued time series can be transferred into a vector time series.

Given an interval-valued financial time series $\left[x^L(t), x^U(t)\right]^T$, $t = 1, 2, \ldots, N$, the midpoint sequence $x^c(t)$ and the interval radius sequence $x^r(t)$ can be obtained by Equation (1) and then these two series can be normalized into interval $[0, 1]$, which is the range matched with the sigmoid function.

In this way, the input-output pairs of interval-valued Elman neural network can be arranged as the form:

$$IOP = \{(x(t), y(t)), t = 1, 2, \ldots, N-m\}, \tag{2}$$

where $x(t) = [x_1^c(t), x_2^c(t), \ldots, x_m^c(t), x_1^r(t), x_2^r(t), \ldots, x_m^r(t)]^T$ and $y(t) = \left[x_{m+1}^c(t), x_{m+1}^r(t)\right]^T$ are input and output for the system, and $x_i^c(t) = x^c(t+i-1)$, $x_i^r(t) = x^r(t+i-1)$.

For training the network and testing the forecasting ability, the set of input-output pairs IOP is divided into training set IOP_1 and testing set IOP_2 as follows:

$$IOP_1 = \{(x(t), y(t)), t = 1, 2, \ldots, L_1\},$$
$$IOP_2 = \{(x(t), y(t)), t = L_1 + 1, \ldots, L_1 + L_2\},$$

where $L_1 + L_2 = N - m$.

In this paper, an adaptive gradient descent algorithm is used to optimize the parameters in the following forms:

$$b(k+1) = b(k) + \alpha(k) \cdot \frac{\partial e}{\partial b}, \tag{3}$$

$$w(k+1) = w(k) + \alpha(k) \cdot \frac{\partial e}{\partial w}, \tag{4}$$

where $e = \sum_{t=1}^{L_1} \left[(x^r_{m+1}(t) - \hat{x}^r_{m+1}(t))^2 + (x^c_{m+1}(t) - \hat{x}^c_{m+1}(t))^2\right]$ is the training error.

In the optimization procedure, the iteration step size $\alpha(k) = \alpha_0 \cdot e(k)$ is a parameter related to time k, which can be updated adaptively with the variation of error. Generally, when the error $e(k)$ increases, the step size should be enlarged to expand search space. On the contrary, when the error $e(k)$ decreases, it means the search space should be lessened. Compared to the traditional gradient descent algorithm with constant step size, the proposed variable step scheme can fasten the searching speed.

After parameter values b and w have been identified, the prediction output of interval-valued Elman network $\hat{y}(t) = \left[\hat{x}^c_{m+1}(t), \hat{x}^r_{m+1}(t)\right]^T$ can be computed.

Next, the adaptive expectation method is applied to improving forecasting capabilities as a compensation phase:

$$\bar{y}(t) = \left[\bar{x}^c_{m+1}(t), \bar{x}^r_{m+1}(t)\right]^T, \tag{5}$$

$$\bar{x}^c_{m+1}(t) = x^c_{m+1}(t-1) + \beta\left(\hat{x}^c_{m+1}(t) - x^c_{m+1}(t-1)\right), \tag{6}$$

$$\bar{x}^r_{m+1}(t) = x^r_{m+1}(t-1) + \beta\left(\hat{x}^r_{m+1}(t) - x^r_{m+1}(t-1)\right), \tag{7}$$

where $\beta \in [0, 1]$ is an adaptive parameter, and thus the output $\hat{y}(t)$ is revised to $\bar{y}(t)$.

Finally, the lower bound and upper bound of the forecasting interval can be obtained as:

$$\bar{x}^L_{m+1}(t) = \bar{x}^c_{m+1}(t) - \bar{x}^r_{m+1}(t), \quad \bar{x}^U_{m+1}(t) = \bar{x}^c_{m+1}(t) + \bar{x}^r_{m+1}(t). \tag{8}$$

The structure of the proposed AEM-Elman model is shown in Figure 3.

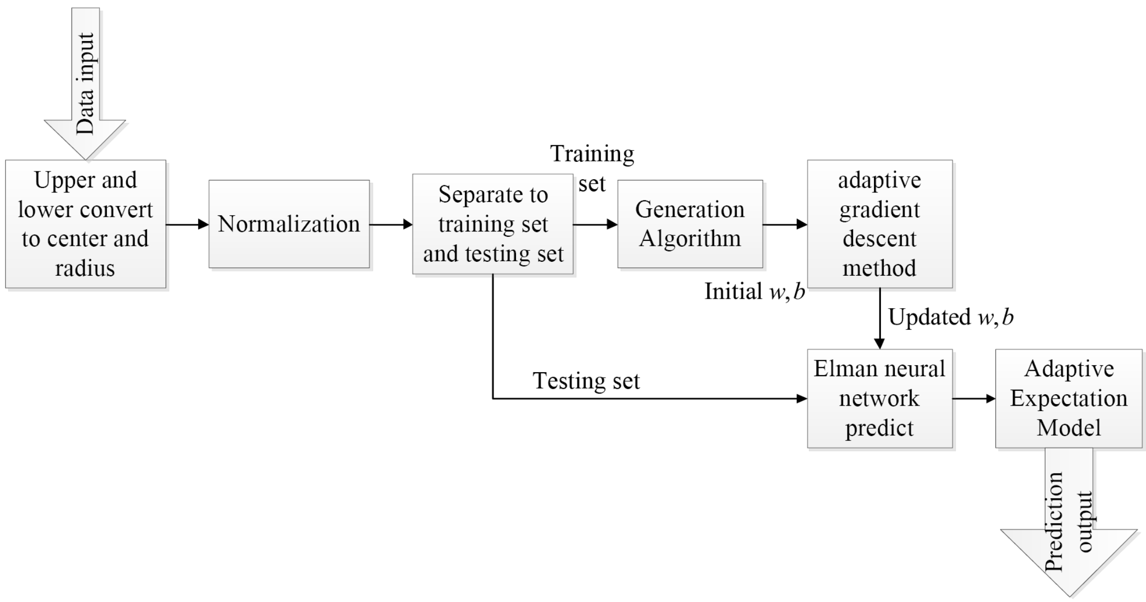

Figure 3. The proposed AEM-Elman neural network model

The initial values of parameters $b(0)$ and $w(0)$ are obtained by using the genetic algorithm. If the error e is less than a certain threshold ε or the number of iterations is larger than a presupposed maximum value, then the training algorithm will terminate and the parameter values b and w will be determined.

In the following, for given interval-valued financial time series $\left[x^L(t), x^U(t)\right]^T$, $t = 1, 2, \ldots, N$, we summarize the main procedure of the proposed algorithm.

Step 1: Set the number of neurons and convert $\left[x^L(t), x^U(t)\right]^T$ to midpoint sequence $x^c(t)$ and interval radius sequence $x^r(t)$ by Equation (1).

Step 2: Arrange the input-output pairs IOP in the form of (2).

Step 3: Use genetic algorithm to obtain the initial values of parameters $b(0)$ and $w(0)$.

Step 4: Use adaptive gradient descent algorithms (3) and (4) to update the values of parameters $b(k)$, $w(k)$ until error e is less than threshold ε or the number of iterations is larger than the maximum iterations.

Step 5: Use adaptive expectation method (5)-(7) to revise the output.

Step 6: Obtain the lower bound and upper bound of forecasting interval by Equation (8).

4. **Numerical Simulation.** In this section, some financial time series are collected for illustrating the validity of the proposed model. In the following two examples, the parameters of interval-valued Elman network are both designed to be $m = 2$ and $n = 2$ for simplicity.

Example 4.1. *Prediction of TAIFEX*

An interval financial time series is collected by the daily variation range of Taiwan futures exchange index (TAIFEX) from August 3 to September 30, 1998, where the daily lowest price and highest price form interval number. The first 35 data are selected as training part, and the rest is the testing set. The parameter in AEM is set as $\beta = 0.5$ by comparative trials in this experiment. The forecast performances of Elman, GA-Elman and AEM-Elman models are listed in Table 1. The evaluation index RMSE of the interval time series is represented by

$$RMSE = \frac{1}{2}\left(\sqrt{\frac{\sum_{t=L_1+1}^{L_1+L_2}\left(x_{m+1}^L(t) - \bar{x}_{m+1}^L(t)\right)^2}{L_2}} + \sqrt{\frac{\sum_{t=L_1+1}^{L_1+L_2}\left(x_{m+1}^U(t) - \bar{x}_{m+1}^U(t)\right)^2}{L_2}}\right).$$

Table 1. Performance comparation for TAIFEX

Experiment No.	RMSE		
	Elman	GA-Elman	AEM-Elman
1	80.1319	70.0038	**63.6987**
2	79.8633	67.6768	**64.4941**
3	101.7084	65.2991	**61.0715**
4	135.4041	60.9368	**58.2537**
5	162.4454	74.8177	**69.8935**
Average	111.9106	67.7468	**63.4823**

And the corresponding lower bound and upper bound time series are shown in Figure 4 and Figure 5 respectively.

From the results, we can find out that the proposed model can achieve higher accuracy than Elman model and GA-Elman model without AEM process. It tells us that Elman neural network combined with compensation technology can improve the forecasting capability of interval-valued data.

Example 4.2. *Prediction of international stock index*

Stock trading indexes in three stock exchanges, that is, the S&P500 of the United States, the FTSE100 of the United Kingdom, and the NIKKEI225 of Japan, are introduced as the experimental data. The samples are the interval time series of the daily lowest and highest values for these indexes. The sample description of experimental data is shown in Table 2. In the simulation, two thirds data are chosen as the training set and the rest of data are the testing samples.

In this example, the interval average relative variance (ARV) is used as the evaluation criterion of the model. The ARV is a common error calculation statistic for interval time

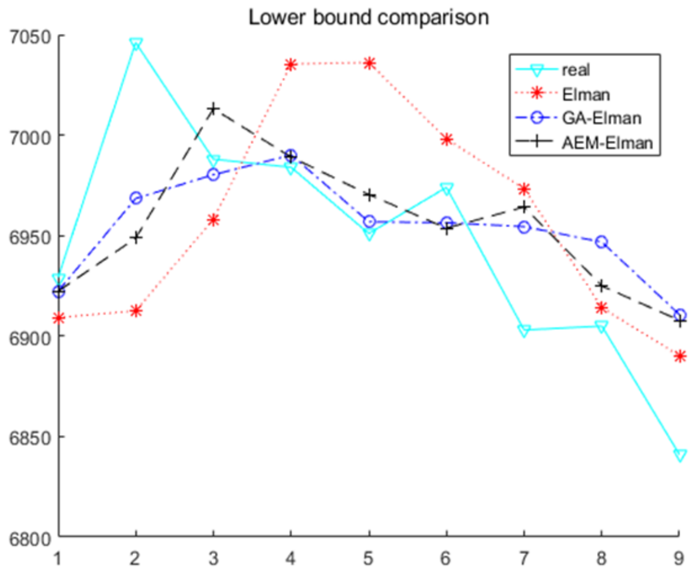

FIGURE 4. Lower bound results

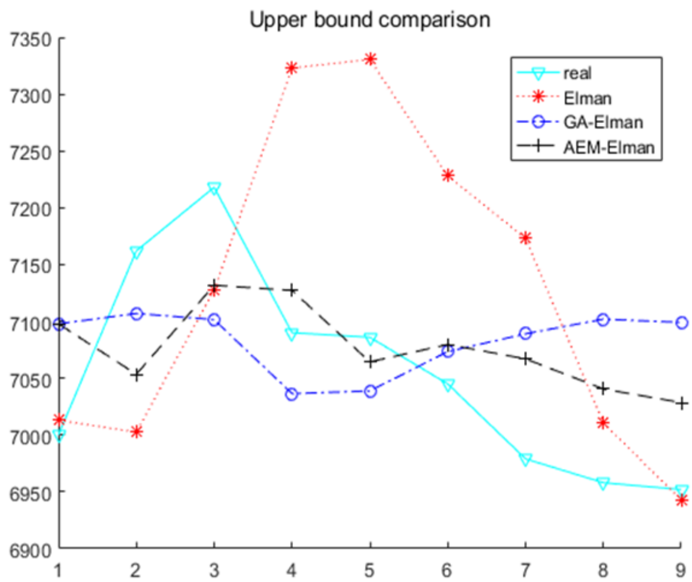

FIGURE 5. Upper bound results

TABLE 2. The size of the experimental samples

Stock index	Period	Sample size
S&P500	July 19, 2010 to August 10, 2012	523
FTSE100	October 15, 2007 to August 10, 2012	1218
NIKKEI225	January 8, 2004 to August 10, 2012	2165

series problem. The expression of ARV is represented by

$$ARV = \frac{\left(\sum_{t=L_1+1}^{L_1+L_2}\left(x_{m+1}^L(t)-\bar{x}_{m+1}^L(t)\right)^2+\sum_{t=L_1+1}^{L_1+L_2}\left(x_{m+1}^U(t)-\bar{x}_{m+1}^U(t)\right)^2\right)}{\left(\sum_{t=L_1+1}^{L_1+L_2}\left(x_{m+1}^L(t)-\bar{\bar{x}}_{m+1}^L(t)\right)^2+\sum_{t=L_1+1}^{L_1+L_2}\left(x_{m+1}^U(t)-\bar{\bar{x}}_{m+1}^U(t)\right)^2\right)}$$

where $\left[x^L(t), x^U(t)\right]^T$ is the target interval value, $\left[\bar{x}^L(t), \bar{x}^U(t)\right]^T$ is the actual output of AEM-Elman network, and $\left[\bar{\bar{x}}^L(t), \bar{\bar{x}}^U(t)\right]^T$ is the average value of actual output.

Table 3 lists the performance comparison for the three stock indexes in terms of the mean results of 50 times simulations. The parameter in AEM is set as $\beta = 0.2$. It can be seen that the proposed AEM-Elman model has achieved higher accuracy than other models.

TABLE 3. Performance comparison for three stock indexes

Stock index	ARV				
	GA-Elman	AEM-Elman	FA-MSVR [11]	PSO-MSVR [11]	GA-MSVR [11]
S&P500	0.5054	**0.0628**	0.299	0.312	0.287
FTSE100	0.6024	**0.0646**	0.301	0.302	0.283
NIKKEI225	0.4579	**0.0413**	0.324	0.309	0.341

5. **Conclusions.** In this paper, an improved AEM-Elman neural network is used to predict interval valued time series. Based on the traditional Elman neural network, the genetic algorithm is used to optimize the initial parameters of the network and the adaptive gradient descent method is designed for training the step-length parameter. Further, a compensation scheme is applied to the Elman neural network. In the future research, how to enhance the interpretability of the proposed model could be considered.

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