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ADAPTIVE CONSENSUS TRACKING CONTROL FOR MULTIPLE AUTONOMOUS UNDERWATER VEHICLES WITH UNCERTAIN PARAMETERS

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Abstract. *Focusing on the problem of consensus tracking control for multi-AUV (multiple autonomous underwater vehicle) system, this paper proposes an adaptive fuzzy finite time control method based on command filtering. First, the algebraic graph theory is combined with leader-follower architecture for describing the communication topology of multi-AUV system. Next, the error compensation mechanism is introduced into the command filtering technology, and they work together to reduce the filtering error and the explosion of complexity caused by backstepping. Finally, the application of finite time and fuzzy logic system improves the convergence rate and the robustness of multi-AUV system. The effectiveness of the proposed algorithm is illustrated by a simulation example.*

Keywords: Adaptive fuzzy control, Multiple AUV system, Command filtering, Graph theory, Finite time

1. **Introduction.** With the increasing importance of marine resources, how to energetically develop marine technology is essential for the world. Marine technology is a general term for all kinds of methods, skills and equipment used to study marine natural phenomena and their changing rules [1]. In view of the abominable underwater environment and the limitation of the depth of human diving, the autonomous underwater vehicles (AUVs) which can carry out integrated operation under water without human remote control have become an important tool for developing the ocean. It has various huge advantages such as small volume, light weight, and low cost. Meanwhile, due to spatial distribution, efficiency and flexibility of task execution, distributed control of multi-AUV system has been paid considerable attention in ocean exploration, oceanographic surveys.

In order to complete the tasks in a reasonable and orderly manner, accurate consensus tracking control of multi-AUV system for acquiring good quality of data has become a hot issue in recent years. In the beginning trend of distributed controller design, leader-follower architecture is used to solve the problems appearing in the communication topology [2]. However, each AUV is independent and there is no direct connection in the system, so it is relative absolutism for information access channels. By integrating the past literature, it is obvious to see graph theory analysis has great advantages for dealing with distributed control problem because it can simplify communication topology and optimize information model of multi-AUV system. It abstracts each AUV as a node, and expresses a certain relationship between AUVs through a directed or undirected graph between the nodes. Then, various methods of control will be combined into the distributed controllers, such as [3, 4].

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Complex undersea environment causes highly coupled nonlinearities, time varying dynamics. Accordingly, many methods are applied to achieving accurate consensus tracking control and overcoming parameter uncertainties resulting from unknown forces. Among them are adaptive control [4], dynamic surface control [5], sliding mode control [6] and so on. After reviewing all the above control methods, it can be clearly seen that adaptive backstepping technology has the most extensive and effective application in the consistency tracking control of multi-AUV system. However, the complexity of mathematical models and operations causes the problem that certain functions must be linear, and it limits the range of its application. Focusing on the external interference and uncertain forces in the nonlinear matter of the multi-AUV system, the fuzzy logic system (FLS) and neural networks (NNs) show great advantages. Meanwhile, for the explosion of complexity condition that results from adaptive backstepping control, command filtered technologies are introduced [7]. Besides, the errors caused by the filter still need to be perfected, and the compensation mechanism is particularly important for the multi-AUV system.

Although AUVs designed by the above control method can achieve consistent or nearly consistent to track the desired trajectory, the convergence time of consistent tracking errors will be infinite. Compared with asymptotic control approaches mentioned above, finite-time control technology can provide faster response, better anti-interference ability and higher tracking precision in nonlinear system. Accordingly, finite time technology is adopted in various different nonlinear systems, such as the multi-agent systems [8], the nonlinear system [9, 10], the attitude synchronization control of spacecraft [11], especially multi-AUV system. Nevertheless, when [12] investigated the finite-time sliding mode control for consensus tracking control of multi-AUV system, its inherent chattering problem will be amplified and hard to be accepted for practical applications, so different control methods are particularly important constructed for this issue. In the sum of the above discussion, it is generally known that adaptive backstepping method can solve the tracking problems of multi-AUV system effectively, but how these techniques combine finite time and command filtering has not yet been reported.

In view of all these methods mentioned above, the controller based on command-filtered backstepping and fuzzy approximation theories is designed for multi-AUV system in this paper, and the proposed algorithm is extended to finite-time control framework. Compared with the control methods for dealing with the uncertain forces and external interference in [4], the filtering processing in [5], and the convergence time in [6, 7], the proposed FLS-based finite-time consensus tracking approach has the following advantages.

(1) In [4] the traditional adaptive control is adopted to deal with the influence of the variation of hydrodynamic parameters. The limitation of the ocean environment and sensors leads to the limited speed of each AUV to obtain global information, but the change of the operating environment is very fast, so it is difficult to produce good results because it is too late to correct. However, this paper introduces the fuzzy approximation theory to approximate the unknown nonlinear forces and external interference, and it improves the robustness and fault tolerance of multi-AUV system.

(2) In [5] the backstepping and dynamic surface technologies are introduced, respectively. The former has the explosion of complexity condition due to the design of virtual controllers, and the traditional filters of the latter will cause errors. The modified error compensation mechanism and the fractional power state feedback are employed in this paper, which guarantee the errors are finite-time stable and uniformly ultimately bounded in the virtual and actual controllers, so the control performance is improved.

(3) Although in [6, 7], various control modes can guarantee the asymptotic convergence of the system, the time of convergence is not ideal. The finite-time technology is adopted to improve the fast response characteristic of the system. It is proved that not only do the AUVs have the ability to realize the desired position and velocity in finite time, but

also the system has strong robustness. Finally, a simulation example is given to verify the effectiveness and robustness of the proposed algorithm.

The rest of this paper is organized as follows. Section 2 describes the graph theory, force analysis and some lemmas. In Section 3, the command-filtered backstepping controllers are designed based on finite time. Section 4 gives stability analysis. Simulation example is given to verify the effectiveness and robustness of the proposed algorithm in Section 5. Finally, some conclusions are presented in Section 6.

2. **Preliminaries and Problem Formulation.**

2.1. **Graph theory.** Consider a distributed multi-AUV system consisting of one leader and n followers, the communication topology can be modeled by a weighted graph $\varsigma = \{M, E, A\}$, where $M = \{m_1, m_2, \ldots, m_n\}$ is the set of nodes, and node i $(i \in M)$ denotes the ith following AUV. $E \subset M \times M$ represents the set of edges, and $A = [a_{ij}] \in R^{n\times n}$ is weighted adjacency matrix of the graph ς. If the ith AUV has access to send information directly to the jth AUV, the edge between them is denoted as $(m_i, m_j) \in E$. Define $a_{ij} > 0$ (usually $a_{ij} = 1$) if $(m_j, m_i) \in E$, else $a_{ij} = 0$, and suppose $a_{ii} = 0$, $\forall i$. In-degree of m_i can be expressed as $d_i = \sum_{j\in N_i}^{n} a_{ij}$ and the sum of it can be represented as $D = \text{diag}\{d_1, d_2, \ldots, d_n\}$. The Laplacian matrix can be expressed as $L = D - A \in R^{n\times n}$. A path between m_1 to m_k is the sequence (m_1, m_2), (m_2, m_3), (m_{k-1}, m_k), where $(m_{j-1}, m_j) \in E$ for $j = 1, 2, \ldots, k$. If each couple of nodes (m_i, m_j) has access, graph ς is called strongly connected. Define a vertex which has access to others as the root node, and if weighted graph ς has a root node, it contains a directed spanning tree.

Choose m_0 to denote leader, the augmented graph $\bar{\varsigma}$ which contains m_0 is defined as $\bar{\varsigma} = \left\{\bar{M}, \bar{E}, \bar{A}\right\}$, where $\bar{M} = \{m_0, m_1, m_2, \ldots, m_n\}$, $\bar{E} \subset \bar{M} \times \bar{M}$. The weights between vertex of leader to vertices of followers are defined as $b_{i0} \geq 0$. If leader has an access to the ith follower, $b_{i0} = 1$, else $b_{i0} = 0$, and diagonal matrices B and H are defined as $B = \text{diag}\{b_{10}, b_{20}, \ldots, b_{n0}\} \in R^{n\times n}$, $H = L + B$. The following assumption on the graph topology is required for consensus tracking control problem of multi-AUV system.

Assumption 2.1. *In multi-AUV system, graph of communication relations $\bar{\varsigma}$ has spanning tree, that is $B \neq 0$ and the root node represents the leader simultaneously.*

Lemma 2.1. [3]: *H is full rank under Assumption 2.1 and each eigenvalue of H will have positive real part.*

2.2. **Force analysis.** Assume the situation that all attitudes of AUVs are fixed, so the distributed multi-AUV system eliminates singular points. Without loss of generality and under Assumption 2.2, assume that the following AUVs are labeled from 1 to n, and the force analysis is given as follows:

$$\begin{cases} \dot{\eta}_i = R_i \mathbf{v}_i \\ M_i \dot{\mathbf{v}}_i = -D_i(\mathbf{v}_i)\mathbf{v}_i - g_i + f_i - d_i \\ y_i = \eta_i \end{cases} \tag{1}$$

where $\eta_i = [x_i, y_i, z_i]^T$ and $\mathbf{v}_i = [u_i, v_i, w_i]^T$ denote position and velocity vectors in the inertial or body-fixed reference frame, respectively. R_i is the transition matrix. $D_i(\mathbf{v}_i)$, M_i, d_i, g_i, $f_i = [X_i, Y_i, Z_i]^T \in R^3$ are defined as water resistance, inertia, interferences, restoring force and motive power, respectively. For angle $\gamma \in R$, denote $s_\gamma = \sin\gamma$, $c_\gamma = \cos\gamma$.

$$R_i = \begin{bmatrix} c_{\psi_i}c_{\theta_i} & -s_{\psi_i}c_{\varphi_i} + s_{\varphi_i}s_{\theta_i}c_{\psi_i} & s_{\psi_i}s_{\varphi_i} + s_{\theta_i}c_{\psi_i}c_{\varphi_i} \\ s_{\psi_i}c_{\theta_i} & c_{\psi_i}c_{\varphi_i} + s_{\varphi_i}s_{\theta_i}s_{\psi_i} & -c_{\psi_i}s_{\varphi_i} + s_{\theta_i}s_{\psi_i}c_{\varphi_i} \\ -s_{\theta_i} & s_{\varphi_i}c_{\theta_i} & c_{\varphi_i}c_{\theta_i} \end{bmatrix} \tag{2}$$

Define $M_i = \text{diag}\left[m_{i1}, m_{i2}, m_{i3}\right]$, $D_i(\mathbf{v}_i) = \text{diag}[d_{L_{i1}} + d_{P_{i1}}|u_i|,\ d_{L_{i2}} + d_{P_{i2}}|v_i|,\ d_{L_{i3}} + d_{P_{i3}}|w_i|]$, m_{ij}, $d_{L_{ij}}$, $d_{P_{ij}} > 0$. $g_i = [(W_i - B_i)s_{\theta_i},\ -(W_i - B_i)c_{\theta_i}s_{\varphi_i},\ -(W_i - B_i)c_{\theta_i}c_{\varphi_i}]^T$, where W_i and B_i represent the gravitational and buoyancy forces, respectively, $i \in M$, $j = 1, 2, 3$.

Assumption 2.2. *Suppose the ith followers have the following properties:* $\|d_i\| \le d_i^*$ *and* $\|g_i\| \le g_i^*$, *where* d_i^*, $d_{L_{ij}}$, $d_{P_{ij}}$, W_i, B_i *and* $g_i(\Theta_i)$ *are bounded positive constant.*

Applying the backstepping method to the above model, choosing $x_{i,1} = \eta_i$, $x_{i,2} = \dot{\eta}_i$, it can be translated to the following equation:

$$\begin{cases} \dot{x}_{i,1} = x_{i,2} \\ \dot{x}_{i,2} = J_i + \bar{f}_i \\ y_i = x_{i,1} \end{cases} \tag{3}$$

where $J_i = \dot{R}_i(\Theta_i)\mathbf{v}_i - R_i(\Theta_i)M_i^{-1}D_i(\mathbf{v}_i)\mathbf{v}_i - R_i(\Theta_i)M_i^{-1}d_i - R_i(\Theta_i)M_i^{-1}g_i(\Theta_i)$, and $\bar{f}_i = R_i(\Theta_i)M_i^{-1}f_i$.

Assumption 2.3. *Let* $\Omega_d \subset \mathbf{R}^n$ *denote an open set that includes the origin,* $J_i^{(p)}$ *are known smooth nonlinear functions bounded on* $\bar{\Omega}_d$ *for* $p = 1, \ldots, (n-i)$ *for system (1).*

Assumption 2.3 ensures that the followers can track the reference signal from any initial positions, the function J_i is stable under the Lipschitz condition.

2.3. Some lemmas.

Lemma 2.2. [11]: *Define function* $F(x)$ *in* $U \in \mathbf{R}^n$ *which satisfies smooth and positive. If it starts from an area* $U_0 \subset \mathbf{R}^n$ *and has the following character:* $\dot{F}(x) + kF^{\gamma}(x) \le 0$, *then it can reach* $F(x) \equiv 0$ *in finite time* t^*, *where* $K > 0$, $0 < \gamma < 1$ *and* $t^* \le F(0)^{1-\gamma}/(K(1-\gamma))$.

Lemma 2.3. [13]: *Suppose smooth and positive function* $F(x)$ *can get the following inequality:* $\dot{F}(x) + \lambda_1 F(x) + \lambda_2 F(x)^{\gamma} \le 0$ *in finite time. Then, the parameters will satisfy* $\lambda_1 > 0$, $\lambda_2 > 0$, $\gamma \in (0, 1)$ *and the setting time is* $T \le t_0 + \frac{1}{\lambda_1(1-\gamma)} \ln \frac{\lambda_1 F(0)^{1-\gamma} + \lambda_2}{\lambda_2}$.

In this brief, the fuzzy logic system will be used to approximate the unknown continuous function $f(x)$ [14]. Let $f(x)$ be the continuous function defined on a set Ω. Then for any scalar $\varepsilon > 0$, the following inequality can be obtained:

$$\sup_{x \in \Omega} \left| f(x) - W^T S(x) \right| \le \varepsilon \tag{4}$$

where $W = [W_1, \ldots, W_N]^T$ is the ideal constant weight vector, and the basis function vector is $S(x) = [p_1(x), p_2(x), \ldots, p_N(x)]^T / \sum_{i=1}^N p_i(x)$, with $N > 1$ being the number of FLS rules and p_i are chosen as Gaussian functions, i.e., for $i = 1, 2, \ldots, N$, $p_i(x) = \exp\left[\frac{-(x-\mu_i)^T(x-\mu_i)}{\gamma_i^2}\right]$, where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$ is the center vector, and γ_i is the width of the Gaussian function.

3. The Design of Finite Time Command Filter Controllers.

In each step of the distributed backstepping controllers design, the errors of each AUV are defined as:

$$\begin{aligned} e_{i,1} &= \sum_{j=1}^{n} a_{ij}(\eta_i - \eta_j) + b_i(\eta_i - \eta_d) = \sum_{j=1}^{n} a_{ij}(x_{i,1} - x_{j,1}) + b_i(x_{i,1} - \eta_d) \\ e_{i,2} &= R_i(\Theta_i)v_i - \alpha_i = x_{i,2} - \alpha_i, \ i \in \varsigma \end{aligned} \tag{5}$$

where given reference tracking trajectory of leader is η_d and α_i are the outputs of second-order Levant differentiators based on finite-time command filter technologies. Assume that the first and second time derivatives, $\dot{\eta}_d$ and $\ddot{\eta}_d$, are smooth, bounded and known functions. The differentiators employ the virtual controller σ_i as the input signals, and obtain their differential signals by quickly filtering the intermediate signals. Then, illustrating the first-order Levant differentiator refers to [15]:

$$\begin{aligned} \dot{\varphi}_{i,1,\chi} &= \tau_{i,1,\chi}, \quad \dot{\varphi}_{i,2,\chi} = -K_{i,2}\mathrm{sign}\left(\varphi_{i,2,\chi} - \tau_{i,1,\chi}\right) \\ \tau_{i,1,\chi} &= -K_{i,1}\left|\varphi_{i,1,\chi} - \sigma_{i,1,\chi}\right|^{\frac{1}{2}}\mathrm{sign}\left(\varphi_1 - \sigma_{i,1,\chi}\right) + \varphi_{i,2\chi} \end{aligned} \tag{6}$$

where $\alpha_i = \varphi_{i,1,\chi} = [\varphi_{i,1,1}, \varphi_{i,1,2}, \varphi_{i,1,3}]^T$ and $\dot{\alpha}_i = \tau_{i,1,\chi} = [\tau_{i,1,1}, \tau_{i,1,2}, \tau_{i,1,3}]^T$ are the outputs. Meanwhile, the input noise is essential due to the bad underwater environment, so the parameters K_1 and K_2 are properly chosen as the parameters of the differentiator for mitigating the effects caused by noise, and the following lemma is introduced.

Lemma 3.1. [15]: *Consider the input noise satisfies the inequality $|\sigma - \sigma_0| \leq \kappa$, and then the following inequalities are established in finite time.*

$$|\varphi_1 - \sigma_0| \leq \varpi_1\kappa = \mu_1, \quad |\tau_1 - \dot{\sigma}_0| \leq \varpi_2\kappa^{\frac{1}{2}} = \mu_2 \tag{7}$$

where ϖ_1 and ϖ_2 are positive constants related to the design of Formula (5), and μ_1 and μ_2 are positive constants.

Remark 3.1. *Note that the inputs σ_i are supposed to be effected by the noise due to the bad underwater environment, and assume that the boundary of noise exists. Normally, if the inputs σ_i of filter (6) are not effected by the noise, then $\mu_1 = 0$.*

Note that the errors caused by the second-order Levant differentiator are magnified which will have impact on the range of error convergence. To remove the influence of $\alpha_i - \sigma_{i,1}$ and guarantee the fast response characteristics of error compensation system distinct from asymptotic convergence in the traditional methods, error compensation mechanism is defined as:

$$\begin{aligned} \dot{\varepsilon}_{i,1} &= -h_{i,1}\varepsilon_{i,1} + (d_i + b_i)(\alpha_i - \sigma_{i,1}) + (d_i + b_i)\varepsilon_{i,2} - z_{i,1}\mathrm{sign}(\varepsilon_{i,1}) \\ \dot{\varepsilon}_{i,2} &= -h_{i,2}\varepsilon_{i,2} - (d_i + b_i)\varepsilon_{i,1} - z_{i,2}\mathrm{sign}(\varepsilon_{i,2}) \end{aligned} \tag{8}$$

where $\varepsilon_{i,m}(0) = 0$ are the error compensating vectors, and $0 < z_{i,m} < 2h_i$ are constants. Meanwhile, it satisfies that $w_{i,m} = e_{i,m} - \varepsilon_{i,m}$, $i \in \varsigma$, $m = 1, 2$.

Then the distributed finite-time virtual controllers $\sigma_{i,m}$ are constructed as follows:

$$\begin{aligned} \sigma_{i,1} &= \frac{1}{(d_i + b_i)}\left(-h_{i,1}e_{i,1} + b_i\dot{\eta}_d + \sum_{j=1}^{n} a_{ij}x_{j,2} - s_{i,1}w_{i,1}^{\gamma}\right) \\ \sigma_{i,2} &= -h_{i,2}e_{i,2} - \frac{w_{i,2}\hat{\theta}_i S_i^T S_i}{2k_i^2} - \frac{1}{2}w_{i,2} + \dot{\alpha}_i - (d_i + b_i)e_{i,1} - s_i w_{i,1}^{\gamma} \end{aligned} \tag{9}$$

where k_i are positive constants and S_i ($i \in \varsigma$) are the basis function vectors chosen from FLS. s_i and γ are positive constants and assumed that $0 < \gamma < 1$ according to the lemma of finite-time. $\hat{\theta}_i$ is the estimation of θ_i which will design later in this paper. Then, Theorem 3.1 is given as follows.

Theorem 3.1. *For the multi-AUV system (1) under Assumptions 2.1-2.3, whose virtual control signals $\sigma_{i,m}$ combine the finite-time command filers (6) and the error compensation mechanism (8), the control law $f_i = M_i R_i^T(\Theta_i)\bar{f}_i$ can guarantee the tracking errors, $\eta_i - \eta_d$, converge to a field containing original point in finite time. Meanwhile, all vectors in the multi-AUV system will achieve bounded in finite time, $i \in \varsigma$.*

4. **Stability Analysis.** In order to illustrate the correctness of Theorem 3.1, the stability analysis is given.

Proof: Three steps are chosen to proof Theorem 3.1, including the recursive control design, adaptive updating law design and the finite-time boundness of error compensating system.

Step 1: Construct the Lyapunov function as:

$$W = W_{i,1} + W_{i,2} + \sum_{i=1}^{N} \frac{1}{2r_i}\tilde{\theta}_1^2 = \frac{1}{2}w_{i,1}^2 + \frac{1}{2}w_{i,2}^2 + \sum_{i=1}^{N} \frac{1}{2r_i}\tilde{\theta}_1^2 \tag{10}$$

Taking the derivative of W_1 yields

$$\dot{W}_{i,1} = w_{i,1}\left[-\sum_{j=1}^{n} a_{ij}x_{j,2} - b_i\dot{\eta}_d - \dot{\varepsilon}_{i,1} + (d_i+b_i)\sigma_{i,1} + (d_i+b_i)(\alpha_i - \sigma_{i,1}) + (d_i+b_i)e_{i,2}\right] \tag{11}$$

Then, replacing the error compensation mechanism $\varepsilon_{i,1}$ and the virtual control signal $\sigma_{i,1}$ with Formulas (8) and (9), the above equation can be translated into:

$$\dot{W}_{i,1} = w_{i,1}\Big[-h_{i,1}w_{i,1} + (d_i+b_i)w_{i,2} + z_{i,1}\text{sign}(\varepsilon_{i,1}) - s_i w_{i,1}^{\gamma}\Big] \tag{12}$$

Taking the derivative of W_2 yields

$$\dot{W}_{i,2} = \dot{W}_{i,1} + w_{i,2}^T w_{i,2} = \dot{W}_{i,1} + w_{i,2}\left(J_i + \bar{f}_i - \dot{\alpha}_i - \dot{\varepsilon}_{i,2}\right) \tag{13}$$

Typically, suppose that the internal and external interferences of the system are bounded. However, the upper bounds of uncertain forces and external disturbances cannot be calculated accurately in practical engineering. It is generally known that FLS has good ability to approximate unknown nonlinear functions, so the FLS is introduced as $J_i = W_i^T S_i(x_i) + \delta_i$, where $x_i = (\dot{\eta}_i, \eta_i)$ are input vectors, W_i are the optimal approximation weight, and $\delta_i > 0$ are approximation errors and satisfy that $|\delta_i| < \xi_i$. By Young's inequality, the following inequalities can be obtained:

$$w_{i,2}J_i \le \frac{1}{2k_i^2}w_{i,2}^2\|W_i\|^2 S_i^T S_i + \frac{1}{2}k_i^2 + \frac{1}{2}w_{i,2}^2 + \frac{1}{2}\xi_i^2.$$

Substituting the error compensation mechanism $\xi_{i,2}$ and virtual control function $\sigma_{i,2}$ into Equation (13), the equation can be further simplified into:

$$\begin{aligned}\dot{W}_{i,2} \le &-\sum_{m=1}^{2} h_{i,m}w_{i,m}^2 + \frac{1}{2}\xi_i^2 + \frac{1}{2}k_i^2 + \frac{1}{2k_i^2}w_{i,2}^2\left(\|W_i\|^2 - \hat{\theta}_i\right)S_i^T S_i \\ &+ \sum_{m=1}^{2} w_{i,m}z_{i,m}\text{sign}(\varepsilon_i) - \sum_{m=1}^{2} s_{i,m}w_{i,m}^{\gamma+1}\end{aligned} \tag{14}$$

Step 2: The adaptive updating laws are designed by using FLS to estimate the uncertain parameters. Choose $\theta_i = \|W_i\|^2$, $\hat{\theta}_i$ is the estimation of θ_i, considering $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$, and the adaptive laws of $\tilde{\theta}_i$ are designed as:

$$\dot{\hat{\theta}}_i = \frac{r_i}{2k_i^2}w_{i,2}^2 S_i^T S_i - \lambda_i\hat{\theta}_i \tag{15}$$

where λ_i and r_i are designed positive constants. For the part of error compensation mechanisms, the following inequalities can be obtained combined with Yang's inequality:

$$w_{i,m}z_{i,m}\text{sign}(\varepsilon_{i,m}) \le \frac{z_{i,m}}{2}w_{i,m}^2 + \frac{z_{i,m}}{2}\left[\text{sign}(\varepsilon_{i,m})\right]^2 \le \frac{z_{i,m}}{2}w_{i,m}^2 + \frac{z_{i,m}}{2} \tag{16}$$

Substituting the adaptive updating laws (15) and (16), the time derivative of W can be computed:

$$\begin{aligned}\dot{W} \le &-\sum_{i=1}^{N}\sum_{m=1}^{2}\left[\left(h_{i,m} - \frac{z_{i,m}}{2}\right)w_{i,m}^2 + s_{i,m}w_{i,m}^{\gamma+1}\right] - \left(\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2\right)^{\gamma+1/2} \\ &+ \sum_{i=1}^{N}\sum_{m=1}^{2}\left(\frac{1}{2}k_i^2 + \frac{1}{2}\xi_i^2 + \frac{z_{i,m}}{2}\right) - \frac{1\lambda_i}{4r_i}\theta_i^2 + \left(\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2\right)^{\gamma+1/2} - \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 + \frac{\lambda_i}{r_1}\theta_i^2\end{aligned}$$

If $\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 \geq 1$, the following inequalities can be obtained:

$$\left(\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2\right)^{\gamma+1/2} - \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 + \frac{\lambda_i\theta_i^2}{r_i} \leq \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 - \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 + \frac{\lambda_i\theta_i^2}{r_i} \leq \frac{\lambda_i\theta_i^2}{r_i}.$$

If $\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 < 1$, the following inequalities can be further obtained:

$$\left(\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2\right)^{\gamma+1/2} - \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 < 1 - \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 < 1.$$

Therefore, combining the two above inequalities yields

$$\left(\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2\right)^{\gamma+1/2} - \frac{\lambda_i}{2r_i}\tilde{\theta}_i^2 + \frac{\lambda_i\theta_i^2}{r_i} \leq \frac{\lambda_i\theta_i^2}{r_i} + 1.$$

Thus, from these inequalities, the time derivative of W can be rewritten as:

$$\begin{aligned}\dot{W} &\leq -\sum_{i=1}^{N}\sum_{m=1}^{2}\left[\left(h_{i,m} - \frac{z_{i,m}}{2}\right)w_{i,m}^2 + s_{i,m}w_{i,m}^{\gamma+1}\right] - \left(\frac{\lambda_i}{2r_i}\tilde{\theta}_i^2\right)^{\gamma+1/2} \\ &\quad + \sum_{i=1}^{N}\sum_{m=1}^{2}\left(\frac{1}{2}k_i^2 + \frac{1}{2}\xi_i^2 + \frac{z_{i,m}}{2}\right) - \frac{1\lambda_i}{4r_i}\tilde{\theta}_i^2 + \frac{\lambda_i\theta_i^2}{r_i} + 1 \\ &\leq -a_0W - b_0W^{\frac{\gamma+1}{2}} + c\end{aligned}$$

where $a_0 = \min\left\{(2h_{i,m} - z_{i,m}), \frac{1}{2}\lambda_i\right\}$, $b_0 = \min\left\{(s_{i,m}) \cdot 2^{\frac{1+\gamma}{2}}\lambda_i^{\frac{1+\gamma}{2}}\right\}$, $c = \sum_{i=1}^{n}\left(\frac{1}{2}k_i^2 + \frac{1}{2}\xi_i^2 + \frac{z_{i,m}}{2}\right) + 1 + \frac{\lambda_i\theta_1^2}{r_1}$, $i = 1, 2, \ldots, n$. The following inequality can be obtained:

$$\dot{W} \leq -\left(a_0 - \frac{c}{2W}\right)W - \left(b_0 - \frac{c}{2W^{\frac{\gamma+1}{2}}}\right)W^{\frac{\gamma+1}{2}} \tag{17}$$

If $2h_{i,m} - z_{i,m} > 0$, $a_0 - \frac{c}{2W} > 0$ and $b_0 - \frac{c}{2W^{\frac{\gamma+1}{2}}} > 0$, the compensated tracking errors $w_{i,m}$ will converge to the region $|w_{i,m}| \leq \max\left\{\sqrt{\frac{c}{a_0}}, \sqrt{2\left(\frac{c}{2b_0}\right)^{\frac{2}{\gamma+1}}}\right\}$ in finite time $T_1 \leq \left[1/\left(a_0 - \frac{c}{2W}\right)\left(1 - \frac{\gamma+1}{2}\right)\right]\ln\left[\left(a_0 - \frac{c}{2W}\right)W^{1-\frac{\gamma+1}{2}}(0)\left(b_0 - \frac{c}{2W^{\frac{\gamma+1}{2}}}\right)\Big/\left(b_0 - \frac{c}{2W^{\frac{\gamma+1}{2}}}\right)\right]$.

Step 3: Combining with the definition of tracking errors, $e_{i,m} = w_{i,m} + \varepsilon_{i,m}$, if $\varepsilon_{i,m}$ is bounded in finite time which can be proved, the conclusion that global tracking errors converge to a field of original point in setting time will establish. Meanwhile, $\varepsilon_n = 0$ has defined in the part of error compensating, so the key is how to prove the bounds of ε_i exist.

$$\bar{W} = \frac{1}{2}\sum_{i=1}^{n}\sum_{m=1}^{2}\varepsilon_{i,m}^T\varepsilon_{i,m} \tag{18}$$

Then, the following equalities can be obtained:

$$\begin{aligned}\dot{\bar{W}} = -\sum_{i=1}^{n}\Bigg(&\sum_{m=1}^{2}\left(h_{i,m}\varepsilon_{i,m}^T\varepsilon_{i,m} - \varepsilon_{i,m}z_{i,m}\mathrm{sign}\left(\varepsilon_{i,m}\right)\right) \\ &+ (d_i + b_i)\,\varepsilon_{i,1}^T\left(\alpha_i - \sigma_{i,1}\right) - (d_i + b_i)\,\varepsilon_{i,2}^T\varepsilon_{i,1} + (d_i + b_i)\,\varepsilon_{i,1}^T\varepsilon_{i,2}\Bigg)\end{aligned} \tag{19}$$

According to Lemma 2.2 and Lemma 2.3, the conclusion that $|(\alpha_i - \sigma_{i,1})| \leq \mu_{i,1}$ can be achieved in finite-time T_2, and then the following equalities can be further obtained:

$$\begin{aligned}\dot{\bar{W}} \leq & -\sum_{i=1}^{n}\sum_{m=1}^{2} h_{i,m}\varepsilon_{i,m}^T\varepsilon_{i,m} - \sum_{i=1}^{n}\sum_{m=1}^{2} z_{i,m}\left|\varepsilon_i\right| + \sum_{i=1}^{n}\left|d_i + b_i\right|\left|\varepsilon_i\right|\left|\alpha_i - \sigma_{i,1}\right| \\ \leq & -h_0\bar{W} - \left(z_0 - \sqrt{2N}\bar{\mu}_1\right)\bar{W}^{\frac{1}{2}}\end{aligned} \tag{20}$$

where $h_0 = 2\min(h_{i,m})$, $z_0 = \sqrt{2}\min(z_{i,m})$ and $\bar{\mu}_1 = \max\{(d_i + b_i)\mu_{i,1}\}$. Then, if $z_0 - \sqrt{2N}\bar{\mu}_1 > 0$ and the parameter z_0 is chosen appropriately according to Lemma 2.2, $\varepsilon_{i,m}$ will achieve stability in finite time T_3, where $T_3 \leq T_2 + \left[1/\left(h_0\left(1 - \frac{1}{2}\right)\right)\right]\ln\left[\left(h_0\bar{W}^{\frac{1}{2}}(T_2) + \left(z_0 - \sqrt{2N}\bar{\mu}_1\right)\right)/\left(z_0 - \sqrt{2N}\bar{\mu}_1\right)\right]$. From the above analysis, if the convergence time is chosen as: $t \geq T_4 = \max\{T_3, T_4\}$, the errors will satisfy: $|e_{i,m}| \leq \max\left\{\sqrt{\frac{c}{a_0}}, \sqrt{2\left(\frac{c}{2b_0}\right)^{\frac{2}{\gamma+1}}}\right\}$. Finally, tracking errors will converge to $|\eta_i - \eta_d| \leq \sqrt{N}\max\left\{\sqrt{\frac{c}{a_0}}, \sqrt{2\left(\frac{c}{2b_0}\right)^{\frac{2}{\gamma+1}}}\right\}/\Lambda_{\min}(H)$ in finite time $t \geq T_4$, where $\Lambda_{\min}(H)$ is the minimum singular value of H.

5. **Simulation Results.** The simulation of finite time command filtered controllers is conducted for the illustration of the above method. Choosing the architecture of 4 AUVs including 1 leader and 3 followers, communication relations of the system can be expressed in Figure 1. The matrices of the second-order multi-AUV system are chosen as follows: the interferences are $d_i = 0.01[\sin(0.3t), \cos(0.6t), \sin(0.05t)]$, the inertia matrix $M_i = \text{diag}\{1.77, 1.43, 1.47\}$, and water resistance $D_i = \text{diag}\{1.21 + 0.93|u_i|,\ 0.91 + 0.92|v_i|,\ 1.53 + 1.21|w_i|\}$. Other parameters of the transformation matrix and restoring force vector are selected as $\varphi_i = \pi/5$, $\theta_i = -\pi/10$, $\psi_i = \pi/12$, $W_i = 1.4$, and $B_i = 1.1$. Combining the communication topology, Laplacian matrices L and B are given as: $L = [0, 0, 0; -1, 1, 0; -1, 0, 1]$, $B = [1, 0, 0; 0, 0, 0; 0, 0, 0]$.

The initial states of each follower are defined as: $\eta_1(0) = [0.09,\ 0.57,\ 0.68]^T$, $\eta_2(0) = [-0.19,\ 0.37,\ 0.48]^T$, $\eta_3(0) = [0.41,\ 0.69,\ -0.09]^T$ and the initial acceleration $\ddot{\eta} = [0, 0, 0]$. Furthermore, choose appropriate controller parameters as: $h_{i,m} = 10$, $s_{i,m} = 20$, $z_{i,m} = 8$, $\gamma = \frac{3}{5}$, $K_{i,1} = 400$, $K_{i,2} = 8000$, $r_i = 1$, $k_i = 1$, $i = 1, 2, 3$. For the FLS, the centers of FLS activation function μ_i are distributed evenly in the range $[2 \times 2]$, the number of neurons is 10 and the width is all set to be 4. In order to observe the performance comparison between the proposed algorithm and the classical command-filtered [15], considering the tracking errors metric as the reference objects, it is defined as: $TEM = \sqrt{\sum_{i=1}^{n}\|e_{i,1}\|^2}$. Besides, choose different $h_{i,m}$ to illustrate the influence caused by parameters on the radiuses of tracking error regions.

Figure 1 shows the communication topology of the multi-AUV system. By using the proposed control method, it can obviously see the results of simulation in Figures 2-4, which display the trajectories of the leader η_d can be tracked well by followers η_i. Meanwhile, the intermediate signals are filtered and the compensation errors can converge to regions near the original point in setting time. From Figure 5, it can clearly see that larger $h_{i,m}$ can guarantee the system has faster convergence rate, and it can be proved that s_i have the same effect. Furthermore, the comparison is made in Figure 6 to better demonstrate the advantages of the finite time in distributed command filtered backstepping control method. The selection of parameters is the same as above approach whether in FLS or command filtered. It can obviously get the conclusion by testing several sets of the control parameters for conventional distributed command filtered backstepping control, the response curves of TEM will have a faster convergence speed under the set of control parameters with $h_{i,m} = 30$ by comparing with other sets of parameters in trail and error. Nevertheless, even in this case, the finite time approach under $h_{i,m} = 10$ has

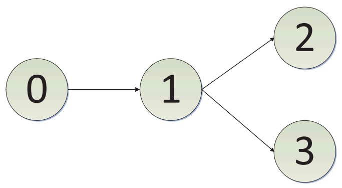

Figure 1. Communication topology

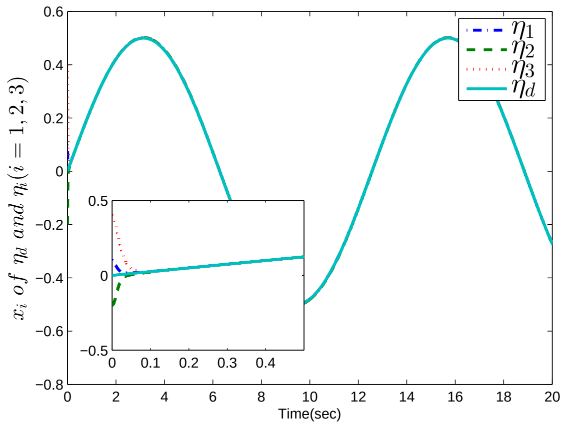

Figure 2. x_i of η_d and η_i. $i = 1, 2, 3$.

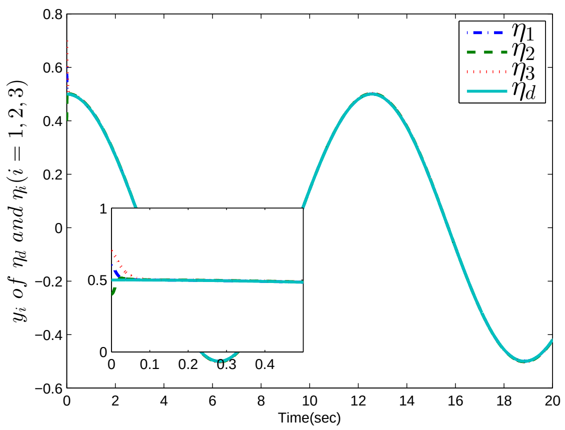

Figure 3. y_i of η_d and η_i. $i = 1, 2, 3$.

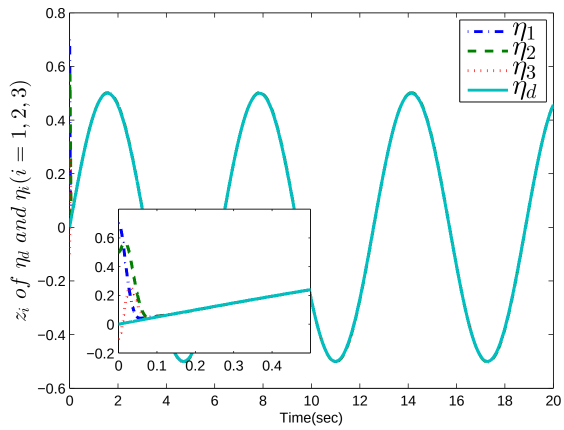

Figure 4. z_i of η_d and η_i. $i = 1, 2, 3$.

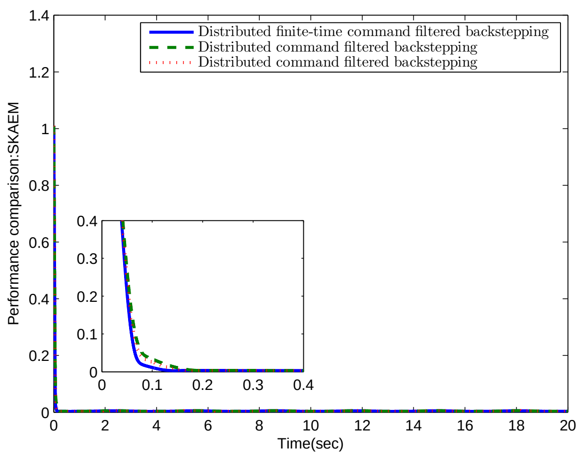

Figure 5. Comparison under different methods

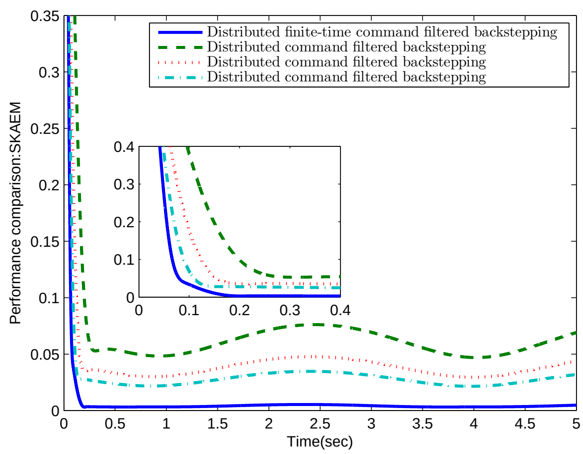

Figure 6. Comparison under different parameters

better tracking performance compared with any set of parameters mentioned above in unfinite time.

6. **Conclusions.** In this paper, a distributed control method based on FLS technique and command filtering has been proposed for multi-AUV system. In order to avoid the

unknown internal and external interferences, the algebraic graph theory and the FLS technique are integrated into the distributed controllers. Moreover, the command filtering is introduced to overcome the explosion of complexity condition appearing in backstepping. In addition, the error compensation mechanism is used by combining the finite-time technology. Then both the errors of compensation system or tracking errors can converge to a desired small neighborhood in finite time. The simulation results demonstrate the effectiveness of the proposed algorithm and and robustness of the multi-AUV system. The future research problems we considered include the algorithm simplification and practical application.

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