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NONLINEAR THRESHOLD EFFECTS BETWEEN STOCK INDEX RETURNS AND TRADING VOLUMES IN CHINA AND HONG KONG

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Abstract. *This study investigates the pre and post 2008 financial crisis nonlinear threshold effects between stock index returns and trading volumes in China and Hong Kong. The main challenge lies in designing a hybrid momentum random threshold GARCH (HMTAR-GARCH) model that could detect the effects of residual information on stock index returns, and that addresses the limitations of the traditional GJR-GARCH model. The empirical results show that the random threshold HMTAR-GARCH model outperforms the GARCH models without a threshold and with a zero threshold. Trading volumes have strong nonlinear and asymmetrical impact on the volatility of stock index returns in both China and Hong Kong under random thresholds. As such, this study provides investors with a reference for their investments for different thresholds amidst fluctuations in the stock market.*

Keywords: Nonlinear threshold, MTAR-GARCH, Stock index returns, Trading volumes

1. **Introduction.** The 2008 financial crisis has had the greatest impact on financial markets in recent history. Several studies have examined the markets pre and post 2008 in order to discuss the effects the financial crisis [1-3]. Some studies have examined the relationships between stock index returns, trading volumes, and volatility [4,5]. Most empirical results have shown a positive autocorrelation between index returns, trading volumes, and volatility, indicating their significant roles in the stock market [4,6]. Many studies have focused on the stock markets in China and Hong Kong [7-10]. China's economic reform has been accompanied by the development of a modern financial system. Several studies suggest that the returns in China's stock market are influenced primarily by regional developed markets, such as, those of Hong Kong and Taipei [8-10]. As such, we examine the nonlinear and asymmetrical relationships between the stock markets of China and Hong Kong using daily data for the period October 1, 2001 to June 30, 2017. Our results have important implications for effective investment and risk management.

The threshold autoregressive (TAR) and momentum threshold autoregressive (MTAR) models have been applied to some financial commodities [11-13]. Boucher applies TAR and MTAR procedures to detecting asymmetric short-run adjustments to the long-run equilibrium between stock index returns and fundamentals [11]. Several studies have applied threshold GARCH models to financial markets [6,14,15]. Koutmos applies the asymmetric autoregressive threshold TAR-GARCH model, finding that both the conditional mean and the conditional variance respond asymmetrically to past information [14]. Chen et al. find that financial markets respond positively to the US stock market [6]. Overall, these studies show an asymmetric adjustment toward a long-run equilibrium in financial markets [6,14].

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There are some shortcomings in previous studies, which estimate TAR and MTAR effects separately. This study attempts to integrate simultaneously the TAR and MTAR effects in the mean and variance equations so as to capture the asymmetric and nonlinear threshold effects of innovative shocks on returns and conditional volatility. Thus, a hybrid TAR-MTAR (HMTAR) GARCH model is proposed for measuring relationships between stock index returns and trading volumes in China and Hong Kong for the pre and post 2008 financial crisis. The remainder of the paper is organized as follows. Section 2 describes the data and methodology. The main empirical results are presented in Section 3. Finally, Section 4 concludes the paper.

2. Data and Methodology.

2.1. Data.

The data are collected from the *Taiwan Economic Journal*, for the period October 1, 2001, to June 30, 2017. The dataset includes 3,817 observations (pre-crisis 1,657; post-crisis 2,160) for the China stock index, and 3,883 observations (pre-crisis 1,691; post-crisis 2,192) for the Hong Kong stock index. Lagged trading volume growth (VG_{t-1}) is defined as $(VOL_{t-1} - VOL_{t-2})/VOL_{t-2}$, where VOL_{t-1} is the lagged stock trading volume. The stock index return (R) is defined as $(P_t - P_{t-1})/P_{t-1}$, where P_t is the stock index market.

2.2. Methodology.

2.2.1. *Unit-root tests.* This study tests for the stationarity property of each time series by applying the augmented Dickey-Fuller (ADF) test [16], Phillips-Perron (PP) test [17], and Kwiatkowski, Philips, Schmidt, and Shin (KPSS) test [18] for the unit root tests.

2.2.2. *Traditional TAR and MTAR models.* Following Enders and Siklos [19], the TAR and MTAR models are formulated as follows:

$$Y_{1t} = \alpha + \beta Y_{2t} + u_t \tag{1}$$

$$\Delta u_t = I_t \rho_1 u_{t-1} + (1 - I_t)\rho_2 u_{t-1} + \sum_{i=1}^{k} \rho_3 \Delta u_{t-i} + \varepsilon_t$$

$$\Delta u_t = M_t \rho_1 u_{t-1} + (1 - M_t)\rho_2 u_{t-1} + \sum_{i=1}^{k} \rho_3 \Delta u_{t-i} + \varepsilon_t, \tag{2}$$

where Y_{1t} and Y_{2t} represent two cointegrated variables at time t; α and β are the unknown parameters; u_t is the residual value; ε_t is a white-noise residual term; ρ_1, ρ_2 and ρ_3 represent regression coefficients; and I_t and M_t are indicator functions. We thus obtain the following (τ_1 and τ_2 are an unknown threshold to be simulated):

$$I_t = \begin{cases} 1, & \text{if } u_{t-1} \geq \tau_1 \\ 0, & \text{if } u_{t-1} < \tau_1. \end{cases} \tag{3}$$

Enders and Siklos [19] further proposed an MTAR model using the first difference of the residual series:

$$M_t = \begin{cases} 1, & \text{if } \Delta u_{t-1} \geq \tau_2 \\ 0, & \text{if } \Delta u_{t-1} < \tau_2. \end{cases} \tag{4}$$

Traditionally, Equations (3) and (4) are estimated separately. As an efficient estimator, a hybrid TAR and MTAR model that combines the two equations is specified in the next section.

2.2.3. *The proposed hybrid MTAR GARCH model (HMTAR-GARCH).* The mean and variance equations are proposed as follows. The TAR coefficients δ_3 and γ_2 measure the asymmetric impacts of positive or negative innovative shocks. The MTAR coefficients δ_4 and γ_3, representing the lagged trading volume growth differencing effect, describe the nonlinear reaction trend during the volatile period. The inclusion of lagged dependent variable terms could mitigate the autocorrelation problem and provides residuals for a white-noise process:

$$R_t = \delta_0 + \delta_1 \cdot R_{t-1} + \delta_2 \cdot VG_{t-1} + \delta_3 \cdot I_t \cdot VG_{t-1} + \delta_4 \cdot M_t \cdot VG_{t-1} + u_t \tag{5}$$

$$h_t = \alpha_0 + \sum_{k=1}^{p} \beta_k \cdot h_{t-k} + \sum_{m=1}^{q} \alpha_m \cdot u_{t-m}^2 + \gamma_1 \cdot VG_{t-1} + \gamma_2 \cdot I_t \cdot VG_{t-1} + \gamma_3 \cdot M_t \cdot VG_{t-1} \tag{6}$$

Here, R_t denotes the stock index returns at time t; u_t is the residual value; h_t is the conditional variance of u_t (also called "volatility of stock index returns"); and δ_0, δ_i, α_0, β_k, α_m, and γ_i are unknown parameters. When δ_4, γ_3 and τ_1 of Equation (3) are equal to zero, Equations (5) and (6) are reduced to the GJR-GARCH model [20].

3. **Empirical Results.** The sample comprises daily data, and the unit-root test results for the stock index returns of the ADF, PP, and KPSS tests provide strong evidence of stationarity.

The results of the three models given in Table 1 show that the HMTAR-GARCH models obtain the best minimum Akaike information criterion (AIC) and Bayesian information criterion (BIC). Model 1 is the traditional GARCH model without a threshold. Model 2 is the GJR-GARCH model with the TAR threshold (τ_1) and MTAR threshold (τ_2) both set to 0. Model 3 is the HMTAR model with random thresholds τ_1 and τ_2.

Table 1. Difference threshold models of performance

2008 Financial Crisis		Model 1 (GARCH)		Model 2 (GJR-GARCH)		Model 3 (HMTAR-GARCH)	
		China	Hong Kong	China	Hong Kong	China	Hong Kong
Pre	AIC	2020.7765	2384.4559	2206.3440	2282.4559	2013.0539	1863.9780
	BIC	2014.1869	2377.8866	2191.7268	2267.8596	1998.4582	1849.3508
	Log-like	−1003.3883	−1185.2279	−1092.1720	−1130.2279	−995.5269	−920.9890
Post	AIC	3539.0764	3002.2385	3326.7927	3033.5786	3258.5355	2969.0868
	BIC	3532.7524	2995.9292	3312.4476	3019.2485	3244.1823	2954.7307
	Log-like	−1762.5382	−1494.1192	−1652.3963	−1505.7893	−1618.2678	−1473.5434

When the lagged residuals are greater than or equal to the threshold τ_1, this study denotes this regime as a "relatively normal period". Alternatively, when the change in lagged residuals is greater than or equal to the threshold τ_2, this is a regime of "relatively normal movement". Table 2 shows the empirical results of models of China's and Hong Kong's stock market performance in the two periods of Model 3. For China's first (pre-2008 financial crisis) period, the mean equation indicates that the lagged trading volume (VG_{t-1}) did not affect stock index returns (R_t) at 0.05 significance level. For the variance equation, the results show that VG_{t-1} has a negative effect (−3.4890%) on volatility of stock index returns (h_t). When the stock market experiences a relatively normal period ($u_{t-1} \geq \tau_1$, $\tau_1 = -2.4081\%$), VG_{t-1} has a positive effect (4.4263%) on the stock index returns of h_t. When the market experiences relatively normal movement ($\Delta u_{t-1} \geq \tau_2$, $\tau_2 = 2.1146\%$), VG_{t-1} has a positive effect (1.3589%) on h_t. In China's second period (post-2008 financial crisis), the mean equation reveals that, similarly to the pre-crisis period, the market experiences relatively normal movement ($\Delta u_{t-1} \geq \tau_2$, $\tau_2 = -3.1672\%$), where VG_{t-1} has a negative effect (−0.0203%) on R_t. For the variance equation, when the

Table 2. HMTAR estimation for stock index returns – Trading volume (%) of Model 3

2008 Financial Crisis			China		Hong Kong	
Pre			$\tau_1 = -2.4081$;	$\tau_2 = 2.1146$	$\tau_1 = 1.4407$;	$\tau_2 = -3.8528$
		Variable	Coeff.	t-value	Coeff.	t-value
	Mean Equation	Intercept(δ_0)	-0.0587*	-2.3877	-0.0721**	-2.7979
		$R_{t-1}(\delta_1)$	0.0324	1.0822	0.0237	0.7743
		$VG_{t-1}(\delta_2)$	2.4040	1.4610	2.5048	1.4720
		$I_t \cdot VG_{t-1}(\delta_3)$	-2.7158	-1.6488	1.1232**	2.7372
		$M_t \cdot VG_{t-1}(\delta_4)$	0.0977	0.2089	-2.8951	-1.7007
	Variance Equation	Intercept(α_0)	0.0399***	3.6410	0.0300**	3.2147
		$h_{t-1}(\beta_1)$	0.8225***	48.2513	0.8498***	54.3267
		$u^2_{t-1}(\alpha_1)$	0.1297***	8.7993	0.1056***	7.7051
		$VG_{t-1}(\gamma_1)$	-3.4890**	-2.5781	-6.7923**	-3.2402
		$I_t \cdot VG_{t-1}(\gamma_2)$	4.4263***	3.2940	-1.1520***	-4.9113
		$M_t \cdot VG_{t-1}(\gamma_3)$	1.3589***	3.9172	7.6012**	3.2601
Post			$\tau_1 = -4.1633$;	$\tau_2 = -3.1672$	$\tau_1 = 0.4651$;	$\tau_2 = 9.2684$
		Variable	Coeff.	t-value	Coeff.	t-value
	Mean Equation	Intercept(δ_0)	0.0209	0.8169	-0.0170	-0.6690
		$R_{t-1}(\delta_1)$	-0.0030	-0.1207	0.0471	1.8975
		$VG_{t-1}(\delta_2)$	0.0066	0.4994	-0.5577***	-7.5182
		$I_t \cdot VG_{t-1}(\delta_3)$	0.0137	0.8250	0.8111***	5.2480
		$M_t \cdot VG_{t-1}(\delta_4)$	-0.0203*	-2.0138	-10.5883	-1.4979
	Variance Equation	Intercept(α_0)	0.0054*	2.0667	0.0195***	2.8004
		$h_{t-1}(\beta_1)$	0.9522***	247.6067	0.8936***	77.3707
		$u^2_{t-1}(\alpha_1)$	0.0466***	10.6992	0.0956***	8.8777
		$VG_{t-1}(\gamma_1)$	-0.0066	-1.1557	0.2305**	3.1064
		$I_t \cdot VG_{t-1}(\gamma_2)$	0.0191**	2.8726	-0.1065	-0.8311
		$M_t \cdot VG_{t-1}(\gamma_3)$	-0.0120***	-3.4882	28.4593***	4.4157

Note: ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

stock market experiences a relatively normal period ($u_{t-1} \geq \tau_1$, $\tau_1 = -4.1633\%$), VG_{t-1} has a positive effect (0.0191%) on h_t. When the stock market experiences relatively normal movement ($\Delta u_{t-1} \geq \tau_2$, $\tau_2 = -3.1672\%$), VG_{t-1} has a negative effect (-0.0120%) on h_t. This indicates that the lagged trading volume of China's stock market has predictive power for the stock index returns and its volatility under a random threshold of a relatively normal period and movement.

Hong Kong's results differ from those of China. The autoregressive nature of the pre-crisis period is significant in the first period of the mean equation. When the market is in a relatively normal period ($u_{t-1} \geq \tau_1$, $\tau_1 = 1.4407\%$), VG_{t-1} has a positive effect (1.1232%) on R_t. For the variance equation, VG_{t-1} has a negative effect (-6.7923%) on h_t. When the market is in a relatively normal period ($u_{t-1} \geq \tau_1$, $\tau_1 = 1.4407\%$), VG_{t-1} has a negative effect (-1.1520%) on h_t. When the stock market experiences relatively normal movement ($\Delta u_{t-1} \geq \tau_2$, $\tau_2 = -3.8528\%$), VG_{t-1} also has a positive effect (7.6012%) on h_t. In the post-crisis period, the mean equation shows that VG_{t-1} has a negative effect (-0.5577%) on R_t. When the market is in a relatively normal period ($u_{t-1} \geq \tau_1$, $\tau_1 = 0.4651\%$), VG_{t-1} has a positive effect (0.8111%) on R_t. However, for the variance equation, the VG_{t-1} has a positive effect (0.2305%) on h_t, in contrast to the pre-crisis period. When the market experiences relatively normal movement ($\Delta u_{t-1} \geq \tau_2$, $\tau_2 = 9.2684\%$), VG_{t-1} has a positive effect (28.4593%) on h_t. This result of post-crisis period has stronger affect than the pre-crisis period, and indicates that lagged trading volume has predictive power for that period's conditional volatility of stock index returns under a random threshold of relatively normal movement.

The results show that in China's stock market, after investors had faced a poor market, they returned to more normal investment behavior and responded to market changes in the expected ways. This result could be attributed to the fact that, after experiencing the 2008 financial crisis, investors' illogical behavior was replaced by normal investment behavior.

For Hong Kong, the lagged trading volume post the 2008 financial crisis had a greater volatility effect than it did pre the crisis. This indicates that, for Hong Kong, when lagged trading volume increases, investors expect this trend to persist, which encourages them to continue increasing their investments in search of higher stock index returns.

Finally, the above TAR effects are similar to those of several related studies [4,8]. In addition, the HMTAR effects for China and Hong Kong's stock markets show that the trading volume affects index returns and its volatility asymmetrically and nonlinearly.

4. **Conclusions.** This study applied a hybrid TAR and MTAR GARCH model to capturing the nonlinear and asymmetrical relationship between stock index returns and trading volumes in China's and Hong Kong's stock markets. The empirical results show from AIC and BIC, that the random threshold HMTAR-GARCH model outperforms the GARCH models without a threshold and with a zero threshold. Both of the China and Hong Kong, the trading volume has greater effect on stock index returns and its volatility. In addition, the TAR and MTAR volatility effects exist in both markets and in pre and post financial crisis, except for TAR effect in Hong Kong during post financial crisis period.

Investors were relatively attentive to the adjustment of trading volume, i.e., smaller effect on the volatility of the stock index returns in China after financial crisis. However, greater volatility effect was observed in Hong Kong stock market. This implies that trading volume in different markets have varying impacts on stock index returns and their volatilities. These results on trading volumes to stock index returns showing asymmetric and nonlinear threshold relationships provide a reference that could help investors adjust their stock market investments effectively. In further research, we plan to focus on bivariate or triple HMTAR-GARCH to capture asymmetric spillover effects between or among international spot and derivative markets.

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