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A COMPROMISE CONFLICT RESOLUTION APPROACH TO THE UNCERTAIN TRANSPORTATION PROBLEM BASED ON FUZZY SET THEORY

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Abstract. *The aim of this study is to explore the interval transportation problem with multiple objectives and develop a compromise conflict resolution approach based on the fuzzy set theory. The two objectives of the multi-objective transportation problem are to minimize the delivery time and to maximize the profit. The study considers the left and the right bounds and the center of given intervals. The center and the left of profit maximization interval can be considered as the expected value and the uncertainty of an interval respectively. This order relation represents the decision maker's preference for the alternative with the higher expected value and less uncertainty. The center and the right of delivery time minimization interval can be considered as the expected value and the uncertainty of an interval respectively. The order relation represents the decision maker's preference for the alternative with the lower expected value and less uncertainty. This study determines the optimal compromise solution of multi-objective interval transportation problem with profit and delivery time objectives by using fuzzy programming algorithm. A numerical example has been provided to illustrate the solution procedure.*
Keywords: Multi-objective interval transportation problem (MITP), Compromise conflict resolution, Fuzzy set theory, Profit maximization, Delivery time minimization

1. **Introduction.** Transportation problem (TP) is a category of linear programming problem which could be solved by applying the simplex technique's simplified version referred to as transportation method. Due to its main use in solving problems that include a certain number of sources of products and a certain number of product destinations, this kind of problem is often referred to as the transportation problem. The basic transportation problem was originally developed by Hitchcock [1]. A transportation problem involves particular origins, for example, factories where products are produced, and a demanded quantity of these manufactured products are supplied to a particular number of destinations. It must be ensured that this is done in a way that maximizes profit or minimizes cost. Therefore, there are manufacturing places as origins and supply places as destinations. In some cases, the origins are called sources whereas the destinations are referred to as sinks. El-Wahed [2] studied a multi-objective transportation problem (MOTP) under fuzziness. As a solution algorithm to the multi-objective non-interval transportation problem, he adopted a fuzzy programming approach and implemented a linear membership function. As regards the studies on the solution of the multiple-objective interval transportation problem (MITP), Keshavarz and Khorram [3] studied

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a fuzzy bi-criteria transportation problem. They used bi-level programming approach to solve it. In their study, the left bound of interval is minimized whereas the right bound is maximized. Furthermore, Ishibuchi and Tanaka [4] introduced the multi-objective programming in optimization of the interval objective function. They explained the order relation which represents the decision maker's preference between interval profit by the right limit, the left limit, the center and the width of an interval. Moreover, Kagade and Bajaj [5] applied the fuzzy method for solving the multi-objective assignment problem with interval cost. They used a hyperbolic membership function for the cost objective. The bounds considered are center and the right bound. Patel and Dhodiya [6] undertook to solve the MITP using the grey situation decision-making theory based on grey numbers to maximize and minimize the objectives. They dealt with the left and right bounds of intervals. This study proposes a fuzzy set-theoretic approach to MITP from the conservative perspective of a decision maker. The method of structural optimization used for solving transport problems and developed by Karganov [7], helps to avoid these shortcomings and find a compromise solution. There were different researches regarding this topic. However, many researchers wanted to apply to the real world, which resulted in developing the TP with multiple objectives. Due to insufficient information regarding the exact value of the objective, there were studies where these objectives are represented by interval numbers. Our solution is based on the fuzzy programming technique which has been programmed and analyzed in MATLAB. The solution approach gives a compromise conflict resolution when increasing the number of objectives and constraints. This feature makes the fuzzy approach more practical than the other approaches such as interactive procedures in solving MITP. Moreover, the approach which we used solves MITP by linear membership function to get the optimal compromise solution. It is easy to implement to solve similar linear multi-objective programming problems.

The remainder of the paper is organized as follows. Section 2 introduces development of MITP with profit maximization and delivery time minimization. Section 3 provides the solution methodology for the MITP. A numerical example problem is carried out with a proposed model from the Russian coal industry in Section 4. The conclusion appears in Section 5.

2. **Development of Multi-Objective Interval Transportation Model.** We constructed the multi-objective interval transportation model with two objectives. Our task is to find the optimal solution to the problem of delivering the available volume of supply to meet demands, where we need to minimize transportation time, which is very important in every logistic system and at the same time maximize the transportation profit. Furthermore, an interval transportation problem constructs the data of supply, demand and objective functions such as cost or other objectives in some intervals. Unlike the standard transportation problem, where cost of transportation is usually fixed at one rate for a delivery of products from certain source to certain destination, in this study we use the concept of intervals for our objectives, because in reality these objectives may change within some range due to many external factors. Depending on these factors, the unit transportation cost can vary from one number to another, which can be represented as an interval:

$$A = [a_L, a_R] = \{a\colon a_L \leq a \leq a_R, a \in R\}$$

where a_L and a_R are, respectively, the left and right limits of A.

Mathematical interval programming models deal with uncertainty and interval coefficients. Interval transportation problems construct the data of supply, demand and objective functions such as cost, and time in some intervals. This problem can be converted into a classical transportation problem by using the concept of right limit, half-width, left limit, and center of an interval. The MITP model is designed to find a solution to a transportation problem with multiple objectives at the same time. Typically, it seeks to achieve the objective of minimizing total cost as one of its several objectives. Other objectives could be a quantity of goods delivered, the safety of delivery, delivery time, etc. The following is a mathematical formulation of an MITP:

$$\textbf{Minimize } z^1 = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c^1_{L_{ij}}, c^1_{R_{ij}}\right] x_{ij} \tag{1}$$

$$\textbf{Minimize } z^k = \sum_{i=1}^{m}\sum_{j=1}^{n}\left[c^k_{L_{ij}}, c^k_{R_{ij}}\right] x_{ij} \quad \text{where } k = 1, 2, \ldots, K \tag{2}$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = b_j,\ j = 1, 2, \ldots, n \quad \text{for all } j \tag{3}$$

$$\sum_{j=1}^{n} x_{ij} = a_i,\ i = 1, 2, \ldots, m \quad \text{for all } i \tag{4}$$

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

$$x_{ij} \geq 0 \quad \text{for all } i \text{ and } j \tag{5}$$

where $\left[c^k_{L_{ij}}, c^k_{R_{ij}}\right]$ $(k = 1, 2, \ldots, K)$ is an interval representing the uncertain cost for the transportation problem. In the MITP a product is to be transported from m sources to n destinations and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$, respectively. In addition, there is a penalty C_{ij} associated with transporting a unit of product from source i to destination j. This penalty may be cost or delivery time, etc. A decision variable x_{ij} represents the unknown quantity to be shipped from source i to destination j. Here, $c_{L_{ij}} = \left(c^1_{L_{ij}}, c^2_{L_{ij}}, \ldots, c^k_{L_{ij}}\right)$ and, $c_{R_{ij}} = \left(c^1_{R_{ij}}, c^2_{R_{ij}}, \ldots, c^k_{R_{ij}}\right)$ represent, respectively, the left bound and right bound of C_{ij} which represents the coefficients related to variable for objective k. Without loss of generality, the following notations are also used: m – number of sources; n – number of destinations; a_i – supply at source i.

The profit and delivery time objectives can be considered as the maximization or minimization of the worst and the average case respectively.

Order relations for profit maximization problem

This order relation $\leq_{LR}$ represents the decision maker's preference for the alternative with the higher minimum profit and maximum profit. There are many pairs of intervals which cannot be compared by left and right bounds. In order to represent the intuition, the order relation by the center and width of the interval is defined

$$f_L(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\Omega_{C_{ij}}x_{ij} - \sum_{i=1}^{m}\sum_{j=1}^{n}\Omega_{W_{ij}}x_{ij} \qquad f_C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\Omega_{ij}x_{ij} \tag{6}$$

Since the center and the width of the interval can be considered as the expected value and the uncertainty of an interval respectively, this order relation represents the decision maker's preference for the alternative with the higher expected value and less uncertainty.

Order relations for delivery time minimization problem

$$f_R(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}t_{C_{ij}}x_{ij} + \sum_{i=1}^{m}\sum_{j=1}^{n}t_{W_{ij}}x_{ij} \qquad f_C(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}t_{ij}x_{ij} \tag{7}$$

The order relation $<_{CW}$ represents the decision maker's preference for the alternative with the lower expected value and less uncertainty, that is, if $A <_{CW} B$, then interval A is preferred to interval B.

3. **Solution Methodology.** In order to determine the compromise solution of MITP with profit and delivery time objectives, we used a fuzzy set approach. Firstly, assign, for each objective, two values U^k and L^k as upper and lower bounds, respectively, for the kth objective.

- L^k is the aspired level of achievement for the objective k
- U^k is the highest acceptable level for achievement for the objective k
- $d^k = U^k - L^k$ is the degradation allowance for the objective k

Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. The main steps of the fuzzy programming technique are as follows [5].

Step 1. Pick the 1st objective function and solve it as a single objective transportation problem. Continue this process k times for k different objective functions.

Step 2. Evaluate the kth objective function at the K optimal solutions ($k = 1, 2, \ldots, K$). For each objective function, determine L^k and U^k from set of optimal solutions.

Step 3. Define the membership function

$$\mu_k\left(F^k(x)\right) = \begin{cases} 1 & \text{if } F^k(x) \leq L_k, \\ \dfrac{U_k - F^k(x)}{U_k - L_k} & \text{if } L^k < F^k(x) < U_k, \\ 0 & \text{if } F^k(x) \geq U_k, \end{cases} \tag{8}$$

where $L_k \neq U_k$, $k = 1, 2, \ldots, K$. If $L_k = U_k$, then $\mu_k\left(F^k(x)\right) = 1$ for any value of k.

Step 4. Construct the fuzzy programming problem and its equivalent linear programming (LP) problem.

Max $\min_{k=1,2,\ldots,K} \mu_k\left(F^k(x)\right)$ Subject to $\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \ldots, m$, $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, \ldots, n$, $x_{ij} \geq 0$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$.	Max β(*Auxiliary variable*) Subject to $\beta \leq \mu_k\left(F^k(x)\right)$, $\beta\left(U_k - L_k\right) \leq \left(U_k - F^k(x)\right)$ $k = 1, 2, \ldots, K$, $\beta\left(U_k - L_k\right) + F^k(x) \leq U_k$, $\dfrac{\beta\left(U_k - L_k\right)}{U_k} + \left(\dfrac{1}{U_k}\right) F^k(x) \leq 1$ $\sum_{j=1}^{n} x_{ij} = a_i$, $i = 1, 2, \ldots, m$, $\sum_{i=1}^{m} x_{ij} = b_j$, $j = 1, 2, \ldots, n$, $0 \leq \beta \leq 1$, $x_{ij} \geq 0 \quad \forall i, j$.

Step 5. Solve linear programming (LP) by using an integer programming technique to get an optimal solution and evaluate the K objective functions at this compromise solution.

4. **Numerical Example.** An illustrative example is carried out to explain the proposed approach and to evaluate its appropriateness. However, it reflects an empirical study for a Russian coal company. Russia is the 3rd largest country for coal reserves in the global coal industry. It accounts for 27% of the whole world's coal reserves. Moreover, Russia is the 6th largest country for coal production with an annual production of 250 million tons of coal. Russia mainly exports coal to four directions: Far East (to countries in Asia such as China, Korea, and Japan), North Europe, West Europe through Baltic Sea and Black Sea (to countries such as Turkey). Based on transportation delivery time and profit, the coal company should decide to which directions and in which quantity it is better to sell coal. Figure 1 shows locations of coal mines in Russia and the export destinations.

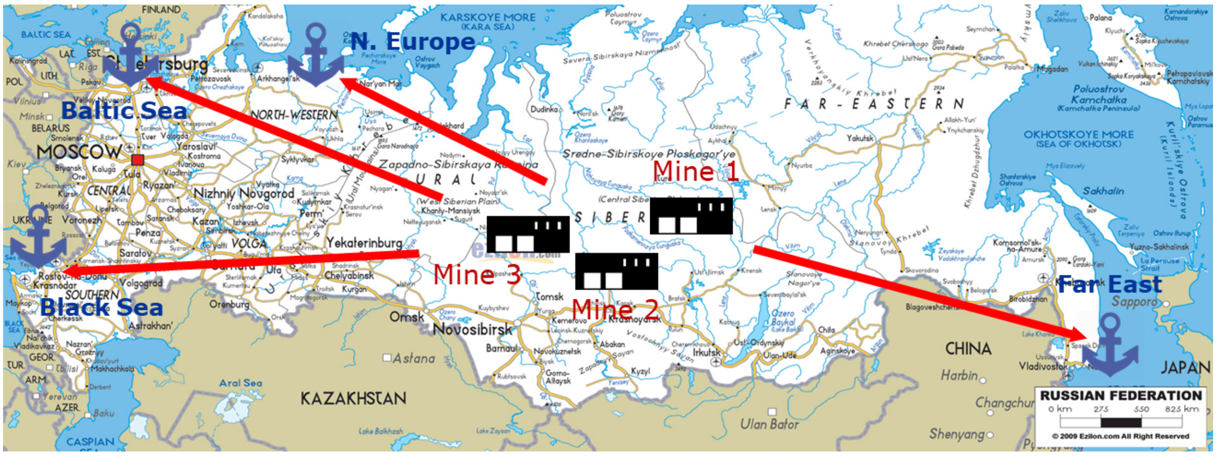

FIGURE 1. The locations of coal mines in Russia and the export destinations

The input data is collected form Patel and Dhodiya's study [6] for further comparison. Tables 1 and 2 show data on delivery times and profit, respectively. Note that MT is denoted as million ton.

TABLE 1. 1st objective: minimization of delivery time

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	$[1, 2]$	$[1, 3]$	$[5, 9]$	$[4, 8]$	8
Mine 2	$[1, 2]$	$[7, 10]$	$[2, 6]$	$[3, 5]$	19
Mine 3	$[7, 9]$	$[7, 11]$	$[3, 5]$	$[5, 7]$	17
Demand (MT)	11	3	14	16	

TABLE 2. 2nd objective: maximization of profit

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	$[3, 5]$	$[2, 6]$	$[2, 4]$	$[1, 5]$	8
Mine 2	$[4, 6]$	$[7, 9]$	$[7, 10]$	$[9, 11]$	19
Mine 3	$[4, 8]$	$[1, 3]$	$[3, 6]$	$[1, 2]$	17
Demand (MT)	11	3	14	16	

The solution to this MITP will be found using the fuzzy programming approach and it consists of the following steps.

Step 1. Each objective function needs to be solved as a single-objective transportation problem (right bound for delivery time and left bound for profit). For the first objective, minimization of delivery time, the right bounds of intervals are taken as it is illustrated in Table 3. For the second objective of profit maximization, the left bounds of intervals are considered, which is shown in Table 4.

TABLE 3. Right bound for the 1st objective

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	2	3	9	8	8
Mine 2	2	10	6	5	19
Mine 3	9	11	5	7	17
Demand (MT)	11	3	14	16	

Table 4. Left bound for the 2nd objective

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	3	2	2	1	8
Mine 2	4	7	7	9	19
Mine 3	4	1	3	1	17
Demand (MT)	11	3	14	16	

Table 5. Center for the 1st objective

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	1,5	2	7	6	8
Mine 2	1,5	8,5	4	4	19
Mine 3	8	9	4	6	17
Demand (MT)	11	3	14	16	

Table 6. Center for the 2nd objective

	Far East	Baltic Sea	Black Sea	N. Europe	Supply (MT)
Mine 1	4	4	3	3	8
Mine 2	5	8	8,5	10	19
Mine 3	6	2	4,2	1,5	17
Demand (MT)	11	3	14	16	

Furthermore, every objective function needs to be solved as a single-objective transportation problem for the center. Tables 5 and 6 incorporate the center of intervals. Center for each cell is found as an average of left and right bounds of intervals.

Then each of these 4 matrixes can be solved as a standard transportation problem.

Step 2. This step comprises determination of each objective's upper and lower bounds (L^k and U^k) according to the set of optimal solutions.

$$F^1\left(X^{tR}\right) = (187, 187, 226, 281), \quad F^2\left(X^{tC}\right) = (149, 149, 182, 229),$$
$$F^3\left(X^{pL}\right) = (207, 207, 243, 243), \quad F^4\left(X^{pC}\right) = (260, 260, 303, 306).$$

From the above pay-off matrix, the following lower and upper limits are established, i.e.:

$$149 \le F^1\left(X^{tR}\right) \le 260; \quad 149 \le F^2\left(X^{tC}\right) \le 260;$$
$$182 \le F^3\left(X^{pL}\right) \le 303; \quad 229 \le F^4\left(X^{pC}\right) \le 306,$$

where $F^1\left(X^{tR}\right)$ is the objective function for the time with right bound, $F^2\left(X^{tC}\right)$ is the objective function for the time with the center, $F^3\left(X^{pL}\right)$ is the objective function for the profit with left bound, and $F^4\left(X^{pC}\right)$ is the objective function for the profit with center.

Step 3. The next task is the definition of the membership function μ of objective functions. The upper and lower bounds of each objective function can be written as follows:

$$\mu_1\left(F^1\left(X^{tR}\right)\right) = \begin{cases} 1 & \text{if } F^1(x) \le 149 \\ \dfrac{260 - F^1(x)}{111} & \text{if } 149 < F^1(x) < 260 \\ 0 & \text{if } F^1(x) \ge 260 \end{cases} \tag{9}$$

$$\mu_2\left(F^2\left(X^{tC}\right)\right) = \begin{cases} 1 & \text{if } F^2(x) \le 149 \\ \dfrac{260 - F^2(x)}{111} & \text{if } 149 < F^2(x) < 260 \\ 0 & \text{if } F^2(x) \ge 260 \end{cases} \tag{10}$$

$$\mu_3\left(F^3\left(X^{pL}\right)\right) = \begin{cases} 1 & \text{if } F^3(x) \leq 182 \\ \dfrac{303 - F^3(x)}{121} & \text{if } 182 < F^3(x) < 303 \\ 0 & \text{if } F^3(x) \geq 303 \end{cases} \tag{11}$$

$$\mu_4\left(F^4\left(X^{pC}\right)\right) = \begin{cases} 1 & \text{if } F^4(x) \leq 229 \\ \dfrac{306 - F^4(x)}{77} & \text{if } 229 < F^4(x) < 306 \\ 0 & \text{if } F^4(x) \geq 306 \end{cases} \tag{12}$$

Step 4. The following thing to do is the construction of the fuzzy programming problem and its equivalent LP (linear programming) problem. By introducing an auxiliary variable β, fuzzy programming problem can be transformed into the following equivalent linear programming (LP) problem: problem: x_{ij} – amount of detail j type (for j consumer), produced on i tool (delivered by i supplier).

$$Max \quad \beta \tag{13}$$

Subject to

$$x_{11} + x_{12} + x_{13} + x_{14} = 8 \tag{14}$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 19 \tag{15}$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 17 \tag{16}$$
$$x_{11} + x_{21} + x_{31} = 11 \tag{17}$$
$$x_{12} + x_{22} + x_{32} = 3 \tag{18}$$
$$x_{13} + x_{23} + x_{33} = 14 \tag{19}$$
$$x_{14} + x_{24} + x_{34} = 16 \tag{20}$$
$$\begin{aligned} &2x_{11} + 3x_{12} + 9x_{13} + 8x_{14} + 2x_{21} + 10x_{22} + 6x_{23} + 5x_{24} + 9x_{31} \\ &+ 11x_{32} + 5x_{33} + 7x_{34} + 111\beta \leq 260 \end{aligned} \tag{21}$$
$$\begin{aligned} &1.5x_{11} + 2x_{12} + 7x_{13} + 6x_{14} + 1.5x_{21} + 8.5x_{22} + 4x_{23} + 4x_{24} + 8x_{31} \\ &+ 9x_{32} + 4x_{33} + 6x_{34} + 111\beta \leq 260 \end{aligned} \tag{22}$$
$$\begin{aligned} &3x_{11} + 2x_{12} + 2x_{13} + x_{14} + 4x_{21} + 7x_{22} + 7x_{23} + 9x_{24} + 4x_{31} + x_{32} \\ &+3x_{33} + x_{34} + 121\beta \leq 303 \end{aligned} \tag{23}$$
$$\begin{aligned} &4x_{11} + 4x_{12} + 3x_{13} + 3x_{14} + 5x_{21} + 8x_{22} + 8.5x_{23} + 10x_{24} + 6x_{31} \\ &+ 2x_{32} + 4.5x_{33} + 1.5x_{34} + 77\beta \leq 306 \end{aligned} \tag{24}$$

Step 5. The last step is to solve this linear programming problem to get an optimal solution and evaluate the objective functions of delivery time and profit. In this study, Matlab has been used for programming the problem. After solving the problem the following results have been obtained as $X = [4, 3, 0, 1, 7, 0, 0, 12, 0, 0, 14, 3]$ in Table 7.

The function values for each objective are delivery time $= [151, 190]$ and Profit $=$ $[200, 254]$, respectively. At this time the overall satisfaction β becomes 0.6306.

Table 7. The compromise solution for MITP

	Far East	Baltic Sea	Black Sea	N. Europe	Supply
Mine 1	4	3	0	1	8
Mine 2	7	0	0	12	19
Mine 3	0	0	14	3	17
Demand	11	3	14	16	

5. **Conclusion.** This study proposed a compromise conflict resolution approach to the transportation problem with multiple objectives, namely, by minimizing the delivery time and maximizing the profit. The problem was converted into a classical multi-objective transportation problem where in order to achieve the delivery time objective the right limit and center of the interval are minimized, and to achieve the profit objective the left limit and center of the interval are maximized. These objective functions are considered as the minimization or maximization of the worst case and the average case. This compromise solution was found based on the fuzzy set theory and analyzed in MATLAB. In addition, the model was simulated for the case from Russian coal company, which shows the compromise solution is more acceptable in a real-world situation when more than one objective needs to be achieved in transporting a product. Further studies can be conducted to provide comparisons between this study and other previous approaches previous and to apply it to solving the real-world transportation problem.

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