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EXPONENTIAL PASSIVE FILTERING FOR A CLASS OF DISCRETE-TIME GENETIC REGULATORY NETWORKS WITH DISCRETE AND DISTRIBUTED DELAYS

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Abstract. *The exponential passive filtering problem for discrete-time genetic regulatory networks with random and distributed delays is investigated. By introducing an appropriate Lyapunov function and Linear Matrix Inequalities (LMIs), a sufficient condition is obtained to ensure the filtering error system is strictly exponentially passive with dissipation $\gamma > 0$. A numerical example is presented to illustrate the effectiveness of the theoretical result.*
Keywords: Exponential passive filtering, Genetic Regulatory Networks (GRNs), Random delays, Distributed delays

1. **Introduction.** Genetic Regulatory Networks (GRNs), structured by networks of regulatory interactions among DNA, mRNA, proteins inhibiting the expression of other genes in order to gain insight into the underlying processes of living systems at the molecular level, have become a crucially important new area of research issue in the biological and biomedical sciences. Recently, a number of different mathematical modelings for GRNs have been proposed. In general, genetic network models can be classified into two types, that is, the discrete model [1, 2] and the continuous model [3, 4]. From these two types of models, biologists hope to obtain actual concentrations of gene products (mRNA and proteins).

In practice, the steady-state values (concentrations of gene products) of the actual GRNs can hardly be obtained. In order to obtain the steady-state values based on available measurement date, the filtering technology has been introduced to solve these problems [5, 6]. A number of filters for functional differential equation models have been considered by some scholars (see, e.g., [7, 8, 9]). Although many results of filtering for GRNs have been proposed, they just consider the continuous-time GRNs model. However, due to the requirement for computer simulation, the design of filter for delayed discrete-time GRNs is of vital importance (see, for instance, [10, 11, 12] and references therein). The set-values filtering for a class of discrete-time GRNs with time-varying parameters, constant delays, and bounded external noise is investigated in [10]. In [11, 12], authors have designed a filter ensuring that the filtering error system is stochastically stable and has a prescribed H_∞ performance. In [13], a class of discrete-time GRNs with parameter uncertainties, time delays, molecular noise and missing values is considered. A set-membership filtering method is proposed to estimate the states of the underlying GRNs. So far, to the author's knowledge, the exponential passive filtering problem of GRNs has little been studied in the literature. This situation motivates the present investigation.

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In this paper, we will design an exponentially passive filter for a class of GRNs with discrete and distributed delays. Based on passivity theory [14] and Lyapunov stability theory [15], a sufficient LMI condition is first established to ensure that the filtering error system is strictly exponentially passive with dissipation γ. Finally, a numerical example is given to show the effectiveness of the proposed approach.

Notation. $E\{\cdot\}$ denotes the expectation operator with respect to some probability measure; $\|\cdot\|$ represents the Euclidean norm for a vector, and the spectral norm for a matrix; $\lambda_{\max}(A)$ (respectively, $\lambda_{\min}(A)$) denotes the largest (respectively, smallest) eigenvalue of A; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $N[a,b] = \{a, a+1, \ldots, b\}$; $\mathrm{diag}(\cdot)$ denotes the diagonal matrix.

2. **Problem Formulation.** Consider a class of discrete-time GRNs with discrete and distributed delays, which can be described as:

$$\begin{cases} x_m(k+1) = Ax_m(k) + Bg(x_p(k-d(k))) + Cg(x_p(k)) + D\displaystyle\sum_{m=1}^{d_1} g(x_p(k-m)) \\ \qquad\qquad + E_1 w(k), \\ x_p(k+1) = Fx_p(k) + Gx_m(k-d(k)) + F_1 v(k), \\ y_m(k) = A_1 x_m(k) + C_1 g(x_p(k)) + E_2 w(k), \\ y_p(k) = A_2 x_p(k) + F_2 v(k), \\ z_m(k) = G_1 x_m(k) + E_3 w(k), \\ z_p(k) = G_2 x_p(k) + F_3 v(k), \\ x_m(k) = \theta_m(k),\ x_p(k) = \theta_p(k),\ k \in N[-d, 0], \end{cases} \tag{1}$$

where $x_m(k) = [x_{m1}(k) \ldots x_{mn}(k)]^T$, $x_p(k) = [x_{p1}(k) \ldots x_{pn}(k)]^T$, where $x_{mi}(k)$ and $x_{pi}(k) \in \mathbb{R}$ are the concentrations of mRNA and protein of the ith gene, respectively; $y_m(k) = [y_{m1}(k) \ldots y_{mq_1}(k)]^T$ and $y_p(k) = [y_{p1}(k) \ldots y_{pq_2}(k)]^T$ represent the expression levels of m-RNAs and proteins, respectively; $z_m(k) = [z_{m1}(k) \ldots z_{ml_1}(k)]^T$ and $z_p(k) = [z_{p1}(k) \ldots z_{pl_2}(k)]^T$ are the signals to be estimated; both $w(k)$ and $v(k)$ are exogenous disturbance signals; $\theta_m(k)$ and $\theta_p(k)$ are the initial conditions of $x_m(k)$ and $x_p(k)$, respectively; $g(x(k)) = [g_1(x_1(k)) \ldots g_n(x_n(k))]^T$, where $g_i(x_i(k))$ denotes the activation function of the ith gene; A, B, C, D, F, G, A_1, A_2, C_1, E_1, E_2, E_3, F_1, F_2, F_3, G_1 and G_2 are constant matrices of appropriate sizes; $d(k)$ denotes the random time delay of mRNAs and proteins, and is assumed to be a Markov chain with state space $\mathcal{N} := \{1, 2, \ldots, d_2\}$; d_1 describes the distributed time delay, $d = \max\{d_1, d_2\}$; $\pi := [\pi_{ij}]$ the transition probability matrix of $d(k)$, where $\pi_{ij} = \mathrm{Prob}\{d(k+1) = j | d(k) = i\}$.

Usually, in complex GRNs, only part of the information of the network components can be obtained. Therefore, to obtain the true states of the GRNs, we need to estimate them from available measurements [16]. The full order linear filter to be designed takes the following form

$$\begin{cases} \hat{x}_m(k+1) = A_f \hat{x}_m(k) + B_f y_m(k), \\ \hat{x}_p(k+1) = C_f \hat{x}_p(k) + D_f y_p(k), \\ \hat{z}_m(k) = G_{1f} \hat{x}_m(k) + H_{1f} y_m(k), \\ \hat{z}_p(k) = G_{2f} \hat{x}_p(k) + H_{2f} y_p(k), \\ \hat{x}_m(k) = 0,\ \hat{x}_p(k) = 0,\ k \in N[-d, 0], \end{cases} \tag{2}$$

where $\hat{x}_m(k)$, $\hat{x}_p(k)$, $\hat{z}_m(k)$ and $\hat{z}_p(k)$ are the estimates of $x_m(k)$, $x_p(k)$, $z_m(k)$ and $z_p(k)$, respectively; A_f, B_f, C_f, D_f, G_{1f}, G_{2f}, H_{1f} and H_{2f} are unknown filter parametric matrices of appropriate dimensions.

Denote

$$\tilde{x}_m(k) = \begin{bmatrix} x_m(k) \\ \hat{x}_m(k) \end{bmatrix},\ \tilde{x}_p(k) = \begin{bmatrix} x_p(k) \\ \hat{x}_p(k) \end{bmatrix},\ e_m(k) = z_m(k) - \hat{z}_m(k),\ e_p(k) = z_p(k) - \hat{z}_p(k).$$

Then the filtering error system can be expressed as

$$\begin{cases} \tilde{x}_m(k+1) = \bar{A}\tilde{x}_m(k) + \bar{B}g(Z\tilde{x}_p(k-d(k))) + \bar{C}g(Z\tilde{x}_p(k)) \\ \qquad + \bar{D}\sum_{m=1}^{d_1} g(Z\tilde{x}_p(k-m)) + \bar{E}_1 w(k), \\ \tilde{x}_p(k+1) = \bar{F}\tilde{x}_p(k) + \bar{G}Z\tilde{x}_m(k-d(k)) + \bar{F}_1 v(k), \\ e_m(k) = \bar{G}_{1f}\tilde{x}_m(k) + \bar{C}_1 g(Z\tilde{x}_p(k)) + \bar{H}_{1f} w(k), \\ e_p(k) = \bar{G}_{2f}\tilde{x}_p(k) + \bar{H}_{2f} v(k), \\ \tilde{x}_m(k) = \tilde{\theta}_m(k),\ \tilde{x}_p(k) = \tilde{\theta}_p(k),\ k \in N[-d, 0], \end{cases} \tag{3}$$

where

$$\tilde{\theta}_m(k) = \begin{bmatrix} \theta_m(k) \\ 0 \end{bmatrix},\ \tilde{\theta}_p(k) = \begin{bmatrix} \theta_p(k) \\ 0 \end{bmatrix},\ \bar{A} = \begin{bmatrix} A & 0 \\ B_f A_1 & A_f \end{bmatrix},\ \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} C \\ B_f C_1 \end{bmatrix},\ \bar{D} = \begin{bmatrix} D \\ 0 \end{bmatrix},\ \bar{F} = \begin{bmatrix} F & 0 \\ D_f A_2 & C_f \end{bmatrix},\ \bar{E}_1 = \begin{bmatrix} E_1 \\ B_f E_2 \end{bmatrix},\ \bar{F}_1 = \begin{bmatrix} F_1 \\ D_f F_2 \end{bmatrix},$$

$$\bar{G} = \begin{bmatrix} G \\ 0 \end{bmatrix},\ \bar{C}_1 = -H_{1f}C_1,\ \bar{G}_{1f} = \begin{bmatrix} G_1 - H_{1f}A_1 & -G_{1f} \end{bmatrix},$$

$$\bar{G}_{2f} = \begin{bmatrix} G_2 - H_{2f}A_2 & -G_{2f} \end{bmatrix},\ \bar{H}_{1f} = E_3 - H_{1f}E_2,\ \bar{H}_{2f} = F_3 - H_{2f}F_2,\ Z = \begin{bmatrix} I & 0 \end{bmatrix}.$$

For convenience, for a nonnegative integer k we define

$$\Theta_k = \{\tilde{x}_m(k), \tilde{x}_m(k-1), \ldots, \tilde{x}_m(k-d_2), \tilde{x}_p(k), \tilde{x}_p(k-1), \ldots, \tilde{x}_p(k-d_2)\}.$$

Assumption 2.1. *The activation functions g_i $(i = 1, 2, \ldots, n)$ satisfy,*

$$g_i(0) = 0,\ 0 \le \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \le l_i,\ \forall s_1, s_2 \in \mathbb{R},\ s_1 \ne s_2,$$

where l_i is a given constant.

The next definitions and lemmas are introduced, which will be used in the proof of the following theorems.

Definition 2.1. *The filtering error system (3) with $w(k) = 0$ and $v(k) = 0$ is said to be exponentially stable if there exist two constants $\alpha > 0$ and $0 < \beta < 1$ such that*

$$\begin{aligned} & E\left\{ \| \tilde{x}_m(K) \|^2 + \| \tilde{x}_p(K) \|^2 \middle| \Theta_0, d(0) \right\} \\ & \le \alpha\beta^K \sup_{s\in N[-d,0]} E\left\{ \| \tilde{x}_m(s) \|^2 + \| \tilde{x}_p(s) \|^2 \middle| \Theta_0, d(0) \right\},\ \forall K \ge 0 \end{aligned} \tag{4}$$

for every initial mode $d(0)$ and every initial state Θ_0.

Definition 2.2. *The filtering error system (3) is said to be strictly exponentially passive with $\gamma > 0$, if system (3) is exponentially stable when $w(k) = 0$ and $v(k) = 0$, and under the zero initial condition, the following relation holds*

$$\begin{aligned} & 2\sum_{k=0}^{K} E\left\{ \begin{bmatrix} e_m(k) \\ e_p(k) \end{bmatrix}^T \begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \middle| \Theta_0, d(0) \right\} \\ & \ge -\gamma \sum_{k=0}^{K} E\left\{ \begin{bmatrix} w(k) \\ v(k) \end{bmatrix}^T \begin{bmatrix} w(k) \\ v(k) \end{bmatrix} \middle| \Theta_0, d(0) \right\},\ \forall K \ge 0 \end{aligned} \tag{5}$$

for all nonzero $w(k)$ or $v(k)$.

Lemma 2.1. *For given a pair of positive integers d and T, and a pair of sequences $\{a_k\}$ and $\{b_i\}$, if all of the following sums are well defined, then the following statements (i) and (ii) are true.*

$$(i)\ \sum_{k=0}^{T-1}\sum_{i=k-d}^{k-1} a_k b_i = \sum_{i=-d}^{-1}\sum_{k=0}^{i+d} a_k b_i + \sum_{i=0}^{T-d-1}\sum_{k=i+1}^{i+d} a_k b_i + \sum_{i=T-d}^{T-2}\sum_{k=i+1}^{T-1} a_k b_i,\ \text{when } T \geq d;$$

$$(ii)\ \sum_{k=0}^{T-1}\sum_{i=k-d}^{k-1} a_k b_i = \sum_{i=-d}^{T-1-d}\sum_{k=0}^{i+d} a_k b_i + \sum_{i=T-d}^{-1}\sum_{k=0}^{T-1} a_k b_i + \sum_{i=0}^{T-2}\sum_{k=i+1}^{T-1} a_k b_i,\ \text{when } T < d.$$

The proof of Lemma 2.1 is omitted, since it can be easily presented by changing the order of summation.

3. Stability Analysis and Passivity Filter Design.

An exponential stability criterion for the filtering error system (3) with $w(k) = 0$ and $v(k) = 0$ is presented in the following theorem.

Theorem 3.1. *The filtering error system (3) with $w(k) = 0$ and $v(k) = 0$ is exponentially stable, if there exist matrices $P_i^T(r) = P_i(r) > 0$, $Q_i^T(r) = Q_i(r) > 0$ $(i = 1, 2, 3,\ r \in \mathcal{N})$, $P_j^T = P_j > 0$ $(j = 2, 3)$, $Q_j^T = Q_j > 0$ $(j = 2, 3, 4)$, $\varsigma := \mathrm{diag}(\varsigma_1, \varsigma_2, \dots, \varsigma_n) > 0$ and $\mu := \mathrm{diag}(\mu_1, \mu_2, \dots, \mu_n) > 0$ such that the following matrix inequalities hold,*

$$\Omega(r) := \tilde{\Omega}(r) + \hat{\Omega}(r) < 0,\ r \in \mathcal{N}, \tag{6}$$

$$\bar{P}_i(r) < P_i,\ \bar{Q}_i(r) < Q_i,\ i = 2, 3,\ r \in \mathcal{N}, \tag{7}$$

where

$$\hat{\Omega}(r) = \Lambda_1^T \bar{P}_1(r)\Lambda_1 + \Lambda_2^T \phi_1(r)\Lambda_2 + \Lambda_3^T \bar{Q}_1(r)\Lambda_3,\ \tilde{\Omega}(r) = \sum_{i=1}^{3}[\Omega_{mi}(r) + \Omega_{pi}(r)] + \sum_{i=4}^{5}\Omega_{pi},$$

$$\Lambda_1 = \bar{A}e_1 + \bar{B}e_5 + \bar{C}e_3 + \bar{D}e_6,\ \Lambda_2 = \Lambda_1 - e_1,\ \Lambda_3 = \bar{F}e_2 + \bar{G}Ze_4,$$

$$\Omega_{m1}(r) = -e_1^T P_1(r)e_1,\ \Omega_{m3}(r) = -\frac{1}{r}\tilde{e}_1^T \tilde{P}_3(r)\tilde{e}_1,\ \Omega_{p4} = d_1 e_3^T Q_4 e_3 - \frac{1}{d_1} e_6^T Q_4 e_6,$$

$$\Omega_{m2}(r) = e_1^T\left[\bar{P}_2(r) + (d_2 - 1)P_2\right]e_1 - e_4^T\left(P_2(r) + \bar{P}_2(r) - P_2\right)e_4 - \frac{1}{r}e_8^T\left(P_2 - \bar{P}_2(r)\right)e_8,$$

$$\Omega_{p1}(r) = -e_2^T Q_1(r)e_2,\ \Omega_{p3}(r) = (e_7 - e_3)^T\phi_2(r)(e_7 - e_3) - \frac{1}{r}\tilde{e}_3^T\tilde{Q}_3(r)\tilde{e}_3,$$

$$\Omega_{p2}(r) = e_3^T\left[\bar{Q}_2(r) + (d_2 - 1)Q_2\right]e_3 - e_5^T\left(Q_2(r) + \bar{Q}_2(r) - Q_2\right)e_5 - \frac{1}{r}e_9^T\left(Q_2 - \bar{Q}_2(r)\right)e_9,$$

$$\Omega_{p5} = -e_3^T\varsigma e_3 - e_2^T Z^T\varsigma L e_3 - e_3^T L\varsigma Z e_2 - e_7\mu e_7 - \Lambda_3^T Z^T\mu L e_7 - e_7^T L\mu Z\Lambda_3,$$

$$L = \frac{1}{2}\mathrm{diag}(-l_1, -l_2, \dots, l_n),\ \bar{P}_i(r) = \sum_{s=1}^{d_2}\pi_{rs}P_i(s),\ \bar{Q}_i(r) = \sum_{s=1}^{d_2}\pi_{rs}Q_i(s),\ i = 1, 2, 3,$$

$$e_i = \begin{bmatrix} 0_{2n\times(i-1)2n} & I_{2n} & 0_{2n\times(9-i)2n}\end{bmatrix}^T,\ i = 1, 2, 4, 8,$$

$$e_i = \begin{bmatrix} 0_{n\times(i-1)n} & I_n & 0_{n\times(9-i)n}\end{bmatrix}^T,\ i = 3, 5, 6, 7, 9,$$

$$\tilde{e}_1 = \begin{bmatrix} e_1 - e_4 \\ \frac{r-1}{r+1}e_1 + e_4 - \frac{2}{r+1}e_8\end{bmatrix},\ \tilde{P}_3(r) = \begin{bmatrix} P_3 + P_3(r) & 0 \\ 0 & 3(P_3 + P_3(r))\end{bmatrix},$$

$$\tilde{e}_3 = \begin{bmatrix} e_3 - e_5 \\ \frac{r-1}{r+1}e_3 + e_5 - \frac{2}{r+1}e_9\end{bmatrix},\ \tilde{Q}_3(r) = \begin{bmatrix} Q_3 + Q_3(r) & 0 \\ 0 & 3(Q_3 + Q_3(r))\end{bmatrix},$$

$$\phi_1(r) = d_2\bar{P}_3(r) + \frac{d_2^2 + d_2}{2}P_3,\ \phi_2(r) = d_2\bar{Q}_3(r) + \frac{d_2^2 + d_2}{2}Q_3.$$

Proof: Choose a Lyapunov functional candidate for the filtering error system (3) with $w(k) = 0$ and $v(k) = 0$ as follows:

$$V(\Theta_k, k, d(k)) = \sum_{i=1}^{3} V_{m,i}(\Theta_k, k, d(k)) + \sum_{i=1}^{4} V_{p,i}(\Theta_k, k, d(k)) \tag{8}$$

with

$$V_{m,1}(\Theta_k, k, d(k)) = \tilde{x}_m^T(k) P_1(d(k)) \tilde{x}_m(k),\ V_{p,1}(\Theta_k, k, d(k)) = \tilde{x}_p^T(k) Q_1(d(k)) \tilde{x}_p(k),$$

$$V_{m,2}(\Theta_k, k, d(k)) = \sum_{i=k-d(k)}^{k-1} \tilde{x}_m^T(i) P_2(d(k)) \tilde{x}_m(i) + \sum_{j=-d_2+1}^{-1} \sum_{i=k+j}^{k-1} \tilde{x}_m^T(i) P_2 \tilde{x}_m(i),$$

$$\begin{aligned} V_{p,2}(\Theta_k, k, d(k)) = & \sum_{i=k-d(k)}^{k-1} g^T(Z\tilde{x}_p(i)) Q_2(d(k)) g(Z\tilde{x}_p(i)) \\ & + \sum_{j=-d_2+1}^{-1} \sum_{i=k+j}^{k-1} g^T(Z\tilde{x}_p(i)) Q_2 g(Z\tilde{x}_p(i)), \end{aligned}$$

$$V_{m,3}(\Theta_k, k, d(k)) = \sum_{j=-d(k)}^{-1} \sum_{i=k+j}^{k-1} \eta^T(i) P_3(d(k)) \eta(i) + \sum_{j=-d_2}^{-1} \sum_{l=j}^{-1} \sum_{i=k+l}^{k-1} \eta^T(i) P_3 \eta(i),$$

$$V_{p,3}(\Theta_k, k, d(k)) = \sum_{j=-d(k)}^{-1} \sum_{i=k+j}^{k-1} \zeta^T(i) Q_3(d(k)) \zeta(i) + \sum_{j=-d_2}^{-1} \sum_{l=j}^{-1} \sum_{i=k+l}^{k-1} \zeta^T(i) Q_3 \zeta(i),$$

$$V_{p,4}(\Theta_k, k, d(k)) = \sum_{j=-d_1}^{-1} \sum_{i=k+j}^{k-1} g^T(Z\tilde{x}_p(i)) Q_4 g(Z\tilde{x}_p(i)),$$

$$\eta(k) = \tilde{x}_m(k+1) - \tilde{x}_m(k),\ \zeta(k) = g(Z\tilde{x}_p(k+1)) - g(Z\tilde{x}_p(k)).$$

By taking the forward difference of the functional $V(\Theta_k, k, d(k))$ along with the trajectories of system (3), one can obtain that $\Delta V(k) = E\{V(\Theta_{k+1}, k+1, d(k+1)) \mid \Theta_k, d(k) = r\} - V(\Theta_k, k, r)$.

$$\Delta V_{m1}(k) = \xi^T(k) \left(\Lambda_1^T \bar{P}_1(r) \Lambda_1 + \Omega_{m1}(r) \right) \xi(k). \tag{9}$$

Additionally, it can be verified from (7) and discrete Wirtinger-based inequality that

$$\Delta V_{m2}(k) \leq \xi^T(k) \Omega_{m2}(r) \xi(k), \tag{10}$$

$$\Delta V_{m3}(k) \leq \xi^T(k) \left(\Lambda_2^T \phi_1(r) \Lambda_2 + \Omega_{m3}(r) \right) \xi(k), \tag{11}$$

$$\Delta V_{p1}(k) = \xi^T(k) \left(\Lambda_3^T \bar{Q}_1(r) \Lambda_3 + \Omega_{p1}(r) \right) \xi(k), \tag{12}$$

$$\Delta V_{p2}(k) \leq \xi^T(k) \Omega_{p2}(r) \xi(k), \tag{13}$$

$$\Delta V_{p3}(k) \leq \xi^T(k) \Omega_{p3}(r) \xi(k), \tag{14}$$

$$\Delta V_{p4}(k) \leq \xi^T(k) \Omega_{p4} \xi(k). \tag{15}$$

In view of Assumption 2.1, we can derive that

$$\xi^T(k) \Omega_{p4} \xi(k) \geq 0. \tag{16}$$

Now, from (9)-(16), we derive

$$\Delta V(k) \leq \xi^T(k) \Omega(r) \xi(k), \tag{17}$$

where

$$\xi(k) = \Big[\tilde{x}_m^T(k) \quad \tilde{x}_p^T(k) \quad g^T(Z\tilde{x}_p(k)) \quad \tilde{x}_m^T(k-r) \quad g^T(Z\tilde{x}_p(k-r)) \quad \sum_{m=1}^{d_1} g^T(Z\tilde{x}_p(k-m)) \quad g^T(Z\tilde{x}_p(k+1)) \quad \sum_{i=k-r}^{k-1} \tilde{x}_m^T(i) \quad \sum_{i=k-r}^{k-1} g^T(Z\tilde{x}_p(i))\Big]^T.$$

Due to (6), Formula (17) results in

$$\Delta V(k) \leq -\lambda_{\min}(-\Omega)\left\{\| \tilde{x}_m(k) \|^2 + \| \tilde{x}_p(k) \|^2\right\}, \tag{18}$$

where $\lambda_{\min}(-\Omega) = \min_{r\in\mathcal{N}}(-\lambda_{\min}(\Omega(r)))$.

On the other hand, it follows from (8) that

$$\begin{aligned} V(\Theta_k, k, r) \leq\ & \rho_1 \left\{\| \tilde{x}_m(k) \|^2 + \| \tilde{x}_p(k) \|^2\right\} + \rho_2 \sum_{i=k-d}^{k-1} \left\{\| \tilde{x}_m(i) \|^2 + \| \tilde{x}_p(i) \|^2\right\} \\ & + \rho_3 \sum_{i=k-d}^{k-1} \left\{\| \tilde{x}_m(i+1) \|^2 + \| \tilde{x}_p(i+1) \|^2\right\}, \end{aligned} \tag{19}$$

where $\lambda_{\max}(\cdot)$ denotes the maximal eigenvalue of matrix,

$$\varphi_1 = \lambda_{\max}(P_2(r)) + (d_2-1)\lambda_{\max}(P_2) + \varphi_2,\ \varphi_2 = 2d_2\lambda_{\max}(P_3(r)) + (d_2^2+d_2)\lambda_{\max}(P_3),$$

$$\begin{aligned} \varphi_3 =\ & 4\lambda_{\max}(Q_2(r))\lambda_{\max}\left(L^TL\right) + 4(d_2-1)\lambda_{\max}(Q_2)\lambda_{\max}\left(L^TL\right) + \varphi_4 \\ & + 4d_1\lambda_{\max}(Q_4)\lambda_{\max}\left(L^TL\right), \end{aligned}$$

$$\varphi_4 = 8d_2\lambda_{\max}(Q_3(r))\lambda_{\max}\left(L^TL\right) + 4\left(d_2^2+d_2\right)\lambda_{\max}(Q_3)\lambda_{\max}\left(L^TL\right),$$

$$\rho_1 = \max\{\lambda_{\max}(P_1(r)), \lambda_{\max}(Q_1(r))\},\ \rho_2 = \max\{\varphi_1, \varphi_3\},\ \rho_3 = \max\{\varphi_2, \varphi_4\}.$$

For a scalar $\theta > 1$, it follows from (18) and (19) that

$$\begin{aligned} & \theta^{k+1}E\{V(\Theta_{k+1}, k+1, d(k+1)) \mid \Theta_k, d(k) = r\} - \theta^k V(\Theta_k, k, r) \\ \leq\ & \left(\theta^k(\theta-1)\rho_1 - \theta^{k+1}\lambda_{\min}(-\Omega)\right)\left(\| \tilde{x}_m(k) \|^2 + \| \tilde{x}_p(k) \|^2\right) \\ & + \rho_2\theta^k(\theta-1)\sum_{i=k-d}^{k-1}\left(\| \tilde{x}_m(i) \|^2 + \| \tilde{x}_p(i) \|^2\right) \\ & + \rho_3\theta^k(\theta-1)\sum_{i=k-d}^{k-1}\left(\| \tilde{x}_m(i+1) \|^2 + \| \tilde{x}_p(i+1) \|^2\right). \end{aligned} \tag{20}$$

Since

$$\begin{aligned} & E\{E\{V(\Theta_{k+1}, k+1, d(k+1)) \mid \Theta_k, d(k)\} \mid \Theta_0, d(0)\} \\ =\ & E\{V(\Theta_{k+1}, k+1, d(k+1)) \mid \Theta_0, d(0)\}, \end{aligned}$$

by taking the conditional expectation $E\{\cdot|\Theta_0, d(0)\}$, and then summing from $k = 0$ to $k = K-1$ on both sides of (20), we obtain

$$\begin{aligned} & \theta^K E\{V(\Theta_K, K, d(K)) \mid \Theta_0, d(0)\} - V(\Theta_0, 0, d(0)) \\ \leq\ & [(\theta-1)\rho_1 - \theta\lambda_{\min}(-\Omega)]\sum_{k=0}^{K-1}\theta^k E\left\{\| \tilde{x}_m(k) \|^2 + \| \tilde{x}_p(k) \|^2 | \Theta_0, d(0)\right\} \\ & + (\theta-1)\rho_2\sum_{k=0}^{K-1}\sum_{i=k-d}^{k-1}\theta^k E\left\{\| \tilde{x}_m(i) \|^2 + \| \tilde{x}_p(i) \|^2 | \Theta_0, d(0)\right\} \\ & + (\theta-1)\rho_3\sum_{k=0}^{K-1}\sum_{i=k-d}^{k-1}\theta^k E\left\{\| \tilde{x}_m(i+1) \|^2 + \| \tilde{x}_p(i+1) \|^2 | \Theta_0, d(0)\right\}. \end{aligned} \tag{21}$$

Due to Lemma 2.1, it is easy to compute that

$$\begin{aligned}&\sum_{k=0}^{K-1}\sum_{i=k-d}^{k-1}\theta^k E\left\{\|\ \tilde{x}_m(i)\ \|^2+\|\ \tilde{x}_p(i)\ \|^2|\ \Theta_0,d(0)\right\}\\ &\le d^2\theta^d\sup_{s\in N[-d,0]}E\left\{\|\ \tilde{x}_m(s)\ \|^2+\|\ \tilde{x}_p(s)\ \|^2|\ \Theta_0,d(0)\right\}\\ &\quad+d\theta^d\sum_{i=0}^{K-2}\theta^i E\left\{\|\ \tilde{x}_m(i)\ \|^2+\|\ \tilde{x}_p(i)\ \|^2|\ \Theta_0,d(0)\right\}.\end{aligned}\tag{22}$$

Similarly, we have

$$\begin{aligned}&\sum_{k=0}^{K-1}\sum_{i=k-d}^{k-1}\theta^k E\left\{\|\ \tilde{x}_m(i+1)\ \|^2+\|\ \tilde{x}_p(i+1)\ \|^2|\ \Theta_0,d(0)\right\}\\ &\le d^2\theta^d\sup_{s\in N[-d,0]}E\left\{\|\ \tilde{x}_m(s)\ \|^2+\|\ \tilde{x}_p(s)\ \|^2|\ \Theta_0,d(0)\right\}\\ &\quad+d\theta^d\sum_{i=1}^{K-1}\theta^i E\left\{\|\ \tilde{x}_m(i)\ \|^2+\|\ \tilde{x}_p(i)\ \|^2|\ \Theta_0,d(0)\right\}.\end{aligned}\tag{23}$$

From (19), we obtain

$$V(\Theta_0,0,d(0))\le(\rho_1+\rho_2 d+\rho_3 d)\sup_{s\in N[-d,0]}E\left\{\|\ \tilde{x}_m(s)\ \|^2+\|\ \tilde{x}_p(s)\ \|^2|\ \Theta_0,d(0)\right\}.\tag{24}$$

It follows from (21)-(24) that

$$\begin{aligned}\theta^K E\{V(\Theta_K,K,d(K))\mid\Theta_0,d(0)\}\le L_1(\theta)&\sup_{s\in N[-d,0]}E\left\{\|\ \tilde{x}_m(s)\ \|^2+\|\ \tilde{x}_p(s)\ \|^2\right\}\\ &+L_2(\theta)\sum_{i=0}^{K-1}\theta^i E\left\{\|\ \tilde{x}_m(i)\ \|^2+\|\ \tilde{x}_p(i)\ \|^2\right\},\end{aligned}\tag{25}$$

where

$$L_1(\theta)=\rho_1+\rho_2 d+\rho_3 d+(\theta-1)\rho_2 d(d+1)\theta^d+(\theta-1)\rho_3 d(d+1)\theta^d,$$
$$L_2(\theta)=(\theta-1)\rho_1-\theta\lambda_{\min}(-\Omega)+(\theta-1)\rho_2 d\theta^d+(\theta-1)\rho_3 d\theta^d.$$

Since $L_2(1)<0$, by the continuity of $L_2(\theta)$ we can choose a scalar $\hbar>1$ such that $L_2(\hbar)\le 0$. Obviously, $L_1(\hbar)>0$. From (25), we have

$$\hbar^K E\{V(\Theta_k,k,d(k))\mid\Theta_0,d(0)\}\le L_1(\hbar)\sup_{s\in N[-d,0]}E\left\{\|\ \tilde{x}_m(s)\ \|^2+\|\ \tilde{x}_p(s)\ \|^2\right\}.\tag{26}$$

From (8), we can obtain

$$E\{V(\Theta_K,K,d(K))\mid\Theta_0,d(0)\}\ge\bar{\rho_1}E\left\{\|\ \tilde{x}_m(K)\ \|^2+\|\ \tilde{x}_p(K)\ \|^2|\ \Theta_0,d(0)\right\},\tag{27}$$

where

$$\bar{\rho_1}=\min\{\lambda_{\min}(P_1(1)),\lambda_{\min}(Q_1(1)),\ldots,\lambda_{\min}(P_1(d_2)),\lambda_{\min}(Q_1(d_2))\}.$$

Let $\alpha=\frac{L_1(\hbar)}{\bar{\rho_1}}$ and $\beta=\frac{1}{\hbar}$. Then $\alpha>0$ and $0<\beta<1$. It follows from (26) and (27) that

$$\begin{aligned}&E\left\{\|\ \tilde{x}_m(K)\ \|^2+\|\ \tilde{x}_p(K)\ \|^2\Big|\ \Theta_0,d(0)\right\}\\ &\le\alpha\beta^T\sup_{s\in N[-d,0]}E\left\{\|\ \tilde{x}_m(s)\ \|^2+\|\ \tilde{x}_p(s)\ \|^2\Big|\ \Theta_0,d(0)\right\}.\end{aligned}$$

Therefore, by Definition 2.1, the delayed discrete-time filter error system (3) with $w(k)=0$ and $v(k)=0$ is exponentially stable, which completes the proof.

Remark 3.1. *Compared with* [18, Theorem 3.1], *we use the so-called Wirtinger's inequality (see* [19, Lemma 1]*) instead of the Jensen's inequality to estimate the forward differences of* $V_{m,3}(\Theta_k, k, d(k))$ *and* $V_{p,3}(\Theta_k, k, d(k))$, *which can obtain less conservative results.*

Similar to [18, Theorem 3.2], the following result can be easily obtained, which gives a method to design the filter in the form of (2).

Theorem 3.2. *For given a scalar* $\gamma > 0$ *and a pair of positive integers* d_1 *and* d_2, *if for each* $r \in \mathcal{N}$, *there exist matrices* $P_i^T(r) = P_i(r) > 0$, $Q_i^T(r) = Q_i(r) > 0$ $(i = 1, 2, 3)$, $P_j^T = P_j > 0$ $(j = 2, 3)$, $Q_j^T = Q_j > 0$ $(j = 2, 3, 4)$,

$$R_k := \begin{bmatrix} R_{k1} & R_{k2} \\ R_{k3} & R_{k2} \end{bmatrix}^T, \ \det R_{k2} \neq 0, \ k = 1, 2,$$

$\varsigma := \mathrm{diag}(\varsigma_1, \varsigma_2, \ldots, \varsigma_n) > 0$, $\mu := \mathrm{diag}(\mu_1, \mu_2, \ldots, \mu_n) > 0$, $\bar{A}_f$, $\bar{B}_f$, $\bar{C}_f$, $\bar{D}_f$, G_{1f}, H_{1f}, G_{2f} *and* H_{2f}, *such that the following LMIs (29) and (30) hold, then the filtering error system (3) is strictly exponentially passive with dissipation* $\gamma > 0$. *Moreover, the desired filter is given by (2) with*

$$A_f = R_{12}^{-1}\bar{A}_f, \ B_f = R_{12}^{-1}\bar{B}_f, \ C_f = R_{22}^{-1}\bar{C}_f, \ D_f = R_{22}^{-1}\bar{D}_f. \tag{28}$$

$$\begin{bmatrix} \Upsilon_{11}(r) & 0 & 0 & \Upsilon_{14} \\ * & \Upsilon_{22}(r) & 0 & \Upsilon_{24} \\ * & * & \Upsilon_{33}(r) & \Upsilon_{34} \\ * & * & * & \Upsilon_{44}(r) \end{bmatrix} < 0, \tag{29}$$

$$\bar{Q}_j(r) < Q_j, \ \bar{P}_j(r) < P_j, \ (j = 2, 3), \tag{30}$$

where

$$\Upsilon_{11}(r) = \bar{P}_1(r) - R_1 - R_1^T, \ \Upsilon_{22}(r) = \phi_1(r) - R_1 - R_1^T, \ \Upsilon_{33}(r) = \bar{Q}_1(r) - R_2 - R_2^T,$$

$$\Upsilon_{14} = R_1^T\Psi_1 + \left(Z + \tilde{Z}\right)^T \left(\bar{B}_f\Psi_2 + \bar{A}_f\Psi_3\right), \ \Upsilon_{24} = R_1^T\Psi_4 + \left(Z + \tilde{Z}\right)^T \left(\bar{B}_f\Psi_2 + \bar{A}_f\Psi_3\right),$$

$$\Upsilon_{34} = R_2^T\Psi_5 + \left(Z + \tilde{Z}\right)^T \left(\bar{D}_f\Psi_6 + \bar{C}_f\Psi_7\right), \ Z = \begin{bmatrix} I & 0 \end{bmatrix}, \ \tilde{Z} = \begin{bmatrix} 0 & I \end{bmatrix},$$

$$\Psi_1 = \begin{bmatrix} AZ & 0 & C & 0 & B & D & 0 & 0 & 0 & E_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_2 = \begin{bmatrix} A_1Z & 0 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 & E_2 & 0 \end{bmatrix},$$

$$\Psi_3 = \begin{bmatrix} \tilde{Z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_4 = \begin{bmatrix} (A - I)Z & 0 & C & 0 & B & D & 0 & 0 & 0 & E_1 & 0 \\ -\tilde{Z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_5 = \begin{bmatrix} 0 & FZ & 0 & GZ & 0 & 0 & 0 & 0 & 0 & 0 & F_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_6 = \begin{bmatrix} 0 & A_2Z & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & F_2 \end{bmatrix},$$

$$\Psi_7 = \begin{bmatrix} 0 & \tilde{Z} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{44}(r) = \begin{bmatrix} \tilde{\Omega}(r) & \bar{\Phi}_1 & \bar{\Phi}_2 \\ * & -\gamma I - \bar{H}_{1f}^T - \bar{H}_{1f} & 0 \\ * & * & -\gamma I - \bar{H}_{2f}^T - \bar{H}_{2f} \end{bmatrix},$$

$$\bar{\Phi}_1 = \begin{bmatrix} -\bar{G}_{1f} & 0 & -\bar{C}_1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$$\bar{\Phi}_2 = \begin{bmatrix} 0 & -\bar{G}_{2f} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

and $\bar{P}_i(r)$, $\bar{Q}_i(r)$, $\tilde{\Omega}(r)$, $\bar{C}_1$, $\phi_1(r)$, $\bar{G}_{1f}$, $\bar{G}_{2f}$, $\bar{H}_{1f}$ *and* $\bar{H}_{2f}$ *are defined as noted previously.*

4. **Illustrative Example.** In this section, we will test the theoretical results of this paper by an example.

Example 4.1. *Consider GRN (1) with parameters as follows:*

$$A=\begin{bmatrix}0.3679&0&0\\0&0.3679&0\\0&0&0.3679\end{bmatrix},\ B=\begin{bmatrix}0&0&-0.126\\-0.126&0&0\\0&-0.126&0\end{bmatrix},$$

$$C=\begin{bmatrix}0.1&0&0.1\\0&-0.1&0.2\\0.02&0&0.2\end{bmatrix},\ D=\begin{bmatrix}0.2&-0.1&0.1\\0&-0.2&0.2\\0&0&0.1\end{bmatrix},\ C_1=\begin{bmatrix}0.1&0.2&0.2\\0.1&-0.2&0.1\\0.3&0.2&0.1\end{bmatrix},$$

$$F=\begin{bmatrix}0.3679&0&0\\0&0.6065&0\\0&0&0.3679\end{bmatrix},\ G=\begin{bmatrix}0.6321&0&0\\0&0.3935&0\\0&0&0.6321\end{bmatrix},$$

$$E_1=\begin{bmatrix}0.3\\0.5\\0\end{bmatrix},\ F_1=\begin{bmatrix}0.6\\0.4\\0.2\end{bmatrix},\ E_2=\begin{bmatrix}0.5\\0.4\\0.2\end{bmatrix},\ F_2=\begin{bmatrix}0.2\\0.6\\0.3\end{bmatrix},$$

$$E_3=0.1,\ F_3=-0.1,\ G_1=G_2=\begin{bmatrix}0.3\\0.2\\0.3\end{bmatrix},\ A_2=A_1=\begin{bmatrix}0.3&0&0\\0&0.2&0\\0&0&0.3\end{bmatrix}.$$

The regulation function is taken as $g(x)=\frac{x^2}{1+x^2}$*. It is easy to know that the derivative of* $g(x)$ *is less than* $l=0.65$*, which shows* $L=\mathrm{diag}(-0.325,-0.325,-0.325)$*. Suppose the bound of the time-delay is* $d_1=1$*,* $d_2=3$*, then* $d(k)\in\mathcal{N}=\{1,2,3\}$*,* $\gamma=2$*. The transition probability matrix* Π *is given by*

$$\Pi=\begin{bmatrix}0.3&0.5&0.2\\0.4&0.3&0.3\\0.2&0.5&0.3\end{bmatrix}.$$

Take

$$w(k)=v(k)=\begin{cases}\sin(0.3k), & k\le 20,\\ 0, & k>20.\end{cases}$$

Solving the matrix inequality (29) in Theorem 3.2 by the Toolbox YALMIP of MATLAB, we can obtain the desired filter gain matrices as follows:

$$A_f=\begin{bmatrix}0.4390&-0.1118&-0.1273\\-0.0679&0.2707&0.0934\\-0.4936&-0.1490&0.3374\end{bmatrix},\ B_f=\begin{bmatrix}-1.3340&-0.7445&0.4208\\-0.1763&-1.9257&0.2864\\-0.4030&-1.2214&-0.6494\end{bmatrix},$$

$$C_f=\begin{bmatrix}-0.2788&-0.4372&0.0185\\0.0477&0.0610&-0.0064\\-0.4510&-0.7585&0.0303\end{bmatrix},\ D_f=\begin{bmatrix}-4.4224&-1.8862&0.0916\\-0.0122&-4.4201&-0.1940\\-1.6843&-3.4882&-3.1874\end{bmatrix},$$

$$H_{1f}=\begin{bmatrix}-1.1891&-2.3000&0.7353\end{bmatrix},\ H_{2f}=\begin{bmatrix}-1.3042&-3.1660&0.2639\end{bmatrix},$$

$$G_{1f}=\begin{bmatrix}-0.4791&-0.3222&-0.2524\end{bmatrix},\ G_{2f}=\begin{bmatrix}-0.3393&-0.5767&0.0151\end{bmatrix}.$$

Let the filtering error system run by random sequence $d(k)$*, and the trajectories and their estimations of the mRNAs and proteins are shown in Figure 1, where the solid line and dotted line describe the state trajectories and estimations of mRNAs and proteins, respectively. The filtering errors are shown in Figure 2. It can be seen from Figure 2 that the filtering error converges to zero in the absence of disturbances, which illustrates the effectiveness of the proposed approach in this paper. In addition, for the GRN under consideration, the LMIs in* [18, Theorem 3.2] *are not feasible, so the method proposed in this paper may be less conservative than one in* [18].

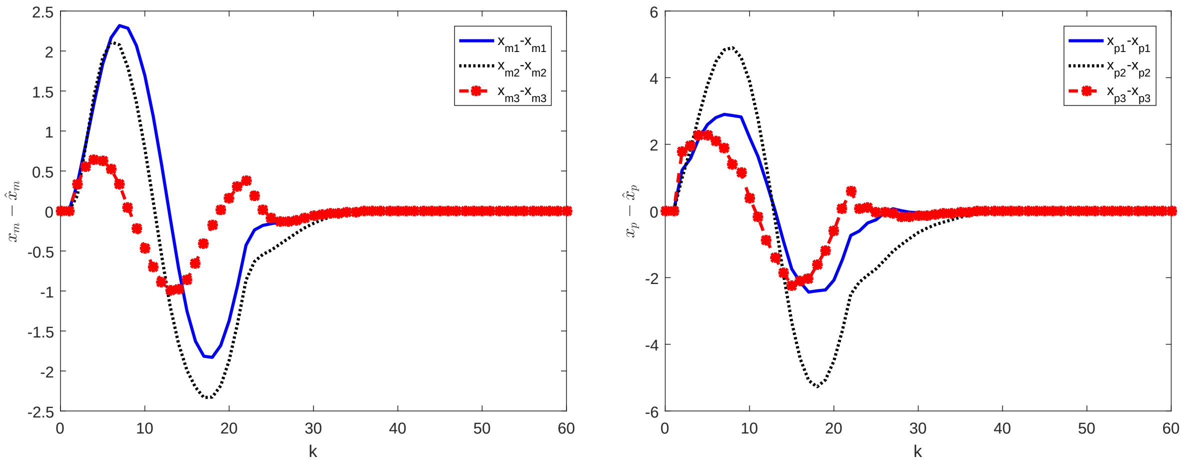

FIGURE 1. Trajectories and estimations of mRNAs and proteins

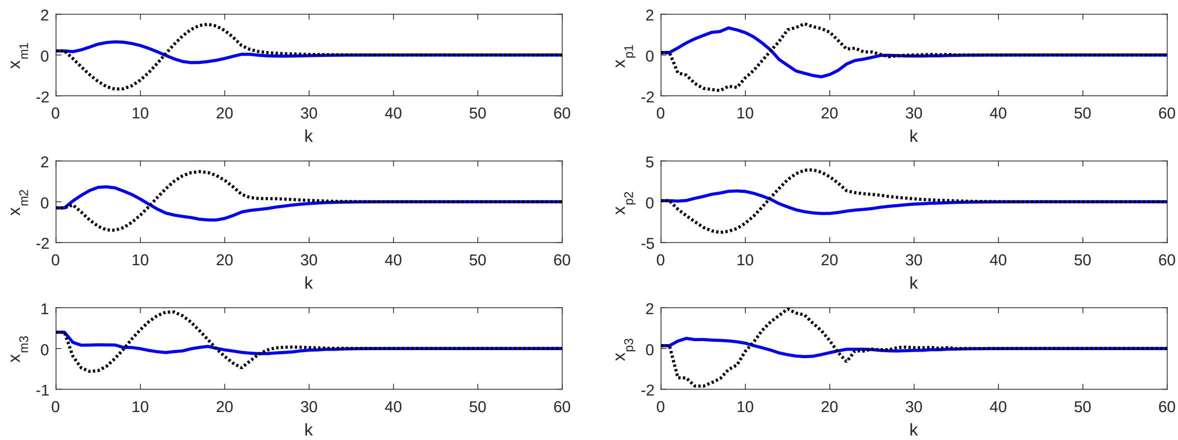

FIGURE 2. Estimation errors of mRNAs and proteins

5. **Conclusions.** This paper investigates the problem of the exponential passive filtering for discrete-time GRNs with discrete and distributed delays. Firstly, an appropriate Lyapunov functional is constructed to give a sufficient condition that the exponentially passive filter error system is strictly exponentially passive with dissipation $\gamma > 0$. Secondly, by the available measurement data, an exponential passive filter is designed to estimate the real concentration of mRNA and protein in GRNs. Finally, a numerical example demonstrates the effectiveness of the proposed exponential passive filter design method. The research method proposed in this paper can also be extended to Markov jumping gene regulation network model and T-S fuzzy gene regulation network model.

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