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STATE SPACE DESIGN METHOD FOR LEFT FILTERED INVERSE SYSTEMS FOR NON-LINEAR SYSTEMS

Yosuke Kimura, Kotaro Hashikura, Takaaki Suzuki and Kou Yamada

Graduate School of Science and Technology
Gunma University
1-5-1 Tenjincho, Kiryu 376-0052, Japan
{ t171b029; k-hashikura; suzuki.taka; yamada }@gunma-u.ac.jp

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Abstract. *We examine a new state space design method for filtered inverse systems for non-linear systems that reproduces approximately the input to the non-linear system. The inverse system is expected to be applied to various fields, and the behavior of the system can be described in a wider operation range than the linear system by using the non-linear system. This method is based on the fusion of the observer and the decoupling method. The proposed method for filtered inverse systems is simplified considerably.*
Keywords: Inverse system, Observer, Decoupling, State space

1. **Introduction.** For time-invariant linear systems, various researches on inverse systems that reproduce input data from the output data have been done [1-11]. Applications of the inverse system to various fields such as model matching control [3], servo system [4], internal model control [13], repetitive control [14,15], model feedback control [16,17] are being considered. The conventional studies of inverse systems are classified into three systems: the system by Silverman [5], the system by Sain and Massey [6], and the system reproducing only low frequency components by Yoshikawa and Sugie [10].

The system by Silverman, for a given system, finds a system that makes the whole system into an identity matrix by series coupling and defines it as the inverse system. The inverse system obtained by this method is ideal, because the input data can be faithfully reproduced from its output data. However, in general, the obtained inverse system is not proper, so it needs a differentiator. Therefore, this method is not realistic.

The system by Sain and Massey, for a given system, finds a system that makes the whole system into an integral function matrix by series coupling of the system and defines it as the inverse system. The inverse system obtained by this method is also impractical because it requires a differentiator when faithfully reproducing the input.

The system by Yoshikawa and Sugie is a method of giving up reproduction of high frequency components of input and only reproducing low frequency components [10]. This system finds a system that makes the whole system into a low-pass filter by series coupling of the proper system and defines it as the inverse system. The system obtained by this method, which is called filtered inverse system, is practical because it does not require a differentiator. In addition, Yamada and Watanabe have proposed a method of constructing this filtered inverse system on a state space that can be easily calculated [12]. The above research is directed to a linear system. An actual system generally has nonlinearity. When linear control is applied, it is difficult to obtain a good reaction in the operating range including the non-linear system. A wider operating range can be obtained by using a non-linear system. However, the inverse systems for non-linear system have not been fully considered yet.

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The purpose of this paper is to propose a design method for left filtered inverse system for non-linear systems. This method is based on decoupling into the observer and state feedback. There is an advantage that it can be handled that control structure and stability uniformly on state space.

2. Inverse Systems.

Consider the non-linear system in

$$\begin{cases} \dot{x} = A(x)x(t) + B(x)u(t) \\ y = C(x)x(t), \end{cases} \tag{1}$$

where $x \in R^n(q)$ is a state variable, $u \in R^p(q)$ is the input, $y \in R^m(q)$ is the output and q is the differential operator with respect to t. That is, qx means

$$qx = \dot{x}. \tag{2}$$

Using the differential operator q, the system in (1) can be written by

$$G(q) = C(qI - A)^{-1}B(\in R^{m\times p}(q)). \tag{3}$$

We assume that the normal rank of $G(q)$ satisfies

$$\mathrm{rank}G(q) = p. \tag{4}$$

The filtered inverse system for (1) is defined as follows [10,12].

Definition 2.1. *(left filtered inverse system)*
When $m \geq p$, the system $G^+(q)$ is called a left inverse system for (1) if $G^+(q)$ is causal and

$$G^+(q)G(q) = \mathrm{diag}\left\{ \frac{1}{(1+qT_1)^{\alpha_1}} \quad \cdots \quad \frac{1}{(1+qT_p)^{\alpha_p}} \right\} \tag{5}$$

is satisfied, where

$$T_i > 0 \ \ (i = 1, \ldots, p)$$

and α_i $(i = 1, \ldots, p)$ is an appropriately positive integer.

3. Left Inverse Systems.

In this section, we propose a state space design method for the left filtered inverse system for the non-linear system for (1).

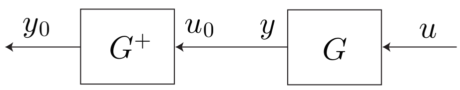

FIGURE 1. G^+G

Consider the system

$$\begin{cases} \dot{\xi} = A_0(\xi)\xi + B_0(\xi)u_0 \\ y = C_0(\xi)\xi + D_0(\xi)u_0, \end{cases} \tag{6}$$

to be a candidate of a left filtered inverse system satisfying (5). Here $\xi \in R^n(q)$, $u_0 \in R^m(q)$ and $y_0 \in R^p(q)$. The series connected system G^+G is shown in Figure 1. From Figure 1, $u_0 = y$, we have

$$\begin{cases} \begin{bmatrix} \dot{\xi} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} A_0(\xi) & B_0(\xi)C(x) \\ 0 & A(x) \end{bmatrix} \begin{bmatrix} \xi \\ x \end{bmatrix} + \begin{bmatrix} 0 \\ B(x) \end{bmatrix} u \\ y_0 = \begin{bmatrix} C_0(\xi) & D_0(\xi)C(x) \end{bmatrix}. \end{cases} \tag{7}$$

Applying transformation as

$$\begin{bmatrix} \xi_0 \\ x_0 \end{bmatrix} = \begin{bmatrix} I & I \\ 0 & I \end{bmatrix} \begin{bmatrix} \xi \\ x \end{bmatrix} \tag{8}$$

to (7) gives

$$\begin{cases} \begin{bmatrix} \dot{\xi}_0 \\ \dot{x}_0 \end{bmatrix} = \begin{bmatrix} A_0(\xi) & -A_0(\xi) + A(x) + B_0(\xi)C(x) \\ 0 & A(x) \end{bmatrix} \begin{bmatrix} \xi_0 \\ x_0 \end{bmatrix} + \begin{bmatrix} B(x) \\ B(x) \end{bmatrix} u \\ y_0 = \begin{bmatrix} C_0(\xi) & D_0(\xi)C(x) \end{bmatrix} \begin{bmatrix} \xi_0 \\ x_0 \end{bmatrix}. \end{cases} \tag{9}$$

We settle A_0 and C_0 as

$$A_0(\xi) = A(x) + B_0(\xi)C(x) \tag{10}$$

and

$$C_0(\xi) = D_0(\xi)C(x), \tag{11}$$

respectively. Equation (9) is rewritten by

$$\begin{cases} \begin{bmatrix} \dot{\xi}_0 \\ \dot{x}_0 \end{bmatrix} = \begin{bmatrix} A_0(\xi) & 0 \\ 0 & A(x) \end{bmatrix} \begin{bmatrix} \xi_0 \\ x_0 \end{bmatrix} + \begin{bmatrix} B(x) \\ B(x) \end{bmatrix} u \\ y_0 = \begin{bmatrix} D_0(\xi)C(x) & 0 \end{bmatrix} \begin{bmatrix} \xi_0 \\ x_0 \end{bmatrix}. \end{cases} \tag{12}$$

This equation is equivalent to

$$\begin{cases} \dot{\xi}_0 = (A(x) + B_0(\xi))\, C(x)\xi_0 + B(x)u \\ y_0 = D_0(\xi)C(x)\xi_0. \end{cases} \tag{13}$$

The rest is to settle $D_0(\xi)$ and $B_0(\xi)$ to make (13) satisfy (5). From (13), using differential operator q, we have

$$y_0 = D_0(\xi)C(x)\left(qI - A(x) - B_0(\xi)C(x)\right)^{-1} B(x)u. \tag{14}$$

Transposed system of (14) is equivalent to system in

$$\begin{cases} \dot{x} = A^T(x) + C^T(x)u \\ y = B^T(x)x \end{cases} \tag{15}$$

controlled by

$$u = B_0^T(\xi)x + D_0^T(\xi)v. \tag{16}$$

Here we denote

$$B(x) = \begin{bmatrix} B_1(x) & \dots & B_p(x) \end{bmatrix}, \quad B_i(x)\ (i = 1, \dots, p), \tag{17}$$

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_p \end{bmatrix} \quad \text{and} \quad y_i\ (i = 1, \dots, p), \tag{18}$$

respectively. Differential equations of y_i from the first order to α_i-st order are written by

$$\begin{cases} \dot{y}_i &= \left(\dot{B}_i^T(x) + B_i^T(x)A^T(x)\right) x = A_{i1}(x)x \\ \ddot{y}_i &= \dot{A}_{i1}(x)x + A_{i1}(x)\dot{x} = \left(\dot{A}_{i1}(x) + A_{i1}(x)A^T(x)\right) x = A_{i2}(x)x \\ &\vdots \\ \overset{(\alpha_i-1)}{y_i} &= \left(\dot{A}_{i(\alpha_i-2)}(x) + A_{i(\alpha_i-2)}(x)A^T(x)\right) x = A_{i(\alpha_i-1)}(x)x \\ \overset{(\alpha_i)}{y_i} &= \left(\dot{A}_{i(\alpha_i-1)}(x) + A_{i(\alpha_i-1)}(x)A^T(x)\right) x + A_{i(\alpha_i-1)}(x)C^T(x)u \\ &= A_{i\alpha_i}(x)x + A_{i(\alpha_i-1)}(x)C^T(x)u, \end{cases} \tag{19}$$

where

$$\begin{cases} A_{i0}(x) = B_i^T(x) \\ A_{ik} = \dot{A}_{i(k-1)}(x) + A_{i(k-1)}A^T(x)\ \ (k = 1, \dots, \alpha_i) \end{cases} \tag{20}$$

and

$$\alpha_i = \min\left(j | A_{i(j-1)}(x)C^T(x) \neq 0; j = 1, \dots, n\right). \tag{21}$$

Multiplying $\overset{(\alpha_i)}{y_i}$, $\overset{(\alpha_i-1)}{y_i}$, ..., $\dot{y}_i$, y_i by 1, $\beta_{i1}, \dots, \beta_{i\alpha_i}$, respectively and summing, we obtain

$$\begin{aligned} \overset{(\alpha_i)}{y_i} + \beta_{i1}\overset{(\alpha_i-1)}{y_i} + \cdots + \beta_{i\alpha_i} y_i = &\left(A_{i\alpha_i}(x) + \beta_{i1}A_{i(\alpha_i-1)}(x) + \cdots + \beta_{i\alpha_i}B_i^T(x)\right)x \\ &+ A_{i(\alpha-1)}C^T(x)u. \end{aligned} \tag{22}$$

We will describe β_{ij}'s design method later. Therefore, when we denote y_H by

$$y_H = \begin{bmatrix} \overset{(\alpha_1)}{y_1} + \beta_{11}\overset{(\alpha_1-1)}{y_1} + \cdots + \beta_{1\alpha_1}y_1 \\ \vdots \\ \overset{(\alpha_p)}{y_p} + \beta_{p1}\overset{(\alpha_p-1)}{y_p} + \cdots + \beta_{p\alpha_p}y_p \end{bmatrix}, \tag{23}$$

we have

$$y_H = \Psi x + \Phi u, \tag{24}$$

where

$$\Psi = \begin{bmatrix} A_{1\alpha_1}(x) + \beta_{11}A_{1(\alpha_1-1)}(x) + \cdots + \beta_{1\alpha_1}B_1^T(x) \\ \vdots \\ A_{p\alpha_p}(x) + \beta_{p1}A_{p(\alpha_p-1)}(x) + \cdots + \beta_{p\alpha_p}B_p^T(x) \end{bmatrix} \tag{25}$$

and

$$\Phi = \begin{bmatrix} A_{1(\alpha_1-1)}C^T(x) \\ \vdots \\ A_{p(\alpha_p-1)}C^T(x) \end{bmatrix}. \tag{26}$$

When Φ has row full rank, that is,

$$\text{rank}\Phi = p, \tag{27}$$

the matrix $\tilde{\Phi}$ exists satisfying

$$\Phi\tilde{\Phi} = I. \tag{28}$$

When u in (16) is given by

$$u = B_0^T(\xi)x + D_0^T(\xi)v = -\tilde{\Phi}\Psi x + \tilde{\Phi}Xv(t), \tag{29}$$

where

$$B_0(\xi) = -\Psi^T\tilde{\Phi}^T, \tag{30}$$

$$D_0(\xi) = X\tilde{\Phi}^T \tag{31}$$

and

$$X = \text{diag}(\beta_{1\alpha_1}, \dots, \beta_{p\alpha_p}), \tag{32}$$

y_H in (24) is written by

$$y_H = \text{diag}(\beta_{1\alpha_1}, \dots, \beta_{p\alpha_p})v. \tag{33}$$

By using differential operator q, this equation is written by

$$y = \text{diag}\left(\frac{\beta_{1\alpha_1}}{q^{\alpha_1} + \beta_{11}q^{\alpha_1-1} + \cdots + \beta_{1\alpha_1}} \quad \cdots \quad \frac{\beta_{p\alpha_p}}{q^{\alpha_p} + \beta_{p1}q^{\alpha_p-1} + \cdots + \beta_{p\alpha_p}} \right)v. \tag{34}$$

When β_{ij} are settled by

$$\beta_{ij} = {}_{\alpha_i}C_j(T_i)^{-j}, \tag{35}$$

we have

$$y = \text{diag}\left\{ \frac{1}{(1+qT_1)^{\alpha_1}} \quad \cdots \quad \frac{1}{(1+qT_p)^{\alpha_p}} \right\}v. \tag{36}$$

Equation (36) implies that we can design G^+ satisfying (5).

In summary, from (10), (11), (30) and (31), the left filtered inverse system $G^{+}(q)$ for $G(q)$ satisfying (5) is given by

$$\begin{cases} \dot{\xi} = A_0(\xi)\xi + B_0(\xi)u_0 = (A(x) + B_0(\xi)C(x))\,\xi + B_0(\xi)u_0 \\ \quad = \Big(A(x) - \Psi^T\tilde{\Phi}^T C(x)\Big)\,\xi - \Psi^T\tilde{\Phi}^T u_0 \\ y = C_0(\xi)\xi + D_0(\xi)u_0 = D_0(\xi)C(x)\xi + D_0(\xi)u_0 = X\tilde{\Phi}^T C(x)\xi + X\tilde{\Phi}^T u_0. \end{cases} \tag{37}$$

4. **Conclusion.** In this paper, we proposed a state space design method of the left filtered inverse system for non-linear systems. If it is possible to construct an inverse system in a non-linear system, application in a wider range can be expected. Since this method uses a noninterference method, it is limited to the configuration of the inverse system. However, we believe that it can be used for IMC (Internal Model Control) applications, where designers can design models to satisfy noninterference conditions.

Application of the proposed method will be presented in another article.

REFERENCES

[1] L. M. Silverman and H. J. Payne, Input-output structure of linear systems with application to the decoupling problem, *SIAM J. on Control*, vol.9, no.2, pp.199-233, 1971.

[2] T. Nakamizo and N. Kobayashi, On decoupling of a linear multivariable system, *Transactions of the Society of Instrument and Control Engineers*, vol.16, no.5, pp.616-622, 1980 (in Japanese).

[3] B. C. Moore and L. M. Silverman, Model matching by state feedback and dynamical compensation, *IEEE Trans. Automatic Control*, vol.17, no.4, pp.491-497, 1972.

[4] T. Yoshikawa and T. Sugie, Filtered inverses with application to servo systems, *Transactions of the Society of Instrument and Control Engineers*, vol.18, no.8, pp.792-799, 1982 (in Japanese).

[5] L. M. Silverman, Inversion of multivariable linear systems, *IEEE Trans. Automatic Control*, vol.14, no.3, pp.270-276, 1969.

[6] M. K. Sain and J. L. Massey, Invertibility of linear time-inverant dynamical systems, *IEEE Trans. Automatic Control*, vol.14, no.2, pp.141-149, 1969.

[7] S. Kamiyama, Reduced system and minimal order inverse system, *Transactions of the Society of Instrument and Control Engineers*, vol.15, no.3, pp.313-319, 1979 (in Japanese).

[8] H. Kaizuka and S. Sugimoto, Minimal order partial inverse systems with respect to inputs and states, *Transactions of the Society of Instrument and Control Engineers*, vol.18, no.3, pp.209-216, 1982 (in Japanese).

[9] T. Nakamizo and N. Kobayashi, The low order inverse system with a specified number of differentiators, *Transactions of the Society of Instrument and Control Engineers*, vol.16, no.1, pp.35-41, 1980.

[10] T. Yoshikawa and T. Sugie, Inverse systems for reproducing linear functions of inputs, *Transactions of the Society of Instrument and Control Engineers*, vol.15, no.7, pp.880-887, 1979 (in Japanese).

[11] T. Mori and M. Ito, On the inverse of linear time-invariant systems, *Transactions of the Society of Instrument and Control Engineers*, vol.14, no.5, pp.494-498, 1978 (in Japanese).

[12] K. Yamada and K. Watanabe, State space design method of filtered inverse systems, *Transactions of the Society of Instrument and Control Engineers*, vol.28, no.8, pp.923-930, 1992 (in Japanese).

[13] K. Yamada, S. Kikuchi, A. Totsuka and W. Kinoshita, A design method of stable delayed inverse systems for the strictly proper digital systems, *Transactions of the Japan Society of Mechanical Engineers, Series C*, vol.70, no.690, pp.407-413, 2004.

[14] K. Yamada, K. Watanabe, M. Tsuchiya and T. Kaneko, Repetitive control systems with filtered inverse, *Transactions of the Society of Instrument and Control Engineers*, vol.29, no.11, pp.1311-1319, 1993.

[15] K. Yamada and K. Watanabe, Robust control design for repetitive control systems with filtered inverse, *Proc. of the 1st Asian Control Conference*, pp.243-246, 1994.

[16] K. Huwa and T. Narikiyo, A construction of an inverse model by norm criterion and its applications, *The transactions of the Institute of Electrical Engineering of Japan*, vol.117, no.6, pp.371-378, 1997.

[17] K. Huwa, T. Narikiyo and Y. Funabashi, A construction of an inverse model with cut-off filter and its application to model feedback control system, *The transactions of the Institute of Electrical Engineering of Japan*, vol.119, no.12, pp.1096-1102, 1999.