ICIC Express Letters ICIC International ©2019 ISSN 1881-803X
Volume **13**, Number **7**, July **2019** pp. **601–607**

THE CHARACTERISTIC OF ALL STRONGLY STABILIZABLE MIMO PLANTS

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Received December 2018; accepted March 2019

Abstract. *When the control system is stabilized by a stable controller, the controller is said to be a strongly stabilizing controller. There exist many design methods of stabilizing controller, but most of the proposed design methods do not consider the stability of stabilizing controllers. Since the instability of stabilizing controller occurs to make the closed-loop system very sensitive to disturbances and reduce the tracking performance to reference inputs, it is required in practice to use the stable stabilizing controller whenever it is possible. Youla et al. showed that the plant is strongly stabilizable if and only if the plant satisfies the parity interlacing property condition and examined a design procedure of stable stabilizing controller. Hoshikawa et al. clarified the parameterization of all strongly stabilizable single-input/single-output plants. The purpose of this paper is to expand the result by Hoshikawa et al. and to clarify the parameterization of all strongly stabilizable multiple-input/multiple-output plants.*
Keywords: Strong stabilization, Strongly stabilizable plants, Closed-loop systems

1. **Introduction.** In this paper, we clarify the characteristic of multiple-input/multiple-output plants which can be stabilized by a stable controller. That is, the parameterization of all plants can be stabilized by a stable controller. The parameterization problem, that is, all stabilizing controllers of a plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] and plants those can be stabilized [11, 12] are sought. Since this parameterization can successfully search for all proper stabilizing controllers, it is used as a tool for many control problems.

For an unstable plant, the parameterization of all stabilizing controllers is solved by Youla et al. [1, 2]. The structure of a parameterization of all stabilizing controllers for unstable plants has full-order state feedback, including a full-order observer [3]. Glaria and Goodwin [4] gave a simple parameterization for single-input/single-output minimum-phase systems. However, two difficulties remained. One is that the parameterization of all stabilizing controllers generally includes improper controllers. In practice, the controller is required to be proper. The other one is that they do not give the parameterization of all internally stabilizing controllers. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for single-input/single-output minimum-phase systems [5].

For a stable plant, the parameterization of all stabilizing controllers has a structure identical to that of Internal Model Control which has advantages, for example, closed-loop stability is assured simply by choosing a stable Internal Model Controller parameter and closed-loop performance characteristics are related directly to controller parameters. It makes on-line tuning of the Internal Model Controller very convenient. However, there

DOI: 10.24507/icicel.13.07.601

exists a question whether or not, stabilizing controllers for unstable plants can be represented by the Internal Model Control structure. For this question, Morari and Zafiriou examined the parameterization of all stabilizing Internal Model Controllers for unstable plants [6]. However, their internal model is not necessarily proper. In addition, their parameterization includes improper Internal Model Controllers. To overcome these problems, Chen et al. proposed the simple parameterization of all proper stabilizing Internal Model Controllers for minimum-phase unstable plants [13]. Zhang et al. [14] proposed a new parameterization, which is needless with the coprime factorization. In this way, the parameterization of all stabilizing controllers is advanced.

Using unstable stabilizing controllers, unstable poles of stabilizing controller make the closed-loop transfer function have zeros in right half plane. It makes the closed-loop system very sensitive to disturbances and reduces the tracking performance to reference inputs [8, 9]. In addition, a breakdown exists in the feedback-loop of feedback control system, that is, the feedback control system becomes feed-forward control system. The unstable poles of stabilizing controller become the unstable poles of the control system. Thus, the control system becomes unstable even if the plant is stable. From above reasons, it is desirable in practice that the control system is stabilized by stable stabilizing controller [9]. Therefore, several design methods of a stable stabilizing controller, which is referred as a strongly stabilizing controller, have been considered [8, 9, 10, 16, 17, 18].

Youla et al. showed that the plant is strongly stabilizable if and only if the plant satisfies the parity interlacing property condition and examined a design procedure of stable stabilizing controller [10]. Wakaiki et al. studied the sensitivity reduction problem with stable controllers for the linear time-invariant multi-input/multi-output distributed parameter system [19]. Wakaiki et al. considered the strong and robust stabilization problem that a class of plants has finitely many simple unstable zeros but possibly infinitely many unstable poles stabilized by a stable controller in the linear time-invariant single-input/single-output infinitely dimensional system [15]. However, they do not clarify the characteristic of strongly stabilizable plants. If the characteristic of strongly stabilizable plants is clarified, we have a possibility to obtain the parameterization of all stable stabilizing controllers. In addition, we have a possibility to clarify the characteristic of strongly stabilizable plants. From this viewpoint, it is desirable to clarify the characteristic of strongly stabilizable plants. From this point of view Hoshikawa et al. clarified the characteristic of strongly stabilizable single-input/single-output plants [12].

There are many multiple-input/multiple-output plants such as the application for vibration test. The vibration environment is a very important environmental factor in the transportation and use of products, and with the complexity of products, intelligence and high performance requirements, especially in the engineering fields of aviation, aerospace, shipbuilding and vehicles, due to the limitations of product space. As well as the increase in reliability requirements, the characteristics of the product's vibration response are becoming more and more demanding. The early uniaxial vibration test method played an important role in product development, and derived a large number of test equipment, test methods and standard specifications, but with the complication of product structure, the weakness and deficiency of uniaxial vibration test become more obvious. Therefore, as early as the 1990s, there were already experiments such as earthquakes and began to experiment with multiple multi-axis vibration technologies. Now, it has gradually started to be applied in many fields. Therefore, to clarify the characteristic of strongly stabilizable multiple-input/multiple-output plants is an important problem to solve.

In this paper, we expand the result in [12] and clarify the characteristic of strongly stabilizable multiple-input/multiple-output plants, that is, the parameterization of all strongly stabilizable multiple-input/multiple-output plants is clarified. This paper is organized as follows. In Section 2, we show the problem considered in this paper. In Section 3, we clarify the characteristic of strongly stabilizable multiple-input/multiple-output plants. In Section 4, we show a numerical example. Section 5 gives concluding remarks.

Notations

R	The set of real numbers.
$R(s)$	The set of real rational functions with s.
RH_∞	The set of stable proper real rational functions.

2. **Problem Formulation.** Consider the control system in

$$\begin{cases} y(s) = G(s)u(s) + d(s) \\ u(s) = C(s)\left(r(s) - y(s)\right) \end{cases}, \tag{1}$$

where $G(s) \in R^{m\times p}(s)$ is the plant, $C(s) \in R^{p\times m}(s)$ is the controller, $y(s) \in R^m(s)$ is the output, $u(s) \in R^p(s)$ is the control input, $d(s) \in R^m(s)$ is the disturbance and $r(s) \in R^m(s)$ is the reference input. The strong stabilization is the control method that makes the plant stable by using the stable stabilizing controller. Therefore, if the plant $G(s)$ in (1) can be stabilized by the stable controller $C(s)$, we call this plant $G(s)$ the strongly stabilizable plant, and we call this controller $C(s)$ the strongly stabilizing controller. If the characteristic of strongly stabilizable multiple-input/multiple-output plants is clarified, we have a possibility to obtain the parameterization of all strongly stabilizing controllers.

The problem considered in this paper is to clarify the characteristic of strongly stabilizable multiple-input/multiple-output plants.

3. **The Characteristic of Strongly Stabilizable Plants.** In this section, we clarify the characteristic of strongly stabilizable plants $G(s)$, that is, the parameterization of all strongly stabilizable plants $G(s)$ is shown.

The characteristic of strongly stabilizable plants is summarized in the following theorem.

Theorem 3.1. *$G(s)$ is assumed to be coprime. The plant $G(s)$ is strongly stabilizable if and only if $G(s)$ is written by the form in*

$$G(s) = \left(I - Q_1(s)Q_2(s)\right)^{-1} Q_1(s), \tag{2}$$

where $Q_1(s) \in RH_\infty^{m\times p}$ and $Q_2(s) \in RH_\infty^{p\times m}$ are any functions.

Proof: First, the necessity is shown. That is, we show that if the stable controller $C(s)$ makes $G(s)$ stable, then $G(s)$ takes the form in (2). From the assumption that $C(s)$ makes $G(s)$ in (1) is stable, $(I + G(s)C(s))^{-1}G(s)$ is included in RH_∞. Therefore, using $Q_1(s) \in RH_\infty^{m\times p}$, $(I + G(s)C(s))^{-1}G(s) \in RH_\infty^{m\times p}$ can be rewritten as

$$(I + G(s)C(s))^{-1}G(s) = Q_1(s). \tag{3}$$

From simple manipulation, we have

$$G(s) = \left(I - Q_1(s)C(s)\right)^{-1} Q_1(s). \tag{4}$$

Since $C(s)$ is stable, using $Q_2(s) \in RH_\infty^{p\times m}$, let $C(s)$ be

$$C(s) = Q_2(s), \tag{5}$$

and (4) is rewritten as

$$G(s) = \left(I - Q_1(s)Q_2(s)\right)^{-1} Q_1(s). \tag{6}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $G(s)$ in (1) takes the form in (2), then the stable controller $C(s)$ makes $G(s)$ stable. When we set $C(s)$ as

$$C(s) = Q_2(s), \tag{7}$$

then $C(s) \in RH_\infty^{p\times m}$ because of $Q_2(s) \in RH_\infty^{p\times m}$. If the stable controller $C(s)$ makes $G(s)$ stable, according to definition of internal stability, the transfer functions $(I+G(s)C(s))^{-1}$, $(I+G(s)C(s))^{-1}G(s)$, $C(s)(I+G(s)C(s))^{-1}$ and $(I+G(s)C(s))^{-1}G(s)C(s)$ are stable. From simple manipulation. The transfer functions are rewritten as

$$(I+G(s)C(s))^{-1}G(s)C(s) = Q_1(s)Q_2(s), \tag{8}$$

$$C(s)(I+G(s)C(s))^{-1} = (I-Q_2(s)Q_1(s))Q_2(s), \tag{9}$$

$$(I+G(s)C(s))^{-1}G(s) = Q_1(s) \tag{10}$$

and

$$(I+G(s)C(s))^{-1} = I - Q_1(s)Q_2(s). \tag{11}$$

Since $Q_1(s) \in RH_\infty^{m\times p}$ and $Q_2(s) \in RH_\infty^{p\times m}$, (8)$\sim$(11) are stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

Lemma 3.1. *When the plant $G(s)$ is written by the form in (2), one of stable controllers to stabilize the plant $G(s)$ is given by*

$$C(s) = Q_2(s). \tag{12}$$

Proof: It is obvious from the sufficiency of proof of Theorem 3.1. □

4. **Numerical Example.** In this section, a numerical example is illustrated to show that the plant written by the form in (2) can be stabilized by using a stable controller.

Consider the problem to make the control system in (1) stable using stable controller, where the plant $G(s)$ is written by

$$G(s) = \begin{bmatrix} \dfrac{s^2-4s-12}{s^3+34s^2+99s+66} & \dfrac{2s^2+12s+16}{s^3+34s^2+99s+66} \\ \dfrac{3s^2+18s+24}{s^3+34s^2+99s+66} & \dfrac{4s^2+14s+12}{s^3+34s^2+99s+66} \end{bmatrix}. \tag{13}$$

$G(s)$ in (13) is rewritten by

$$\begin{aligned} G(s) &= \left(I - \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{2}{s+1} \\ \dfrac{3}{s+1} & \dfrac{4}{s+1} \end{bmatrix} \begin{bmatrix} \dfrac{s+1}{s+2} & \dfrac{2s+2}{s+2} \\ \dfrac{3s+3}{s+2} & \dfrac{4s+4}{s+2} \end{bmatrix} \right)^{-1} \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{2}{s+1} \\ \dfrac{3}{s+1} & \dfrac{4}{s+1} \end{bmatrix} \\ &= (I - Q_1(s)Q_2(s))^{-1} Q_1(s), \end{aligned} \tag{14}$$

where

$$Q_1(s) = \begin{bmatrix} \dfrac{1}{s+1} & \dfrac{2}{s+1} \\ \dfrac{3}{s+1} & \dfrac{4}{s+1} \end{bmatrix} \tag{15}$$

and

$$Q_2(s) = \begin{bmatrix} \dfrac{s+1}{s+2} & \dfrac{2s+2}{s+2} \\ \dfrac{3s+3}{s+2} & \dfrac{4s+4}{s+2} \end{bmatrix}. \tag{16}$$

This equation implies that $G(s)$ in (13) is written by the form in (2), that is, $G(s)$ in (13) is strongly stabilizable.

From Lemma 3.1, one of strongly stabilizing controllers $C(s)$ for $G(s)$ in (14) is given by

$$C(s) = \begin{bmatrix} \dfrac{s+1}{s+2} & \dfrac{2s+2}{s+2} \\ \dfrac{3s+3}{s+2} & \dfrac{4s+4}{s+2} \end{bmatrix}. \tag{17}$$

$C(s)$ in (17) is obviously stable. Therefore, if $C(s)$ in (17) makes $G(s)$ in (14) stable, then $C(s)$ in (17) is a strongly stabilizing controller for $G(s)$ in (14).

In order to confirm that $C(s)$ in (17) is a strongly stabilizing controller for $G(s)$ in (14), the response of the output

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{18}$$

for the step reference input

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{19}$$

is shown in Figure 1. Here, the solid line shows the response of $y_1(t)$ and the dotted line shows that of $y_2(t)$. Figure 1 shows that the controller $C(s)$ in (17) makes the plant $G(s)$ in (14) stable, that is, $C(s)$ is a strongly stabilizing controller for $G(s)$ in (14).

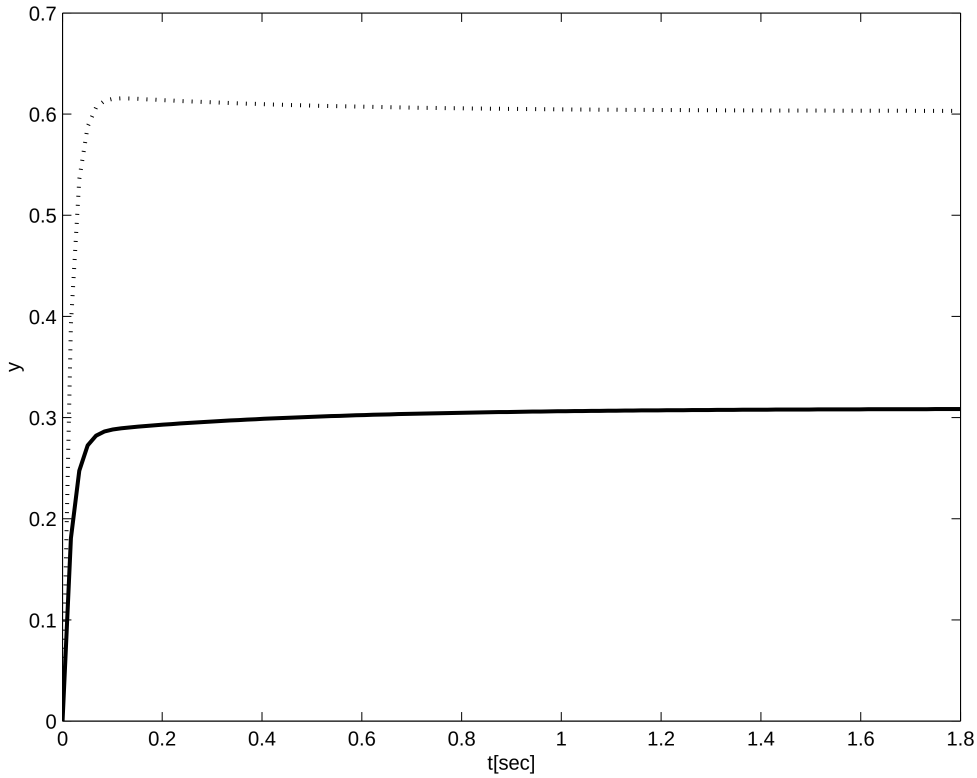

FIGURE 1. Response of the output $y(t)$ of the control system in (1) for the step reference input $r(t)$

In this way, we find that if the plant $G(s)$ is written by the form in (2), the plant is strongly stabilizable.

A breakdown exists in the feedback-loop of feedback control system, that is, the feedback control system becomes feed-forward control system. The unstable poles of stabilizing controller become the unstable poles of the control system. Thus, the control system becomes unstable even if the plant is stable. In order to verify that the strong stabilization is reliable stabilization and it can be against emergency well such as a breakdown existing in the feedback-loop. This situation is considered that the stable controller $C(s)$ in (17) stabilizes the plant $G(s)$ in (13) and after 5 seconds the feedback-loop has breakdown. In this situation, the response of the output $y(t)$ in (18) for the step reference input $r(t)$ in (19) is shown in Figure 2. Here, the solid line shows the response of $y_1(t)$ and the dotted line shows that of $y_2(t)$. Figure 2 shows that the strong stabilization can still keep the system stable, even if a breakdown exists in the feedback-loop.

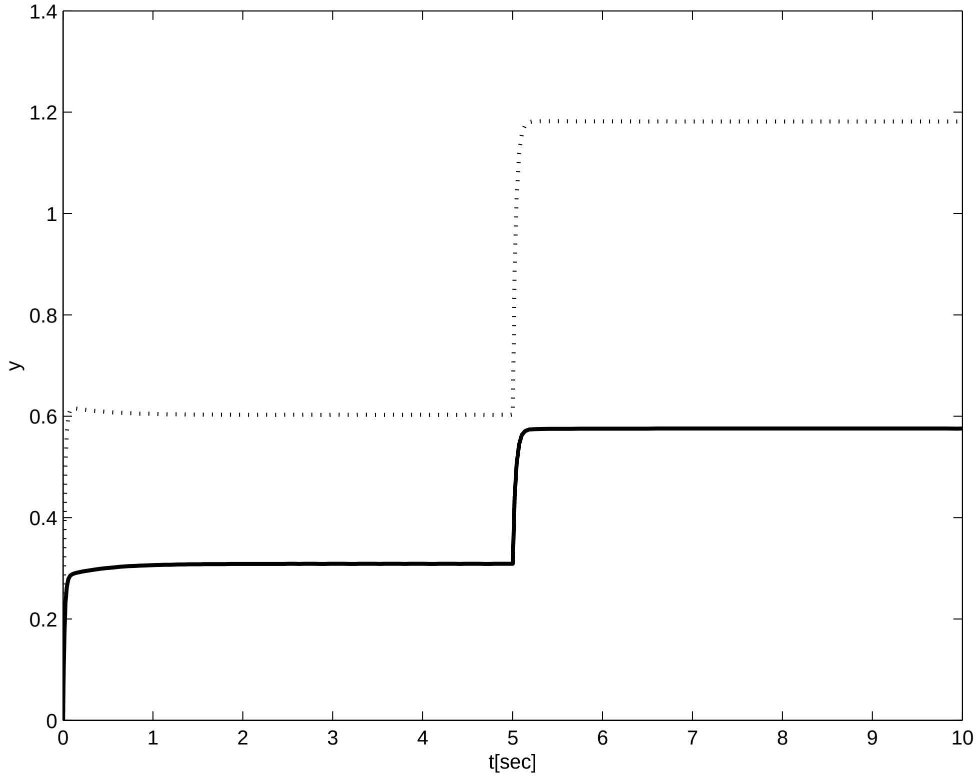

Figure 2. Response of the output $y(t)$ of the control system in (1) for the step reference input $r(t)$

5. **Conclusion.** In this paper, we clarified the characteristic of strongly stabilizable multiple-input/multiple-output plants. That is, we showed that if the plant $G(s)$ is written by the form in (2), the plant can be stabilized by stable controllers. In addition, we showed a numerical example to illustrate that the plant written by the form in (2) can be stabilized by using a stable stabilizing controller. We will present the parameterization of all strongly stabilizing controllers in another article.

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