ICIC Express Letters
Volume **13**, Number **7**, July **2019**
ICIC International ©2019 ISSN 1881-803X
pp. 625–633

MODEL STRUCTURE IDENTIFICATION AND PARAMETER ESTIMATION FOR UNSTABLE PROCESS IN CLOSED-LOOP

Lianming Sun[1], Xinyu Liu[1] and Akira Sano[2]

[1]Faculty of Environmental Engineering
The University of Kitakyushu
1-1, Hibikino, Wakamatsu-ku, Kitakyushu, Fukuoka 808-0135, Japan
sun@kitakyu-u.ac.jp; z8dcb001@eng.kitakyu-u.ac.jp

[2]Faculty of Science and Technology
Keio University
3-14-1, Hiyoshi, Kouhoku-ku, Yokohama, Kanagawa 223-8522, Japan
sano@sd.keio.ac.jp

Received December 2018; accepted March 2019

Abstract. *Identification of an unstable process has to be performed in closed-loop manner. When the sensitivity of the closed-loop is very low, while the external test signal is restricted to a small range, the numerical computation of the closed-loop identification suffers from severe numerical problems in model structure identification and parameter estimation. In this paper the cyclo-stationarity is applied to dealing with the poor numerical conditions. The model structure is determined by both the mean squared error and the cyclo-stationarity detection, and the parameter estimation is performed through optimization of the criteria in both the time and subspace domains. The effectiveness is illustrated through the simulation example and the identification experiment.*
Keywords: System identification, Output over-sampling, Cyclo-stationarity, Model structure selection

1. **Introduction.** In many practical applications to deal with an unstable process, a mathematical model is expected to be identified from the experimental data collected in the closed-loop manner, where the unstable process is stabilized by a feedback controller. When little priori information is available for performing identification, sometimes not only the estimation of model parameters, but identification of model structure is required to be performed under the environment with unknown noise terms. If the experimental data are informative, the model structure is conventionally determined by using some information criteria such as the final prediction-error (FPE), Akaike information criterion (AIC), or the minimum description length (MDL), and then the model parameters are estimated through optimizing a specified criterion by some algorithms in time domain, or the spatial methods using the subspace properties [1].

The informativeness is a fundamental condition in system identification, where the process input and output signals must have sufficient frequency excitations, and their power is expected to suppress the affection of noise or disturbance. Nevertheless, in order to stabilize the unstable process, the feedback controller usually makes the sensitivity to the external signals be low in some frequency bands. Moreover, the closed-loop performance sometimes requires that the unstable process works around the stationary points, so the external test signals cannot be arbitrarily added into the closed-loop. As a result, the experimental data collected from closed-loop may have less frequency components in some frequency bands, and suffer severe ill-conditioned numerical problem. Another difficulty is the correlation of experimental data with the noise terms. Since the input signal added

DOI: 10.24507/icicel.13.07.625

to the process is generated by the feedback controller, both the input and output signals are correlated with the noise in closed-loop, i.e., the parameter estimation is an error-in-variables problem [2], which may cause large estimation bias unless the correlation of both the input and output signals with the noise terms is removed or compensated. Accordingly, the conventional methods are performed under very poor numerical conditions, and affected by the noise easily; thus, the identification performance of unstable process characteristics in the conventional methods degrades largely.

On the other hand, the output over-sampling based identification approach has been demonstrated to improve the performance of closed-loop identification where the input signal is held by a zero-order holder, while the output signal is sampled several times with the holding period of the input signal [3]. Consequently, the input and output signals have cyclo-stationarity compared with the signals in the conventional methods. It has been illustrated that the characteristics of cyclo-stationarity can be applied to detecting the model information of the over-sampled stable or unstable processes under poor numerical conditions, and to decreasing the affection of the noise term [4, 5], but the approach to determining the model structure has not been explicit yet. In this paper, an efficient cyclo-stationarity detection approach is presented, then, a novel information criterion is developed to determine the model structure by applying cyclo-stationarity and mean square of prediction error, and thus the model structure can be determined more appropriately than the conventional methods. Moreover, the cyclo-stationary information is introduced into the identification algorithm to mitigate the severe numerical problem, so the proposed algorithm has high identification performance for the unstable process in closed-loop experiment conditions.

The rest of the paper is organized as follows. In the next section, the main identification problems of the unstable process in closed-loop are summarized. In Section 3, the cyclo-stationary characteristics of the process input and output signals are analyzed, and the estimation algorithm for the cyclo-stationarity is presented. Then the identification algorithm for model structure determination and parameter estimation is illustrated in Section 4, and some simulation examples and the identification experiment of a magnetic levitation system are shown in Section 5. Finally, the conclusion and the future research work are given in Section 6.

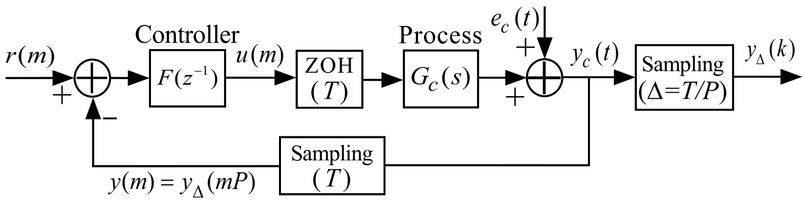


FIGURE 1. Illustration of output over-sampling in closed-loop identification

2. **Problem Statement.**

The diagram of the unstable process in closed-loop considered in this paper is illustrated in Figure 1. The continuous time unstable process $G_c(s)$ with unstable poles is regulated by a digital controller $F(z^{-1})$, where z^{-1} is a backward shift operator. The control input $u(m)$ is added to the process through a zero-order holder with holding period T, i.e., the control input is a piece-wise signal with rate $1/T$, and correspondingly the process model can be described by a discrete-time model with respect to the interval T. Assume that the process output $y_c(t)$ is disturbed by a stochastic noise $e_c(t)$, and then the input-output description of the process can be given by

$$y(m) = G\big(z^{-1}\big)u(m) + e(m), \tag{1}$$

where $y(m)$ and $e(m)$ are the samples of output $y_c(t)$ and the noise term $e_c(t)$ at instant

mT, respectively. $G\big(z^{-1}\big)$ is the discrete-time transfer function of $G_c(s)$, and the backward shift operator z^{-1} corresponds to the interval T.

If the reference signal $r(m)$ is informative, the conventional indirect closed-loop identification algorithms construct two models first, one is $r(m) \sim u(m)$ model, the other one is $r(m) \sim y(m)$ model, and then the process model $G\big(z^{-1}\big)$ is extracted from these two models [6]. When the informativeness of $r(m)$ is not available for identification, the direct method is the possible choice to identify the process model using the data of $u(m)$ and $y(m)$, and thus it requires that $u(m)$ and $y(m)$ can sufficiently offer independent excitations to guarantee the informativeness of experimental data. It might be achieved by using high order controller or switching between different control laws to reduce the linear dependence of $u(m)$ and $y(m)$ in the conventional direct methods. Nevertheless, the simple controller such as proportional-integral-differential (PID) controller is widely used in many process control systems, whereas the switching control laws may yield fluctuation in the process output. An alternative approach to guaranteeing the informativeness is the output over-sampling scheme, which is illustrated in Figure 1. The controller remains the same as that in the usual operation, while the output $y_c(t)$ is sampled at the rate P/T, which is faster than the rate of input signal $1/T$. Here P is an integer indicating the over-sampling rate. Let the output sampling interval be indicated as $\Delta = T/P$, then the over-sampled output is $y_\Delta(k)$, whereas only the output signals at the instants $mT = mP\Delta$ are fed back into the controller. For the simplicity of notation, the process input at instants $k\Delta$ is denoted as $u_\Delta(k)$, and $u_\Delta(mP) = u_\Delta(mP+1) = \cdots = u_\Delta((m+1)P-1) = u(m)$. The output over-sampling is easily implemented in the digital control systems [7].

It is demonstrated that both $u_\Delta(k)$ and $y_\Delta(k)$ have cyclo-stationarity with respect to the interval Δ, which is quite different from the stationary properties in the conventional methods. The cyclo-stationarity can provide independent information for closed-loop identification, which will be shown in the next section.

3. **Cyclo-Stationarity Property Analysis and Estimation.** Assume that $r(m)$ is pseudo stationary stochastic signal, $e_\Delta(k)$ and $e(m)$ are stationary signals, and then $u_\Delta(k)$ and $y_\Delta(k)$ are cyclo-stationary signals whose correlation function $\mathcal{R}_{x_1,x_2}(k,\tau)$

$$\mathcal{R}_{x_1,x_2}(k,\tau) = E\left\{x_1(k+\tau)x_2(k)\right\}, \tag{2}$$

where $E\{\cdot\}$ indicates the operation of expectation, satisfying that

$$\mathcal{R}_{x_1,x_2}(k,\tau) = \mathcal{R}_{x_1,x_2}(k+P,\tau) \neq \mathcal{R}_{x_1,x_2}(k+1,\tau) \neq \cdots \neq \mathcal{R}_{x_1,x_2}(k+P-1,\tau), \tag{3}$$

where $x_1(k)$ and $x_2(k)$ are $u_\Delta(k)$ or $y_\Delta(k)$. It is seen that the correlation function $\mathcal{R}_{x_1,x_2}(k,\tau)$ is a periodic function in k, quite different from the correlation functions of stationary signals, i.e., $\mathcal{R}_{x_1,x_2}(k,\tau) = \mathcal{R}_{x_1,x_2}(k+1,\tau) = \mathcal{R}_{x_1,x_2}(k+2,\tau) = \cdots$.

3.1. **Definition of cyclo-stationary relation functions.** Consider the periodic correlation function $\mathcal{R}_{x_1,x_2}(k,\tau)$ with period P in k. Then its Fourier transform with respect to k is defined as

$$\mathcal{C}_{x_1,x_2}(\alpha,\tau) = \lim_{N\to\infty} \frac{1}{N}\sum_{k=0}^{N-1} \mathcal{R}_{x_1,x_2}(k,\tau)e^{-i\alpha k}, \tag{4}$$

where $0 \leq \alpha < 2\pi$. Following (4), the cyclo-stationary relation function satisfies

$$\mathcal{C}_{x_1,x_2}(\alpha,\tau) = \begin{cases} \dfrac{1}{P}\displaystyle\sum_{k=mP}^{(m+1)P-1} \mathcal{R}_{x_1,x_2}(k,\tau)e^{-i\alpha_p k}, & \alpha = \alpha_p \\ 0, & \text{others} \end{cases} \tag{5}$$

where $\alpha_p \in \mathcal{A}_P$, $\mathcal{A}_P = \left\{\alpha_p | \alpha_p = p\frac{2\pi}{P}, p = 0, 1, \ldots, P-1\right\}$. Moreover, the Fourier transform of (5) yields the cyclo-stationary spectral density function

$$\mathcal{S}_{x_1,x_2}(\alpha, \omega) = \sum_{\tau=-N/2+1}^{N/2} \mathcal{C}_{x_1,x_2}(\alpha, \tau) e^{-j\omega\tau}. \tag{6}$$

It can be seen that if the cyclo-stationary spectral density function is estimated by the definitions given above, the estimation will be time consuming since the correlation functions $\mathcal{C}_{x_1,x_2}(k, \tau)$ for both the possible k and τ should be estimated. Correspondingly, a fast estimation algorithm is desired in the practical applications.

To decrease the computation load and the affection of noise term, the correlation functions are calculated from the average periodograms of the experimental data.

3.2. **Estimation algorithm of cyclo-stationary functions.** Assume that the experimental data of two cyclo-stationary signals $x_1(k)$ and $x_2(k)$ are recorded, where the cyclic period is P. Let $x_{n,p}(m) = x_n(mP + p)$, where $n = 1, 2$, $p = 0, 1, \ldots, P-1$. Then, for every n or p, $x_{n,p}(m)$ can be treated as stationary signal. Consequently, from the fast Fourier transform (FFT) of $x_{n,p}(m)$, the correlation functions of the corresponding signals are given by

$$\begin{aligned} \mathcal{R}_{p_1,p_2}^{(m_P)}(\tau_P) &= E\left\{x_{1,p_1}(m_P M + \tau_P) x_{2,p_2}(m_P M)\right\} \\ &\approx \frac{1}{N} \sum_{l=-\frac{N}{2}+1}^{\frac{N}{2}} \frac{1}{N} X_{1,p_1}^{(m_P)}(e^{i\omega_l}) \left(X_{2,p_2}^{(m_P)}(e^{i\omega_l})\right)^* e^{i\omega_l \tau_P}, \end{aligned} \tag{7}$$

where M is the length of shift data block, N is the FFT size which is an integer number of power of 2. Following the definition of $x_{n,p}(m)$, the following equation

$$\begin{aligned} \mathcal{R}_{p_1,p_2}^{(m_P)}(\tau_P) &= E\left\{x_1\big((m_P M + \tau_P)P + p_1\big) x_2(m_P M + p_2)\right\} \\ &= \mathcal{R}_{x_1,x_2}\left(m_P M + p_2, \tau_P P + p_1 - p_2\right) \end{aligned} \tag{8}$$

holds for $x_1(k)$ and $x_2(k)$. Define a correlation matrix $\mathbf{\Phi}_{\mathcal{R}}$ as follows:

$$\begin{bmatrix} \mathcal{R}_{0,0}^{(0)}(0) & \mathcal{R}_{1,0}^{(0)}(0) & \mathcal{R}_{2,0}^{(0)}(0) & \cdots & \mathcal{R}_{P-1,0}^{(0)}(0) & \mathcal{R}_{0,0}^{(0)}(1) & \mathcal{R}_{1,0}^{(0)}(1) & \cdots \\ \mathcal{R}_{1,1}^{(0)}(0) & \mathcal{R}_{2,1}^{(0)}(0) & \mathcal{R}_{3,1}^{(0)}(0) & \cdots & \mathcal{R}_{0,1}^{(0)}(1) & \mathcal{R}_{1,1}^{(0)}(1) & \mathcal{R}_{2,1}^{(0)}(1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathcal{R}_{P-1,P-1}^{(0)}(0) & \mathcal{R}_{0,P-1}^{(0)}(1) & \mathcal{R}_{1,P-1}^{(0)}(1) & \cdots & \mathcal{R}_{P-2,P-1}^{(1)}(1) & \mathcal{R}_{P-1,P-1}^{(0)}(1) & \mathcal{R}_{0,P-1}^{(0)}(2) & \cdots \\ \mathcal{R}_{0,0}^{(1)}(0) & \mathcal{R}_{1,0}^{(1)}(0) & \mathcal{R}_{2,0}^{(1)}(0) & \cdots & \mathcal{R}_{P-1,0}^{(1)}(0) & \mathcal{R}_{0,0}^{(1)}(1) & \mathcal{R}_{1,0}^{(1)}(1) & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}.$$

Then, performing Fourier transform with respect to every column of $\mathbf{\Phi}_{\mathcal{R}}$ yields the matrix $\mathbf{\Phi}_{\mathcal{C}}$ of cyclo-stationary correlation functions.

$$\mathbf{\Phi}_{\mathcal{C}}^T = \begin{bmatrix} \mathcal{C}_{x_1,x_2}(0,0) & \mathcal{C}_{x_1,x_2}\left(\frac{2\pi}{N}, 0\right) & \cdots & \mathcal{C}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N}, 0\right) \\ \mathcal{C}_{x_1,x_2}(0,1) & \mathcal{C}_{x_1,x_2}\left(\frac{2\pi}{N}, 1\right) & \cdots & \mathcal{C}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N}, 1\right) \\ \mathcal{C}_{x_1,x_2}(0,2) & \mathcal{C}_{x_1,x_2}\left(\frac{2\pi}{N}, 2\right) & \cdots & \mathcal{C}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N}, 2\right) \\ \vdots & \vdots & \cdots & \vdots \\ \mathcal{C}_{x_1,x_2}(0,-1) & \mathcal{C}_{x_1,x_2}\left(\frac{2\pi}{N}, -1\right) & \cdots & \mathcal{C}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N}, -1\right) \end{bmatrix}. \tag{9}$$

Correspondingly, performing Fourier transform with respect to every column of $\mathbf{\Phi}_{\mathcal{C}}^T$ leads to the cyclo-stationary spectrum as

$$\mathbf{\Phi}_{\mathcal{S}} = \begin{bmatrix} \mathcal{S}_{x_1,x_2}(0,0) & \mathcal{S}_{x_1,x_2}\left(\frac{2\pi}{N},0\right) & \cdots & \mathcal{S}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N},0\right) \\ \mathcal{S}_{x_1,x_2}\left(0,\frac{2\pi}{N}\right) & \mathcal{S}_{x_1,x_2}\left(\frac{2\pi}{N},\frac{2\pi}{N}\right) & \cdots & \mathcal{S}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N},\frac{2\pi}{N}\right) \\ \mathcal{S}_{x_1,x_2}\left(0,\frac{4\pi}{N}\right) & \mathcal{S}_{x_1,x_2}\left(\frac{2\pi}{N},\frac{4\pi}{N}\right) & \cdots & \mathcal{S}_{x_1,x_2}\left(\frac{4\pi}{N},\frac{4\pi}{N}\right) \\ \vdots & \vdots & \cdots & \vdots \\ \mathcal{S}_{x_1,x_2}\left(0,\frac{2(N-1)\pi}{N}\right) & \mathcal{S}_{x_1,x_2}\left(\frac{2\pi}{N},\frac{2(N-1)\pi}{N}\right) & \cdots & \mathcal{S}_{x_1,x_2}\left(\frac{2(N-1)\pi}{N},\frac{2(N-1)\pi}{N}\right) \end{bmatrix}. \tag{10}$$

From the estimation of $\mathbf{\Phi}_{\mathcal{C}}(\alpha,\tau)$ or $\mathbf{\Phi}_{\mathcal{S}}(\alpha,\omega)$, the cyclo-stationary components can be detected by comparing the terms associated to the angles α_p.

4. **System Identification Algorithm.** The approach to determining the model structure is investigated first.

4.1. **Model structure identification.** In some applications the model structure is required to be determined for identification. The information criterion for model structure identification in the conventional methods generally uses the mean square error (MSE) of the process model residue, where the signals are treated as stationary ones. However, under the situation where the external test signals are restricted when stabilizing the unstable process in closed-loop, MSE has little explicit difference between the local minima, and thus it is hard to determine the appropriate model structure.

In the output over-sampling scheme, the information on cyclo-stationarity is applied. Let the residue of the process model be denoted as $\varepsilon_\Delta\left(k,\hat{\boldsymbol{\theta}}_\Delta\right)$

$$\varepsilon_\Delta\left(k,\hat{\boldsymbol{\theta}}_\Delta\right) = \frac{1}{\hat{H}_\Delta(q^{-1})}\left(\hat{A}_\Delta(q^{-1})y_\Delta(k) - \frac{\hat{B}_\Delta(q^{-1})}{\hat{F}_\Delta(q^{-1})}u_\Delta(k)\right), \tag{11}$$

where $\hat{H}_\Delta\left(q^{-1}\right)$ is the estimated noise model, $\hat{A}_\Delta\left(q^{-1}\right)$ contains all the unstable poles of the estimated model $\hat{G}_\Delta\left(q^{-1}\right) = \hat{B}_\Delta\left(q^{-1}\right)/\hat{A}_\Delta\left(q^{-1}\right)\hat{F}_\Delta\left(q^{-1}\right)$, and $\hat{\boldsymbol{\theta}}_\Delta$ is the model parameter vector. Following the definition of $\varepsilon_\Delta\left(k,\hat{\boldsymbol{\theta}}_\Delta\right)$, if there is less cyclo-stationary components in $\varepsilon_\Delta\left(k,\hat{\boldsymbol{\theta}}_\Delta\right)$, the estimated process model is near to the true one, while the estimated noise model can approximate the noise term if the residue has small mean square errors.

Then, the model orders are determined by

$$\underset{n_a,n_b,n_f,n_h}{\arg\min}\left(\log\left(\frac{2n}{N}\sigma_\varepsilon^2\right) + \log M_S + |\log\eta_1| + |\log\eta_2|\right), \tag{12}$$

where σ_ε^2 is the variance of $\varepsilon_\Delta(k,\hat{\boldsymbol{\theta}}_\Delta)$, M_S is the maximum of $\mathcal{S}_{\varepsilon_\Delta,\varepsilon_\Delta}(\alpha_p,\omega)$, η_1 indicates the magnitude ratio of $\mathcal{S}_{\varepsilon_\Delta,\varepsilon_\Delta}(\alpha,\omega)$ at $\alpha=\alpha_p$ vs $\alpha\neq\alpha_p$, whereas η_2 indicates distribution difference of the spectrum. They are given by

$$\eta_1 = \frac{\frac{1}{P-1}\sum\limits_{p=1}^{P-1}\sum\limits_{l=-N/2+1}^{N/2}\left|\mathcal{S}_{\varepsilon_\Delta,\varepsilon_\Delta}(\alpha_p,\omega_l)\right|}{\frac{1}{\bar{P}}\sum\limits_{\bar{p}\notin\mathcal{A}_P}\sum\limits_{l=-N/2+1}^{N/2}\left|\mathcal{S}_{\varepsilon_\Delta,\varepsilon_\Delta}(\alpha_{\bar{p}},\omega_l)\right|}, \quad \eta_2 = \frac{\frac{1}{P-1}\sum\limits_{p=1}^{P-1}\mathrm{cov}\left(\mathcal{S}_{\varepsilon_\Delta,\varepsilon_\Delta}(\alpha_p,\omega_l)\right)}{\frac{1}{\bar{P}}\sum\limits_{\bar{p}\notin\mathcal{A}_P}\mathrm{cov}\left(\mathcal{S}_{\varepsilon_\Delta,\varepsilon_\Delta}(\alpha_{\bar{p}},\omega_l)\right)}, \tag{13}$$

where $\bar{P}$ is the number of non-cyclo-stationary angles.

4.2. **Model parameter estimation.** In order to apply the information of cyclo-stationarity, define the correlation matrix $\mathcal{R}_{\boldsymbol{\phi}_\varepsilon,\boldsymbol{\phi}}(k,\tau)$

$$\mathcal{R}_{\boldsymbol{\phi}_\varepsilon,\boldsymbol{\phi}}(k,\tau) = E\left\{\boldsymbol{\phi}_\varepsilon(k+\tau)\boldsymbol{\phi}^T(k)\right\}. \tag{14}$$

Then, it is cyclo-stationary, i.e., its Fourier transform with respect to k given by

$$\mathcal{C}_{\boldsymbol{\phi}_\varepsilon,\boldsymbol{\phi}}(\alpha,\tau) = \frac{1}{M}\sum_{m=0}^{M-1}\left(\sum_{p=0}^{P-1} e^{-ip\alpha}\mathcal{R}_{\boldsymbol{\phi}_\varepsilon,\boldsymbol{\phi}}(mP+p,\tau)\right) \tag{15}$$

has non-zero values for $\alpha \in \mathcal{A}_P$, while it is 0 for $\alpha \notin \mathcal{A}_P$, where

$$\boldsymbol{\phi}(k) = \begin{bmatrix} y_\Delta(k) \\ \vdots \\ y_\Delta(k-n_1) \\ -u_\Delta(k-1) \\ \vdots \\ -u_\Delta(k-n_1) \end{bmatrix}, \qquad \boldsymbol{\phi}_\varepsilon(k) = \begin{bmatrix} \varepsilon_\Delta(k+\tau) \\ \varepsilon_\Delta(k+\tau-1) \\ \varepsilon_\Delta(k+\tau-2) \\ \vdots \\ \varepsilon_\Delta(k+\tau-n_2) \end{bmatrix}, \tag{16}$$

where n_1 is larger than the process model order, n_2 is larger than $n_1 + n_H + \max(n_a + n_f, n_b)$, $\tau = -P, \ldots, n_2 - \max(n_a + n_f, n_b)$. Then, the column rank of $\mathcal{C}_{\boldsymbol{\phi}_e,\boldsymbol{\phi}}(\alpha_p,\tau)$ is $n_1 + \max(n_a + n_f, n_b)$, and the orthogonal vectors of $\mathcal{C}_{\boldsymbol{\phi}_e,\boldsymbol{\phi}}(\alpha,\tau)$ are the coefficients of $A_\Delta(q^{-1})F_\Delta(q^{-1})X_\Delta(q^{-1})$ and $B_\Delta(q^{-1})X_\Delta(q^{-1})$, where $X_\Delta(q^{-1})$ is a common polynomial [4, 8]. Then, arranging the orthogonal vectors of $\left(\mathcal{C}^H_{\boldsymbol{\phi}_e,\boldsymbol{\phi}}(\alpha_p,\tau)\mathcal{C}_{\boldsymbol{\phi}_e,\boldsymbol{\phi}}(\alpha_p,\tau)\right)$ yields such a matrix $\boldsymbol{\Omega}$ that [4, 9]

$$\hat{\boldsymbol{\theta}}_{G_\Delta} = \arg\min_{\hat{\boldsymbol{\theta}}_{G_\Delta}} \begin{bmatrix} 1 & \hat{\boldsymbol{\theta}}^T_{G_\Delta} \end{bmatrix} \boldsymbol{\Omega} \begin{bmatrix} 1 \\ \hat{\boldsymbol{\theta}}_{G_\Delta} \end{bmatrix}. \tag{17}$$

Then the criterion function for the parameter estimation is as follows:

$$J\left(\hat{\boldsymbol{\theta}}_\Delta\right) = J_{\mathrm{T}}\left(\hat{\boldsymbol{\theta}}_\Delta\right) + \lambda J_{\mathrm{S}}\left(\hat{\boldsymbol{\theta}}_{G_\Delta}\right), \tag{18}$$

$$J_{\mathrm{T}}\left(\hat{\boldsymbol{\theta}}_\Delta\right) = \frac{1}{2N}\sum_{k=1}^{N}\varepsilon^2_\Delta(k),\ J_{\mathrm{S}}\left(\hat{\boldsymbol{\theta}}_{G_\Delta}\right) = \frac{1}{2}\begin{bmatrix} 1 & \hat{\boldsymbol{\theta}}^T_{G_\Delta} \end{bmatrix} \boldsymbol{\Omega} \begin{bmatrix} 1 \\ \hat{\boldsymbol{\theta}}_{G_\Delta} \end{bmatrix}, \tag{19}$$

where λ is a coefficient to accommodate the value of $J_{\mathrm{S}}\left(\hat{\boldsymbol{\theta}}_{G_\Delta}\right)$ with $J_{\mathrm{T}}\left(\hat{\boldsymbol{\theta}}_\Delta\right)$. Since $J_{\mathrm{S}}\left(\hat{\boldsymbol{\theta}}_{G_\Delta}\right)$ is composed of the normalized $\boldsymbol{\Omega}$, λ can be chosen by the mean eigenvalues of the Hessian matrices which will be given in (21). Let the initial value of $\hat{\boldsymbol{\theta}}_\Delta$ be $\hat{\boldsymbol{\theta}}^{(0)}_\Delta$. In the $(l+1)$th iteration, the parameter vector can be estimated by

$$\hat{\boldsymbol{\theta}}^{(l+1)}_\Delta = \hat{\boldsymbol{\theta}}^{(l)}_\Delta - \mu \boldsymbol{H}^{-1}_{\mathrm{ess}}\left[\frac{dJ_{\mathrm{T}}\left(\hat{\boldsymbol{\theta}}_\Delta\right)}{d\hat{\boldsymbol{\theta}}_\Delta} + \lambda\frac{dJ_{\mathrm{S}}\left(\hat{\boldsymbol{\theta}}_{G_\Delta}\right)}{d\hat{\boldsymbol{\theta}}_\Delta}\right], \tag{20}$$

where μ $(0 \le \mu \le 1)$ is the step-size of the Gauss-Newton algorithm, and the Hessian matrix is given by

$$\boldsymbol{H}_{\mathrm{ess}} = \boldsymbol{H}_{\mathrm{ess,T}} + \lambda\boldsymbol{H}_{\mathrm{ess,S}} = \frac{1}{N}\sum_{k=0}^{N-1}\frac{d\varepsilon\left(k,\hat{\boldsymbol{\theta}}_\Delta\right)}{d\hat{\boldsymbol{\theta}}_\Delta}\left(\frac{d\varepsilon\left(k,\hat{\boldsymbol{\theta}}_\Delta\right)}{d\hat{\boldsymbol{\theta}}_\Delta}\right)^H + \lambda\boldsymbol{\Omega}_{\boldsymbol{\theta}_{G_\Delta}}. \tag{21}$$

The first term in (21) is calculated from the data in the time domain similarly as the conventional methods, whereas the second term is the one associated with the subspace information of the cyclo-stationarity. The definite matrix $\boldsymbol{\Omega}_{\boldsymbol{\theta}_{G_\Delta}}$ corresponding to $\hat{\boldsymbol{\theta}}_{G_\Delta}$ in $\boldsymbol{\Omega}$ can improve the numerical condition of the Hessian matrix in the numerical optimization.

5. Numerical Simulation Examples.

5.1. Cyclo-stationarity estimation. Two signals $x_1(k)$ and $x_2(k)$

$$\begin{aligned} x_1(k) = \{ & x_0(0) + w_1(0), x_0(0) + w_1(1), \ldots, x_0(0) + w_1(P-1), \\ & x_0(1) + w_1(P), x_0(1) + w_1(P+1), \ldots, x_0(1) + w_1(2P-1), \\ & x_0(2) + w_1(2P), x_0(2) + w_1(2P+1), \ldots \}, \end{aligned} \tag{22}$$

$$x_2(k) = \left(1 - 1.25q^{-1} + 0.375q^{-2}\right) x_1(k) + w_2(k) \tag{23}$$

are considered in the numerical simulations, where $x_0(m)$, $m = 0, 1, \ldots$, are the samples of a pseudo stationary signal with zero mean and variance σ_x^2, while $w_1(k)$ and $w_2(k)$ are the stationary noise with zero mean, variance $\sigma_{w_1}^2$, $\sigma_{w_2}^2$, respectively. Here P is chosen as 3, and $\sigma_{w_1}^2 = \sigma_{w_2}^2 = 0.5$. It is clear that both $x_1(k)$ and $x_2(k)$ are cyclo-stationary signals.

The estimated cyclo-stationary spectrum is shown in Figure 2, where the spectral components arise only at the angles $\alpha_p = 0$, $2\pi/3$ and $4\pi/3$. The index $\eta_1 = 9.1986$ shows that the cyclo-stationary components can be detected from the spectrum easily.

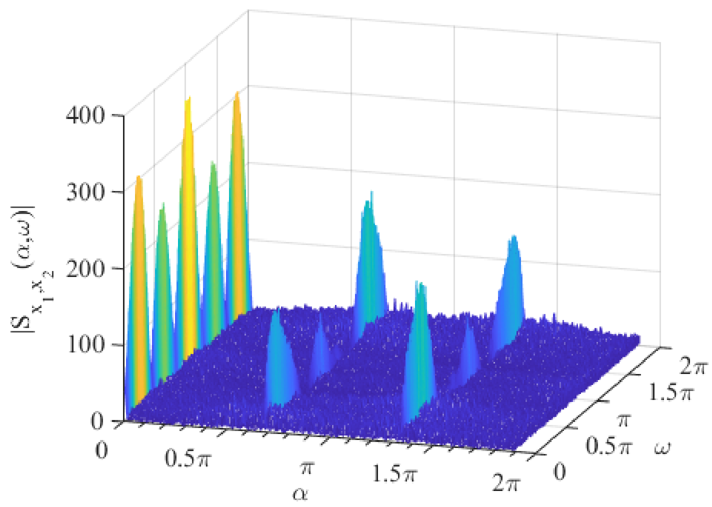
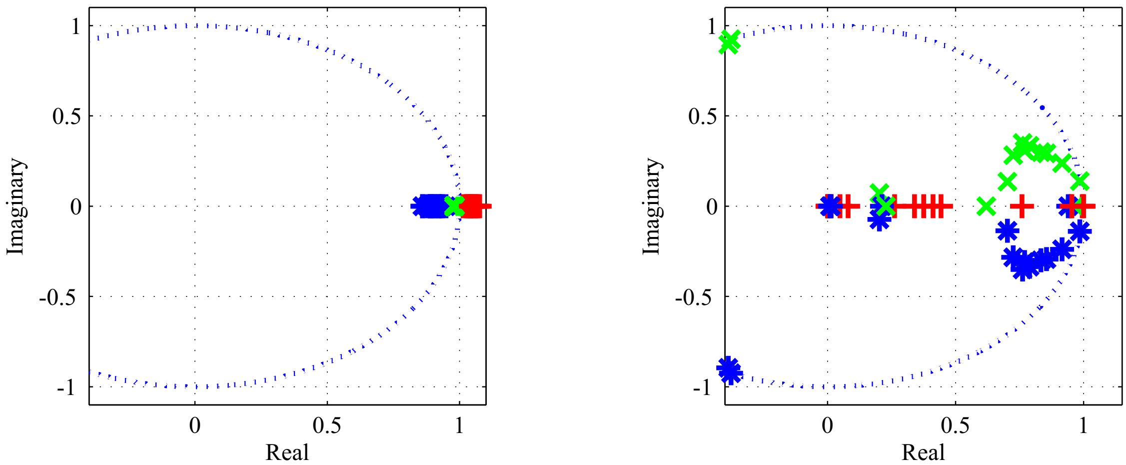


FIGURE 2. Estimated cyclo-stationary spectrum $\mathcal{S}_{x_1,x_2}(\alpha, \omega)$

5.2. Identification experiment of magnetic levitation. In the identification experiment, the magnetic levitation is stabilized by a digital PID controller with the control interval $T = 0.0024$s. Correspondingly, the process can be approximated by a discrete-time transfer function model with respect to the interval T, whose nominal theoretical poles are 1.0886, 0.9682, 0.9202 [10]. There are several poles of the closed-loop close to the unit circle, and the sensitivity to the external exciting signals is very low in the high frequency band, whereas the reference is a constant which does offer little information for identification, hence the numerical conditions are very poor for identification, and many conventional methods fail to work. The noise is assumed as a stationary stochastic process, which is caused by the measurement noise, the disturbance of the ball's rotation, and the air floating, etc. The structure of noise model is unknown and required to be identified from the experimental data.

The experimental data are collected for 100 seconds in one identification experiment. The sampling rate P is chosen as $P = 2$. The spectral components contained in the sampled data concentrate in a narrow frequency band; therefore, the model structure and the parameters are hard to be identified by the conventional methods under the extremely ill-conditioned numerical conditions. The identification experiments are performed independently for 15 times using the experimental data sampled with interval $\Delta = T/2$. In every experiment, the model structure determination of the noise model and estimation of model parameters are executed. The estimated poles of the transfer function are plotted in Figure 3(a). It is shown that by detecting the cyclo-stationarity of prediction errors, the selected model structure can describe the dominant characteristics of both the unstable magnetic levitation process and noise process; by introducing the cyclo-stationary information into the numerical optimization of parameter estimation, the number condition of $\boldsymbol{H}_{\text{ess}}$ is improved. Consequently, the proposed algorithm can estimate the unstable poles from the experimental data under severe numerical conditions in closed-loop manner.



(a) Proposed algorithm (b) General PEM

FIGURE 3. Estimated poles in 15 experiments [10]

As a comparison, the results obtained by the conventional method such as PEM method under the same experiment conditions are also illustrated in Figure 3(b). It is seen that the conventional methods fail to detect the model structure and unstable dynamics from the ill-conditioned experiment data just by considering the mean square error, and the noise model structure does not appropriately describe the main characteristics of the noise process. As a result, the performance of numerical computation is so poor that the numerical optimization is influenced by the noise largely and converges to local minima, and as a result the estimated poles are scattered inside the unit circle.

6. **Conclusions.** The algorithm of model structure identification and parameter estimation is investigated in this paper for the unstable process in closed-loop. It is illustrated that the cyclo-stationarity can be detected by an efficient algorithm using fast Fourier transform. Under the severe numerical conditions where the sensitivity of the closed-loop is very low to the external signals, while the test signal is unavailable, the proposed algorithm applies the information of cyclo-stationarity to determining the model structure and parameter estimation, and therefore, it improves the identification performance greatly. The effectiveness of the algorithm has been illustrated by the numerical simulation and identification experiment. The performance improvement of numerical optimization and the model error evaluation will be investigated in the future work.

REFERENCES

[1] L. Ljung, *System Identification – Theory for the User*, Prentice Hall, Englewood Cliffs, NJ, 1999.
[2] T. Söderström, *The Errors-in-Variables Problem for Dynamic Systems*, Springer, 2018.

[3] L. Sun, H. Ohmori and A. Sano, Output inter-sampling approach to closed-loop identification, *IEEE Trans. Automatic Control*, vol.46, no.12, pp.1936-1941, 2001.
[4] L. Sun and A. Sano, Temporal-spatial information based approach to direct closed-loop identification, *Trans. of the Society of Instrument and Control Engineers*, vol.53, pp.346-354, 2017.
[5] L. Sun, C. Zhou and A. Sano, New prediction error method based on cyclostationarity for system identification, *ICIC Express Letters*, vol.11, no.1, pp.189-196, 2017.
[6] P. M. J. van den Hof and R. J. P. Schrama, Identification and control – Closed-loop issues, *Automatica*, vol.31, no.12, pp.1751-1770, 1995.
[7] M. Fang and Y. Zhu, Asymptotic variance expression in output over-sampling based closed-loop identification, *Proc. of the 17th IFAC Symposium on System Identification*, vol.48, no.28, pp.110-115, 2015.
[8] K. J. Åström and T. Söderstrom, Uniqueness of the maximum likelihood estimates of the parameters of an ARMA model, *IEEE Trans. Automatic Control*, vol.19, no.6, pp.769-773, 1974.
[9] E. Moulines, P. Duhamel, J. Cardoso and S. Mayrargue, Subspace methods for the blind identification of multi-channel FIR filters, *IEEE Trans. Signal Processing*, vol.43, pp.516-526, 1995.
[10] L. Sun, X. Liu and A. Sano, Direct closed-loop identification approach to magnetic levitation system, *Proc. of the 18th IFAC Symposium on System Identification*, Stockholm, Sweden, pp.610-615, 2018.