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PARAMETERIZATION OF SIMULTANEOUSLY STABILIZING CONTROLLERS FOR TWO-STAGE COMPENSATOR SYSTEMS

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Abstract. *This paper aims to show the relationship between compensators which can stabilize a plant in two-stage compensator systems. It is possible to design two-stage compensator systems which are guaranteed to be stable when one of the controllers is broken, and there is changed characteristic of the system depending on input characteristics. Previous works have shown the necessary and sufficient conditions for stable two-stage compensator systems but did not give all stabilizing controllers for the systems. In this paper, we examine the relationship between controllers which can simultaneously stabilize the plant.*
Keywords: Two-stage compensator systems, Closed-loop systems, Input characteristics, Parameterization

1. **Introduction.** Using two-stage compensator systems has some advantages, compared with the other multiple-stage compensator systems. For example, we can achieve decoupling and sensitivity minimization with lower cost, compactness, efficiency and so on. From these advantages, two-stage compensator systems are practically used for such as DC-DC converters. Zhu and Lehman discussed the control design of two-stage compensator systems for high voltage input, low voltage/large current output applications and presented multi-loop control designs for two-stage converters [1]. By using two-stage topology, Khajehoddin et al. proposed a new method to analyze and design bus voltage controller which optimizes both transient response and steady-state harmonics while allowing the reduction of the bus energy storage component size [2]. And they achieved reduction of about 80% of bus capacitor without lowering efficiency. In this paper, we examine the relation between simultaneously stabilizing controllers which can stabilize a plant in two-stage compensator systems. When compensators stabilize a plant in two-stage compensator systems, it is possible to design redundantly stable two-stage compensator systems which stay stable even when one of the controllers is broken, and there are changed characteristics of the system depending on input characteristics. Typically, the two-stage compensator systems consist of a conservative controller and an aggressive controller. The conservative controller lowers the sensitivity against input characteristics. On the other hand, the aggressive controller is effective in stabilizing systems against high-frequency

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input. In addition, the two-stage compensator system considered in this paper has a switch which changes the characteristic of the system, in the outer-loop. The problem of designing two-stage compensator systems is to select an appropriate controller which can stabilize an original plant. Given a plant, the first stage designs a stabilizing controller for the original plant. The second stage designs a stabilizing controller for the inner-loop system. Vidyasagar solved this problem in a two-stage compensator system by the factorization approach [3]. Mori also solved the problem by the factorization approach [4, 5, 6]. They gave the necessary and sufficient conditions for stable two-stage compensator systems. However, they did not explain all stabilizing controllers for two-stage compensator systems. There exist many papers considering the parameterization problem. At first, Yu and Yan proposed the parameterization of all simultaneously stabilizing controllers for two time-varying linear systems in discrete time [7]. Glaría and Goodwin proposed the parameterization for the class of all stabilizing controllers for linear minimum phase plants [8]. Chang and Yousuff proposed the parameterization of observer-based controllers [9]. Hencey and Alleyne gave the parameterization of robust controllers for linear time-varying systems [10]. The parameterization of all strongly stabilizable plants was introduced by Hoshikawa et al. [11]. They proposed the parameterization of strongly stabilizable plants which can be stabilized by a stable controller. Satoh and Yamada clarified the parameterization of all robust stabilizing repetitive controllers for single-input/single-output continuous time non-minimum phase systems [12]. The parameterization for the class of all proper internally stabilizing controllers for multiple-input/multiple-output minimum phase systems was given by Yamada et al. [13]. Sakanushi et al. expanded this idea and clarified the parameterization of all robust stabilizing simple repetitive controllers for multiple-input/multiple-output plants [14]. Li et al. clarified the parameterization of all simultaneous stabilizing controllers with robust servo characteristic for the step signal [15]. Zhang et al. proposed the parameterization of controllers for single-input/single-output and multiple-input/multiple-output plants with time delay [16]. Mori proposed parameterization of all strictly causal stabilizing controllers and stabilizing controllers over commutative rings with application to multidimensional systems [17, 18]. However, to the best of our knowledge, the parameterization of all stabilizing controllers for two-stage compensator systems has not been presented.

In this paper, we clarify the relation between simultaneously stabilizing controllers in two-stage compensator systems. In other words, we parameterize all stabilizing controllers for two-stage compensator systems. This paper is organized as follows. In Section 2, we show the problem statement. In Section 3, we describe the relation between simultaneously stabilizing controllers. Section 4 gives conclusions.

2. **Problem Statement.**

Consider the control system in Figure 1, where $G(s) \in R(s)$ is the plant, and $C_1(s) \in R(s)$ and $C_2(s) \in R(s)$ are the controllers. Here $R(s)$ denotes the set of real rational functions with s. We call the controller $C_1(s)$ conservative controller and it is assumed to be stable. And we call the other controller $C_2(s)$ aggressive controller and it is not necessarily stable. When we design controllers $C_1(s)$ and $C_2(s)$, $C_1(s)$ is settled, and then the class of $C_2(s)$ is determined. Both $C_1(s)$ and $C_2(s)$ need to make the control system in Figure 1 stable. If the relation between $C_1(s)$ and $C_2(s)$ is clarified, there is a possibility to make $C_1(s)$ and $C_2(s)$. However, no paper examines the relation between $C_1(s)$ and $C_2(s)$.

The problem considered in this paper is to clarify the relation between controllers $C_1(s)$ and $C_2(s)$.

3. **Main Results.**

In this section, we firstly give the existence of a stable compensator system with controllers which simultaneously stabilize the plant. Then, we will provide

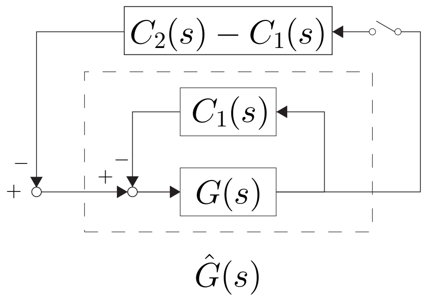

Figure 1. Block diagram of two-stage compensator systems

the class of simultaneously stabilizing controllers $C_1(s)$ and $C_2(s)$, that is, the parameterization of all simultaneously stabilizing controllers is shown. The class of simultaneously stabilizing controllers is clarified in the following theorems.

Theorem 3.1. *Both controllers, $C_1(s)$ and $C_2(s)$ simultaneously stabilize a plant $G(s)$ if and only if $C_2(s) - C_1(s)$ stabilizes*

$$\hat{G}(s) = \frac{G(s)}{1 + C_1(s)G(s)} \in RH_\infty. \tag{1}$$

Here RH_∞ denotes the set of stable proper real rational functions.

Proof: First, the necessity is shown. That is, we show that if $C_1(s)$ and $C_2(s)$ simultaneously stabilize a plant $G(s)$, then $C_2(s) - C_1(s)$ stabilizes $\hat{G}(s)$ in (1). From the assumption that $C_1(s)$ and $C_2(s)$ stabilize $G(s)$, thus, transfer functions, $1/(1 + C_1(s)G(s))$, $C_1(s)/(1 + C_1(s)G(s))$, $G(s)/(1 + C_1(s)G(s))$, $C_1(s)G(s)/(1 + C_1(s)G(s))$, $1/(1 + C_2(s)G(s))$, $C_2(s)/(1 + C_2(s)G(s))$, $G(s)/(1 + C_2(s)G(s))$ and $C_2(s)G(s)/(1 + G(s)C_2(s))$ are stable.

From simple manipulation and (1), we have

$$\frac{1}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = \frac{1 + G(s)C_1(s)}{1 + G(s)C_2(s)}, \tag{2}$$

$$\frac{\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = \frac{G(s)}{1 + C_2(s)G(s)}, \tag{3}$$

$$\frac{C_2(s) - C_1(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = \frac{(1 + C_1(s)G(s))\,C_2(s)}{1 + C_2(s)G(s)} - \frac{(1 + C_1(s)G(s))\,C_1(s)}{1 + C_2(s)G(s)}, \tag{4}$$

and

$$\frac{(C_2(S) - C_1(S))\,\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = \frac{C_2(s)G(s)}{1 + C_2(s)G(s)} - \frac{C_1(s)G(s)}{1 + C_2(s)G(s)}. \tag{5}$$

From the assumption that $1/(1+C_2(s)G(s))$, $C_2(s)/(1+C_2(s)G(s))$, $G(s)/(1+C_2(s)G(s))$ and $C_1(s)$ are stable, transfer functions in (2), (3), (4) and (5) are stable. In addition, $G(s)/(1 + C_1(s)G(s)) \in RH_\infty$ gives us $\hat{G}(s)$ in (1). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $C_2(s) - C_1(s)$ stabilizes $\hat{G}(s)$ in (1), then $C_1(s)$ and $C_2(s)$ stabilize $G(s)$. From the assumption that we assume that $C_2(s) - C_1(s)$ stabilizes $\hat{G}(s)$ in (1), $1\Big/\Big(1 + (C_2(s) - C_1(s))\hat{G}(s)\Big) \in RH_\infty$, $(C_2(s) - C_1(s))\Big/\Big(1 + (C_2(s) - C_1(s))\hat{G}(s)\Big) \in RH_\infty$, $\hat{G}(s)\Big/\Big(1 + (C_2(s) - C_1(s))\hat{G}(s)\Big) \in RH_\infty$, $\Big((C_2(s) - C_1(s))\hat{G}(s)\Big)\Big/\Big(1 + (C_2(s) - C_1(s))\hat{G}(s)\Big) \in RH_\infty$ hold.

Equation (1) gives

$$G(s) = \frac{\hat{G}(s)}{1 - C_1(s)\hat{G}(s)}. \tag{6}$$

From this equation and simple manipulation, we have

$$\frac{1}{1 + C_1(s)G(s)} = 1 - C_1(s)\hat{G}(s), \tag{7}$$

$$\frac{G(s)}{1 + C_1(s)G(s)} = \hat{G}(s), \tag{8}$$

$$\frac{C_1(s)}{1 + C_1(s)G(s)} = C_1(s)\left(1 - C_1(s)\hat{G}(s)\right), \tag{9}$$

$$\frac{C_1(s)G(s)}{1 + C_1(s)G(s)} = C_1(s)\hat{G}, \tag{10}$$

$$\frac{1}{1 + C_2(s)G(s)} = \frac{1 - C_1(s)\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)}, \tag{11}$$

$$\frac{G(s)}{1 + C_2(s)G(s)} = \frac{\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)}, \tag{12}$$

$$\frac{C_2(s)}{1 + C_2(s)G(s)} = \frac{C_2(s)\left(1 - C_1(s)\hat{G}(s)\right)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)}, \tag{13}$$

and

$$\frac{C_2(s)G(s)}{1 + C_2(s)G(s)} = \frac{C_2(s)\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)}. \tag{14}$$

Since $C_1(s) \in RH_\infty$, $\hat{G}(s) \in RH_\infty$ and the assumption that $C_2(s) - C_1(s)$ stabilizes $\hat{G}(s)$, all transfer functions in (7) $\sim$ (14) are stable. From the above, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

Next, we will present the main results.

Theorem 3.2. *$C_1(s)$ and $C_2(s)$ simultaneously stabilize a plant $G(s)$ if and only if $C_2(s) - C_1(s)$ is written by the form in*

$$C_2(s) - C_1(s) = \frac{Q_1(s)}{1 - Q_1(s)Q_2(s)}, \tag{15}$$

where $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$ are any functions.

Proof: First, the necessity is shown. That is, we show that if $C_1(s)$ and $C_2(s)$ simultaneously stabilize $G(s)$, then $C_2(s) - C_1(s)$ takes the form in (15). From Theorem 3.1, this is equivalent to that if $C_2(s) - C_1(s)$ stabilizes $\hat{G}(s)$ in (1), then $C_2(s) - C_1(s)$ takes the form in (15). From the assumption that $C_2(s) - C_1(s)$ stabilizes $\hat{G}(s)$ in (1), all of the transfer functions, $1\Big/\left(1 + (C_2(s) - C_1(s))\hat{G}(s)\right)$, $\hat{G}(s)\Big/\left(1 + (C_2(s) - C_1(s))\hat{G}\right)$, $(C_2(s) - C_1(s))\Big/\left(1 + (C_2(s) - C_1(s))\hat{G}(s)\right)$ and $(C_2(s) - C_1(s))\hat{G}(s)\Big/\left(1 + (C_2(s) - C_1(s))\hat{G}(s)\right)$ belong to RH_∞.

Therefore, using $Q_1(s) \in RH_\infty$, $(C_2(s) - C_1(s))\Big/\left(1 + (C_2(s) - C_1(s))\hat{G}(s)\right)$ can be rewritten as

$$\frac{C_2(s) - C_1(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = Q_1(s). \tag{16}$$

From simple manipulation, we have

$$C_2(s) - C_1(s) = \frac{Q_1(s)}{1 - Q_1(s)\hat{G}(s)}. \tag{17}$$

Since $\hat{G}(s)$ is stable, using $Q_2(s) \in RH_\infty$ and let $\hat{G}(s) = Q_2(s)$, (16) is rewritten as

$$C_2(s) - C_1(s) = \frac{Q_1(s)}{1 - Q_1(s)Q_2(s)}. \tag{18}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $C_2(s) - C_1(s)$ takes the form in (15), then $C_2(s) - C_1(s)$ makes $\hat{G}(s)$ stable. When we set $\hat{G}(s)$ as

$$\hat{G}(s) = Q_2(s), \tag{19}$$

then $\hat{G}(s) \in RH_\infty$ because of $Q_2(s) \in RH_\infty$. Then transfer functions, $1\Big/\Big(1 + (C_2(s) - C_1(s))\hat{G}(s)\Big)$, $\hat{G}(s)\Big/\Big(1+(C_2(s)-C_1(s))\hat{G}\Big)$, $(C_2(s)-C_1(s))\Big/\Big(1+(C_2(s)-C_1(s))\hat{G}(s)\Big)$ and $(C_2(s) - C_1(s))\hat{G}(s)\Big/\Big(1 + (C_2(s) - C_1(s))\hat{G}(s)\Big)$ are rewritten as

$$\frac{1}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = 1 - Q_1(s)Q_2(s), \tag{20}$$

$$\frac{C_2(s) - C_1(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = Q_1(s), \tag{21}$$

$$\frac{\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = (1 - Q_1(s)Q_2(s))\,Q_2(s), \tag{22}$$

$$\frac{(C_2(s) - C_1(s))\,\hat{G}(s)}{1 + (C_2(s) - C_1(s))\,\hat{G}(s)} = Q_1(s)Q_2(s). \tag{23}$$

Since $Q_1(s) \in RH_\infty$ and $Q_2(s) \in RH_\infty$, (20), (21), (22) and (23) are stable. Thus, the sufficiency has been shown.

Therefore, we have proved Theorem 3.2. □

4. **Conclusions.** In this paper, we describe the relationship between simultaneously stabilizing controllers of two-stage compensator systems. A pair of simultaneously stabilizing controller $C_1(s)$ and $C_2(s)$ has a relationship satisfying (15). Numerical examples and an application of the results will be reported in the subsequent article.

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