ICIC Express Letters
Volume **13**, Number **8**, August **2019**

ICIC International ©2019 ISSN 1881-803X
pp. 735–742

H_∞ BOUNDARY CONTROL FOR STOCHASTIC DELAY REACTION-DIFFUSION SYSTEMS WITH MARKOVIAN SWITCHING

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Received January 2019; accepted April 2019

Abstract. *This paper deals with the H_∞ boundary control for stochastic delay Markovian reaction-diffusion systems (SDMRDSs). First, boundary controllers are designed to get mean square H_∞ performance. By constructing of an integral Lyapunov-Krasovskii functional and utilizing of Poincaré inequality, a sufficient condition of mean square H_∞ performance for SDMRDSs is established. The effects of transition rates and diffusion item on mean square H_∞ performance are shown through the derived sufficient condition. Finally, a numerical example is presented to demonstrate the effectiveness of the theoretical result.*

Keywords: Stochastic delay reaction-diffusion systems, Markovian switching, Boundary control, Mean square H_∞ performance

1. **Introduction.** In the past few decades, stochastic reaction-diffusion systems have attracted considerable attention. Many phenomena in nature, such as the fur pattern of animals, chemical reactions, and neural networks can be described by stochastic reaction-diffusion systems [1, 2]. Additionally, time delays usually occur in a variety of engineering systems and biological process, and often result in instability and poor performance. For example, Woolley et al. [3] have shown that a small delay in the reactions can make structure change for raction-diffusion patterning systems. For more prior work on time delay systems, we refer to the reader [4, 5, 6] and the references therein.

When sudden failure, environmental change and components repair occur, Markovian jump systems [7] are suitable mathematical models to represent systems with these abrupt. Zhang et al. have systematically investigated stability problem of Markovian jump linear systems, and refined many results concerned with partly unknown transition probabilities and time-varying delays [8, 9, 10]. However, they did not consider the environment noise. Mao et al. [11] have studied stochastic differential delay equations with Markovian switching by using M-matrix method, but have not taken reaction-diffusion item into account.

In recent years, stability of stochastic reaction-diffusion systems has become a focus of research in both theoretical and practical areas, for instance, asymptotical stability [12], exponential stability [13], and finite-time stability [14]. However, many natural phenomena often affected by random effects such as the noise and environment disturbance, and inevitable external disturbance may degrade the performance of the system to a large extent. H_∞ performance is a good indicator to reflect external interference. Therefore, some burgeoning results about H_∞ performance for disturbed systems have been available so far [15, 16, 17, 18]. In [17, 18], Shi et al. did some research on H_∞ control for Markovian jump systems with parametric uncertainty and delay. Very recently, Li et al. [16] have

DOI: 10.24507/icicel.13.08.735

investigated H_∞ control of continuous Markov jump systems with incomplete transition probabilities and interval time-varying delay.

Diffusion is a usual phenomenon and H_∞ control for reaction-diffusion systems is an interesting topic. However, there are very few works dealing with this problem. Several researchers [19, 20, 21] have only studied the H_∞ synchronization for reaction diffusion systems. From cost viewpoint, boundary control is an effective strategy to deal with reaction-diffusion systems [22, 23, 24]. For reaction-diffusion systems with Markovian switching, there is little literature to study this topic. Although we have studied H_∞ boundary control of reaction-diffusion systems [25, 26], stochastic Markovian reaction-diffusion systems have not been fully investigated yet. Therefore, the problem of H_∞ boundary control for stochastic delay reaction-diffusion systems with Markovian switching has attracted our attention.

Motivated by the above discussion, in this paper, we investigate the H_∞ boundary control for stochastic delay reaction-diffusion systems with Markovian switching (SDM-RDSs). Boundary controllers are designed at first, and we derive a sufficient criterion on H_∞ boundary control for SDMRDSs by Lyapunov functional method and inequality techniques. The relationship between Markov process and mean square H_∞ performance of SDMRDSs is shown through our obtained criterion. The effect of diffusion item is also presented. At last, an example is employed to illustrate the effectiveness of the obtained result.

Throughout this paper, the following mathematical notations are used. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. I_n stands for the n-dimensional identity matrix. A^{-1} and A^{T} denote the inverse and transposition of matrix A, respectively. $\|\cdot\|$ denotes the Euclidean norm for vector. Moreover, let $W^{l,2}([0,1];\mathbb{R}^n)$ be a Sobolev space of absolutely continuous n-dimensional vector functions $\omega(x) : [0,1] \to \mathbb{R}^n$ with square integrable derivatives $\frac{\mathrm{d}^l\omega(x)}{\mathrm{d}x^l}$ of the order $l \geq 1$.

2. **Problem Statement and Preliminaries.** In this paper, we consider a class of stochastic delay Markovian reaction-diffusion systems

$$\begin{aligned} \mathrm{d}y(x,t) = &\left[f(r(t), y(x,t-\tau)) + D(r(t))\frac{\partial^2 y(x,t)}{\partial x^2} + v(x,t)\right]\mathrm{d}t \\ &+ g(r(t), y(x,t), y(x,t-\tau))\mathrm{d}B(t),\ x \in (0,1),\ t > 0, \end{aligned} \tag{1}$$

where $f : S \times \mathbb{R}^n \to \mathbb{R}^n$ and $g : S \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$, and x, t are the spatial variable and time variable, respectively. $y(x,t) = (y_1(x,t), y_2(x,t), \ldots, y_n(x,t))^{\mathrm{T}} \in \mathbb{R}^n$ is the system state. $\tau > 0$ is the time delay. $D(r(t))$ represents the positive definite diffusion-matrix in mode $r(t)$. $B(t)$ is an m-dimensional Brownian motion. $v(x,t)$ is the external disturbance, which is square integrable, i.e.,

$$\int_0^{t_f}\int_0^1 v^{\mathrm{T}}(x,t)v(x,t)\mathrm{d}x\mathrm{d}t < \infty,$$

for a positive time constant t_f.

Let $\{r(t), t \geq 0\}$ be a right-continuous Markov process on the complete probability space $(\Omega, \mathcal{F}, P)$ taking values in a finite state space $S = \{1, 2, \ldots, N\}$ with a generator $\Gamma = (p_{ij})_{N\times N}$, $(i, j \in S)$ given by

$$P\{r(t+\Delta t) = j | r(t) = i\} = \begin{cases} p_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + p_{ii}\Delta t + o(\Delta t), & i = j, \end{cases} \tag{2}$$

where $\Delta t > 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t) = 0$, here $p_{ij} \geq 0$ ($\forall i \neq j$) is the transition rate from mode i to mode j if $i \neq j$, while $p_{ii} = -\sum_{i\neq j} p_{ij} < 0$.

In mode $r(t) = i$, we shall simply write $f(r(t), y(x,t-\tau)) = f_i(y(x,t-\tau))$, $D(r(t)) = D_i$ and $g(r(t), y(x,t), y(x,t-\tau)) = g_i(y(x,t), y(x,t-\tau))$. Therefore, the system (1) can be rewritten as follows

$$\begin{aligned} \mathrm{d}y(x,t) = &\left[f_i(y(x,t-\tau)) + D_i\frac{\partial^2 y(x,t)}{\partial x^2} + v(x,t)\right]\mathrm{d}t \\ &+ g_i(y(x,t), y(x,t-\tau))\mathrm{d}B(t),\ x \in (0,1),\ t > 0. \end{aligned} \tag{3}$$

The initial values are given as follows

$$y(x,s) = 0,\ s \in [-\tau, 0], \tag{4}$$

and the Neumann boundary conditions are imposed

$$\left.\frac{\partial y(x,t)}{\partial x}\right|_{x=0} = 0,\ \left.\frac{\partial y(x,t)}{\partial x}\right|_{x=1} = u_i(t), \tag{5}$$

where $u_i(t)$ is the boundary control input.

The following definition, assumption and lemmas are necessary for our theoretical analysis.

Definition 2.1. *System (3) achieves finite horizon H_∞ performance in the mean square if for a given positive time constant $0 < t_f < \infty$ and a disturbance attenuation level $\gamma > 0$, when $y(x,t) = 0$, $t \in [-\tau, 0]$, the following inequality holds*

$$\mathbb{E}\left(\int_0^{t_f}\int_0^1 y^{\mathrm{T}} y \mathrm{d}x\mathrm{d}t\right) \le \gamma^2 \mathbb{E}\left(\int_0^{t_f}\int_0^1 v^{\mathrm{T}} v \mathrm{d}x\mathrm{d}t\right).$$

Assumption 2.1. *The functions f_i and g_i satisfy the following conditions. That is, there exists $L > 0$ such that*

$$\|f_i(x_1) - f_i(x_2)\|^2 \le L\|x_1 - x_2\|^2 \tag{6}$$

for all $x_1, x_2 \in \mathbb{R}^n$ and $f_i(0) = 0$; and there are, moreover, $L_1 > 0$ and $L_2 > 0$ such that

$$\operatorname{trace}\left(g_i(u,v)^{\mathrm{T}} g_i(u,v)\right) \le L_1 u^{\mathrm{T}} u + L_2 v^{\mathrm{T}} v,\ \forall u, v \in \mathbb{R}^n. \tag{7}$$

Lemma 2.1. *(Poincaré inequality* [24]*) Let $z \in W^{1,2}([0,1]; \mathbb{R}^n)$ be a vector function with $z(0) = 0$ or $z(1) = 0$. Then, for a positive matrix R, one has the following integral inequality*

$$\int_0^1 z^{\mathrm{T}}(s) R z(s)\mathrm{d}s \le \frac{4}{\pi^2}\int_0^1 \left(\frac{\mathrm{d}z(s)}{\mathrm{d}s}\right)^{\mathrm{T}} R \left(\frac{\mathrm{d}z(s)}{\mathrm{d}s}\right)\mathrm{d}s. \tag{8}$$

Lemma 2.2. *(*[27]*) For any vector $x, y \in \mathbb{R}^n$ and one positive definite matrix $Q > 0$, the following inequality holds*

$$2x^{\mathrm{T}} y \le x^{\mathrm{T}} Q^{-1} x + y^{\mathrm{T}} Q y. \tag{9}$$

3. **Main Results.** In this section, we present an H_∞ boundary control criterion by Lyapunov functional method and Poincaré inequality.

First of all, boundary controllers for system (3) are designed as follows

$$u_i(t) = K_i \int_0^1 y(x,t)\mathrm{d}x, \tag{10}$$

where $K_i \in \mathbb{R}^{n\times n}$ denotes the control gain.

For convenience, we suppress (x,t) and denote $y(x,t)$ and $y(x,t-\tau)$ by y and y_τ when it does not cause confusion.

Now, we are on the point to give the main result.

Theorem 3.1. *There exist $\varepsilon > 0$, $q_i > 0$, if matrices K_i, $1 \le i \le N$ such that the following matrices*

$$\Psi(i) = \begin{pmatrix} M_i + q_i\left(D_i K_i + K_i^{\mathrm{T}} D_i\right) + q_i K_i^{\mathrm{T}} D_i K_i & 0 \\ 0 & \left(\varepsilon^{-1} L q_i + q_i L_2 - 1\right) I_n \end{pmatrix}$$

are negative semi-definite, where $M_i = \left(1 + q_i + q_i\varepsilon + q_i\gamma^{-2} + q_iL_1 + \sum_{j\in S} p_{ij}q_j\right) I_n$, *then system (3) can achieve finite horizon* H_∞ *performance in the mean square.*

Proof: Consider the following integral Lyapunov-Krasovskii functional

$$V(y(\cdot,t),i) = \int_0^1 q_i y^{\mathrm{T}} y \mathrm{d}x + \int_0^1 \int_{t-\tau}^t y^{\mathrm{T}}(x,s)y(x,s)\mathrm{d}s\mathrm{d}x.$$

Using the generalized Itô formula (see [28]) and computing $\mathrm{d}V$ along system (3), we can obtain

$$\begin{aligned}\mathrm{d}V(y(\cdot,t),i) = \int_0^1 & \left[y^{\mathrm{T}} y - y_\tau^{\mathrm{T}} y_\tau + 2q_i y^{\mathrm{T}}\left(f_i(y_\tau) + D_i\frac{\partial^2 y}{\partial x^2} + v\right)\right.\\ &+ \mathrm{trace}\left[g_i^{\mathrm{T}}(y,y_\tau) I_n q_i g_i(y,y_\tau)\right]\\ &\left.+ \sum_{j\in S} p_{ij}\left(q_j y^{\mathrm{T}} y + \int_{t-\tau}^t y^{\mathrm{T}}(x,s)\, y(x,s)\,\mathrm{d}s\right)\right]\mathrm{d}x\mathrm{d}t \\ &+ \int_0^1 2q_i y^{\mathrm{T}} g_i(y,y_\tau)\,\mathrm{d}x\mathrm{d}B(t).\end{aligned} \tag{11}$$

For the given constants γ and t_f, we have $V(y(\cdot,0),i) = 0$ when $y(x,t) = 0,\ t \in [-\tau, 0]$, then

$$\begin{aligned}&\mathbb{E}\int_0^{t_f}\int_0^1 q_i\left(y^{\mathrm{T}} y - \gamma^2 v^{\mathrm{T}} v\right)\mathrm{d}x\mathrm{d}t\\ &= \mathbb{E}\int_0^{t_f}\int_0^1 q_i\left(y^{\mathrm{T}} y - \gamma^2 v^{\mathrm{T}} v\right)\mathrm{d}x\mathrm{d}t + \mathbb{E}\int_0^{t_f}\mathrm{d}V + \mathbb{E}V(y(\cdot,0),i) - \mathbb{E}V(y(\cdot,t_f),i)\\ &\le \mathbb{E}\int_0^{t_f}\int_0^1 q_i\left(y^{\mathrm{T}} y - \gamma^2 v^{\mathrm{T}} v\right)\mathrm{d}x\mathrm{d}t + \mathbb{E}\int_0^{t_f}\int_0^1 \left[y^{\mathrm{T}} y - y_\tau^{\mathrm{T}} y_\tau + 2q_i y^{\mathrm{T}} f_i(y_\tau)\right.\\ &\quad \left.+ 2q_i y^{\mathrm{T}} D_i\frac{\partial^2 y}{\partial x^2} + 2q_i y^{\mathrm{T}} v + \mathrm{trace}\left[g_i^{\mathrm{T}}(y,y_\tau) I_n q_i g_i(y,y_\tau)\right] + \sum_{j\in S} p_{ij}q_j y^{\mathrm{T}} y\right]\mathrm{d}x\mathrm{d}t\\ &= \mathbb{E}\int_0^{t_f}\int_0^1 \left[q_i\left[-\gamma^2\left(v - \gamma^{-2}y\right)^{\mathrm{T}}\left(v - \gamma^{-2}y\right)\right] + q_i y^{\mathrm{T}} y + q_i\gamma^{-2}y^{\mathrm{T}} y + y^{\mathrm{T}} y\right.\\ &\quad - y_\tau^{\mathrm{T}} y_\tau + 2q_i y^{\mathrm{T}} f_i(y_\tau) + 2q_i y^{\mathrm{T}} D_i\frac{\partial^2 y}{\partial x^2} + \mathrm{trace}\left[g_i^{\mathrm{T}}(y,y_\tau) I_n q_i g_i(y,y_\tau)\right]\\ &\quad \left.+ \sum_{j\in S} p_{ij}q_j y^{\mathrm{T}} y\right]\mathrm{d}x\mathrm{d}t\\ &\le \mathbb{E}\int_0^{t_f}\int_0^1 \left[q_i y^{\mathrm{T}} y + q_i\gamma^{-2}y^{\mathrm{T}} y + y^{\mathrm{T}} y - y_\tau^{\mathrm{T}} y_\tau + 2q_i y^{\mathrm{T}} f_i(y_\tau) + 2q_i y^{\mathrm{T}} D_i\frac{\partial^2 y}{\partial x^2}\right.\\ &\quad \left.+ \mathrm{trace}\left[g_i^{\mathrm{T}}(y,y_\tau) I_n q_i g_i(y,y_\tau)\right] + \sum_{j\in S} p_{ij}q_j y^{\mathrm{T}} y\right]\mathrm{d}x\mathrm{d}t.\end{aligned} \tag{12}$$

According to Assumption 2.1, there exists $\varepsilon > 0$ such that

$$2y^{\mathrm{T}} f_i(y_\tau) = y^{\mathrm{T}} f_i(y_\tau) + f_i^{\mathrm{T}}(y_\tau)y \le \varepsilon y^{\mathrm{T}} y + \varepsilon^{-1} f_i^{\mathrm{T}}(y_\tau)f_i(y_\tau) \le \varepsilon y^{\mathrm{T}} y + \varepsilon^{-1} L y_\tau^{\mathrm{T}} y_\tau. \tag{13}$$

Integrating by parts and employing the boundary conditions (5), it yields

$$\begin{aligned}\int_0^1 y^{\mathrm{T}} D_i\frac{\partial^2 y}{\partial x^2}\mathrm{d}x &= y^{\mathrm{T}} D_i\frac{\partial y}{\partial x}\bigg|_{x=0}^{x=1} - \int_0^1 \frac{\partial y^{\mathrm{T}}}{\partial x}D_i\frac{\partial y}{\partial x}\mathrm{d}x\\ &= \int_0^1 y^{\mathrm{T}}(1,t)D_iK_i y\mathrm{d}x - \int_0^1 \frac{\partial y^{\mathrm{T}}}{\partial x}D_i\frac{\partial y}{\partial x}\mathrm{d}x.\end{aligned}$$

Letting $\bar{y}(x,t) = y(x,t) - y(1,t)$, we have $\bar{y}(1,t) = y(1,t) - y(1,t) = 0$ and

$$\frac{\partial y^{\mathrm{T}}}{\partial x} D_i \frac{\partial y}{\partial x} = \frac{\partial (y - y(1,t))^{\mathrm{T}}}{\partial x} D_i \frac{\partial (y - y(1,t))}{\partial x}.$$

With Lemma 2.1, we get

$$\int_0^1 y^{\mathrm{T}} D_i \frac{\partial^2 y}{\partial x^2} \mathrm{d}x \le \int_0^1 y^{\mathrm{T}}(1,t) D_i K_i y \mathrm{d}x - \frac{1}{4}\pi^2 \int_0^1 \bar{y}^{\mathrm{T}} D_i \bar{y} \mathrm{d}x. \tag{14}$$

Combining (12), (13) and (14), we get

$$\begin{aligned}
&\mathbb{E}\int_0^{t_f}\int_0^1 q_i\left(y^{\mathrm{T}}y - \gamma^2 v^{\mathrm{T}} v\right)\mathrm{d}x\mathrm{d}t \\
\le\ &\mathbb{E}\int_0^{t_f}\int_0^1 \Bigg[q_i y^{\mathrm{T}}y + q_i\gamma^{-2}y^{\mathrm{T}}y + y^{\mathrm{T}}y - y_\tau^{\mathrm{T}}y_\tau + \varepsilon q_i y^{\mathrm{T}}y + \varepsilon^{-1}q_i L y_\tau^{\mathrm{T}}y_\tau \\
&+ 2q_i y^{\mathrm{T}}(1,t) D_i K_i y - \frac{\pi^2}{2}q_i\bar{y}^{\mathrm{T}}D_i\bar{y} + q_i L_1 y^{\mathrm{T}}y + q_i L_2 y_\tau^{\mathrm{T}}y_\tau + \sum_{j\in S} p_{ij}q_j y^{\mathrm{T}}y\Bigg]\mathrm{d}x\mathrm{d}t \\
=\ &\mathbb{E}\int_0^{t_f}\int_0^1 \Bigg[q_i y^{\mathrm{T}}y + q_i\gamma^{-2}y^{\mathrm{T}}y + y^{\mathrm{T}}y - y_\tau^{\mathrm{T}}y_\tau + \varepsilon q_i y^{\mathrm{T}}y + \varepsilon^{-1}q_i L y_\tau^{\mathrm{T}}y_\tau \\
&+ 2q_i\left(y^{\mathrm{T}} - \bar{y}^{\mathrm{T}}\right) D_i K_i y - \frac{\pi^2}{2}q_i\bar{y}^{\mathrm{T}}D_i\bar{y} + q_i L_1 y^{\mathrm{T}}y + q_i L_2 y_\tau^{\mathrm{T}}y_\tau + \sum_{j\in S} p_{ij}q_j y^{\mathrm{T}}y\Bigg]\mathrm{d}x\mathrm{d}t \\
=\ &\mathbb{E}\int_0^{t_f}\int_0^1 \Bigg[y^{\mathrm{T}}\left(\left(1 + q_i + q_i\varepsilon + q_i\gamma^{-2} + q_i L_1\right)I_n + 2q_i D_i K_i\right)y \\
&+ y_\tau^{\mathrm{T}}\left(\varepsilon^{-1}q_i L + q_i L_2 - 1\right)I_n y_\tau - 2q_i\bar{y}^{\mathrm{T}}D_i K_i y - \frac{\pi^2}{2}q_i\bar{y}^{\mathrm{T}}D_i\bar{y} + \sum_{j\in S} p_{ij}q_j y^{\mathrm{T}}y\Bigg]\mathrm{d}x\mathrm{d}t.
\end{aligned} \tag{15}$$

By virtue of Lemma 2.2, we get

$$-2q_i\bar{y}^{\mathrm{T}}D_i K_i y \le q_i\bar{y}^{\mathrm{T}}D_i\bar{y} + q_i y^{\mathrm{T}}K_i^{\mathrm{T}}D_i K_i y. \tag{16}$$

Substituting (16) into (15) gives

$$\begin{aligned}
&\mathbb{E}\int_0^{t_f}\int_0^1 q_i\left(y^{\mathrm{T}}y - \gamma^2 v^{\mathrm{T}} v\right)\mathrm{d}x\mathrm{d}t \\
\le\ &\mathbb{E}\int_0^{t_f}\int_0^1 \Bigg[y^{\mathrm{T}}\Bigg(\Bigg(1 + q_i + q_i\varepsilon + q_i\gamma^{-2} + q_i L_1 + \sum_{j\in S}p_{ij}q_j\Bigg)I_n + 2q_i D_i K_i \\
&+ q_i K_i^{\mathrm{T}}D_i K_i\Bigg)y + y_\tau^{\mathrm{T}}\left(\varepsilon^{-1}q_i L + q_i L_2 - 1\right)I_n y_\tau + \bar{y}^{\mathrm{T}}\left(q_i D_i - \frac{\pi^2}{2}q_i D_i\right)\bar{y}\Bigg]\mathrm{d}x\mathrm{d}t \\
\le\ &\mathbb{E}\int_0^{t_f}\int_0^1 \left[y^{\mathrm{T}}\left(M_i + 2q_i D_i K_i + q_i K_i^{\mathrm{T}}D_i K_i\right)y\right. \\
&\left.+ y_\tau^{\mathrm{T}}\left(\varepsilon^{-1}q_i L + q_i L_2 - 1\right)I_n y_\tau\right]\mathrm{d}x\mathrm{d}t \\
=\ &\frac{1}{2}\mathbb{E}\int_0^{t_f}\int_0^1 \left[y^{\mathrm{T}}\left(2M_i + 2q_i D_i K_i + 2q_i K_i^{\mathrm{T}}D_i + 2q_i K_i^{\mathrm{T}}D_i K_i\right)y\right. \\
&\left.+ 2y_\tau^{\mathrm{T}}\left(\varepsilon^{-1}q_i L + q_i L_2 - 1\right)I_n y_\tau\right]\mathrm{d}x\mathrm{d}t \\
=\ &\mathbb{E}\int_0^{t_f}\int_0^1 \begin{pmatrix} y^{\mathrm{T}} & y_\tau^{\mathrm{T}}\end{pmatrix}\Psi(i)\begin{pmatrix} y \\ y_\tau\end{pmatrix}\mathrm{d}x\mathrm{d}t \le 0.
\end{aligned} \tag{17}$$

Since $q_i > 0$, we must have

$$\mathbb{E}\int_0^{t_f}\int_0^1 \left(y^{\mathrm{T}}y - \gamma^2 v^{\mathrm{T}}v\right)\mathrm{d}x\mathrm{d}t \le 0. \tag{18}$$

That is the desired result.

Remark 3.1. *When system (3) reduces to a one-dimensional system, from the second inequality in (17), it is calculated that the control gain* K_i *exists if* $q_iD_i \geq M_i$. *More precisely, the value range of* K_i *satisfies* $-1-\sqrt{1-\frac{M_i}{q_iD_i}} \leq K_i \leq -1+\sqrt{1-\frac{M_i}{q_iD_i}}$. *What we should point out is that the* M_i *can be negative if we choose the appropriate value of* p_{ij}. *The smaller* M_i *is, the wider the value range of* K_i *is. Besides, when* M_i *is negative, we observe that the smaller the diffusion coefficient* D_i *is, the wider the value of* K_i *is. Thus, the transition rates and diffusion item do have an effect on mean square* H_∞ *performance for SDMRDSs.*

4. **Numerical Example.** Let $r(\cdot)$ be a right-continuous Markov process taking values in $S=\{1,2\}$ with generator

$$\Gamma = (p_{ij})_{2\times 2} = \begin{pmatrix} -2 & 2 \\ 1 & -1 \end{pmatrix}. \tag{19}$$

Consider the one-dimensional stochastic delay reaction-diffusion system with Markovian switching of the form

$$\begin{aligned} \mathrm{d}y(x,t) = &\left[A_iy(x,t-0.1)+D_i\frac{\partial^2 y(x,t)}{\partial x^2}+0.14\cos(\pi t)+0.1\sin(2x)\right]\mathrm{d}t \\ &+[C_iy+H_iy(x,t-0.1)]\mathrm{d}B(t),\ x\in(0,1),\ t>0. \end{aligned} \tag{20}$$

When $r(t)=1$, we take $A_1=0.36$, $D_1=10$, $C_1=-0.15$, $H_1=-0.65$, that is

$$\begin{aligned} \mathrm{d}y(x,t) = &\left[0.36y(x,t-0.1)+10\frac{\partial^2 y(x,t)}{\partial x^2}+0.14\cos(\pi t)+0.1\sin(2x)\right]\mathrm{d}t \\ &-[0.15y+0.65y(x,t-0.1)]\mathrm{d}B(t),\ x\in(0,1),\ t>0, \end{aligned} \tag{21}$$

and when $r(t)=2$, we take $A_2=0.31$, $D_2=5$, $C_2=0.17$, $H_2=-0.51$, that is

$$\begin{aligned} \mathrm{d}y(x,t) = &\left[0.31y(x,t-0.1)+5\frac{\partial^2 y(x,t)}{\partial x^2}+0.14\cos(\pi t)+0.1\sin(2x)\right]\mathrm{d}t \\ &+[0.17y-0.51y(x,t-0.1)]\mathrm{d}B(t),\ x\in(0,1),\ t>0. \end{aligned} \tag{22}$$

Take the zero initial value, i.e.,

$$y(x,t)=0,\ t\in[-0.1,0],\ x\in(0,1). \tag{23}$$

The boundary controllers for system (20) are designed as follows

$$u_1(t)=u_2(t)=-1\int_0^1 y(x,t)\mathrm{d}x. \tag{24}$$

By letting $\gamma=1$, $t_f=1$, we take $\epsilon=1$, $q_1=0.65$, $q_2=1$. And we verify that all the conditions stated in Theorem 3.1 have been satisfied. Thus, system (20) achieves finite horizon H_∞ performance in the mean square.

To show the effectiveness of our control design, numerical calculation gives

$$\frac{\mathbb{E}\int_0^1\int_0^1 y^{\mathrm{T}}y\mathrm{d}x\mathrm{d}t}{\mathbb{E}\int_0^1\int_0^1 v^{\mathrm{T}}v\mathrm{d}x\mathrm{d}t} = 0.5915^2 < 1^2 = \gamma^2, \tag{25}$$

which is in accordance with the theoretical result.

For comparison, we take control strategy out, i.e., $K_1=K_2=0$, numerical calculation gives

$$\frac{\mathbb{E}\int_0^1\int_0^1 y^{\mathrm{T}}y\mathrm{d}x\mathrm{d}t}{\mathbb{E}\int_0^1\int_0^1 v^{\mathrm{T}}v\mathrm{d}x\mathrm{d}t} = 1.1831^2 > 1^2 = \gamma^2. \tag{26}$$

The above result indicates that mean square H_∞ performance is not valid. Therefore, our boundary control strategy is achieved.

5. **Conclusions.** In this paper, H_∞ boundary control of stochastic reaction-diffusion systems with Markovian switching and time delays has been investigated. First of all, suitable boundary controllers have been constructed. Moreover, by choosing Lyapunov functional and using Poincaré inequality, a sufficient condition ensuring mean square H_∞ performance for stochastic delay reaction-diffusion systems with Markovian switching are given under the given controllers. The effects of transition rate and diffusion item on mean square H_∞ performance are presented. Besides, a numerical simulation is performed to substantiate the effectiveness of the obtained result and the rationality of controller design. It should be pointed out that the effect of time delays is not significant due to the simple form of auxiliary functional. We can further study the effect of time delays or time-varying delays on stochastic reaction-diffusion systems with Markovian switching.

Acknowledgment. This work is supported by Natural Science Foundations of Shandong Province under Grant ZR2018MF018.

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