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NEW CONSTRUCTIONS OF ALMOST DIFFERENCE SET PAIRS BASED ON CHINESE REMAINDER THEORY AND CYCLOTOMY

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Abstract. *Almost difference set pair, proposed as a mathematical tool for the study of binary sequence pairs, has significant applications in cryptology and coding theory. In this letter, two new construction methods for almost difference set pairs over Z_{2v} are presented based on Chinese remainder theory and cyclotomic class of order 2, where $v = 2f + 1$ is an odd prime. These new construction methods will greatly expand the number of almost different set pairs and provide more discrete signals for practical engineering demand.*
Keywords: Almost difference set pairs, Generalized cyclotomy, Binary sequence pairs, Chinese remainder theory

1. **Introduction.** Sequences pairs with ideal correlation have many applications in Code Division Multiple Access (CDMA) systems, radar, signal processing and source coding in spectrum communication systems [1]. Almost Difference Set Pair (ADSP) is a mathematical tool to construct sequence pairs with three-level correlation, which is first presented in [2]. However, the set H of the ADSPs in [2] only contains two elements. Later, a new definition of ADSP was proposed in [3], which is more universal than that in [2]. Meanwhile, it was proved that an ADSP is equivalent to a binary sequence pair with three-level correlation. Therefore, ADSP has drawn the attention of researches in spread spectrum communication, coding theory and applied mathematics. So far, the known constructions of ADSPs are as follows.

(1) ADSPs with period $4q$, $4nq$, $4nq(q+2)$ and $4n(2^t-1)$ are constructed in [3] based on cyclotomy and interleaving technique, where q is an odd prime and $gcd(q,n) = 1$.

(2) Several classes of ADSPs of period $N = 2q$ are constructed in [4] based on cyclotomic classes of order 4, and the corresponding binary sequence pairs of these ADSPs have good out-phase cross-correlation value $\{2, -2\}$. It is the first time that optimal binary sequence pairs with out-phase cross-correlation value $\{2, -2\}$ have been constructed.

(3) The ADSPs on two times prime v of residual class ring Z_{2v} are constructed in [5] based on cyclotomic classes and Chinese Remainder Theorem.

(4) Recently, the properties of Whiteman generalized cyclotomic classes were studied and the constructions of ADSP with period $N = pq$ were proposed in [6] based on Whiteman generalized cyclotomy.

In this letter, we will present two new construction methods for ADSPs with period $N = 2v$ based on Chinese remainder theory and cyclotomic classes of order 2, where $v = 2f + 1$ is an odd prime. The ADSPs constructed by these new methods have new parameter forms that have not been found by previous construction methods. So, these new construction methods will greatly enrich the theoretical research of ADSP and provide more binary sequence pairs with three-level correlation for engineering needs.

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This letter is organized as follows. The notations and the related lemmas needed for the remainder of this letter will be introduced in Section 2. The new construction methods of ADSPs and examples are given in Section 3. Finally, the concluding statement for this letter is made in Section 4.

2. Preliminaries.

2.1. Almost difference set pair.

Here and here after, $|S|$ denotes the number of elements in set S and 'a mod b' denotes the least nonnegative integer which is congruent to 'a modulo b'.

Definition 2.1. [7] *Let B, B' be two subsets of the residue class ring Z_N, and then the difference function $d_{B,B'}(\theta)$ was defined as*

$$d_{B,B'}(\theta) = |B' \cap (B + \theta)|. \tag{1}$$

where $B + \theta = \{g + \theta : g \in B\}$ and $g + \theta = ((g + \theta) \bmod N)$.

Definition 2.2. [3] *Let T be a subset of Z_N^*, $t = |T|$, $k = |B|$, $k' = |B'|$ and $c = |B \cap B'|$, and then (B, B') is called a $(N, k, k', c, \lambda, t)ADSP$ if arbitrary nonzero element g satisfies the equation*

$$d_{B,B'}(\theta) = \begin{cases} \lambda, & \text{if } g \in T \\ \lambda + 1, & \text{if } g \in Z_N^* - T \end{cases}$$

where λ is a constant and $Z_N^* = Z_N \setminus \{0\}$.

2.2. Cyclotomy.

Definition 2.3. [8] *Let $GF(v)$ be a finite field, where $v = ef + 1$ is an odd prime, e and f be positive integers, and ω be a primitive element of v, and define*

$$H_i^e = \{\omega^{i+et}, t = 0, 1, \ldots, f-1\}, \quad 0 \le i \le e-1,$$

and then these H_i^e are called cyclotomic classes of order e with respect to $GF(v)$.

It is clear that $|H_i^e| = f$, and it has been proved that $Z_v = \cup_{i=0}^{e-1} H_i^e \cup \{0\}$ and $|H_i^e \cap H_j^e| = 0$ if $i \ne j$.

Definition 2.4. [8] *For $0 \le i, j \le e-1$, let*

$$(i, j)_e = \left|(H_i^e + 1) \cap H_j^e\right|,$$

and then these constants $(i, j)_e$ are called cyclotomic numbers of order e.

Lemma 2.1. [7,8] *Let $g \in H_k^e$, $k = 0, \ldots, e-1$, then*
(1) $|H_j^e \cap (H_i^e + g)| = (i - k, j - k)_e$, where $i - k$ and $j - k$ take modulo e.
(2) $|H_i^e \cap \{g\}| = \begin{cases} 1, & \text{if } k = i \\ 0, & \text{otherwise.} \end{cases}$
(3) When f is odd,

$$|\{0\} \cap (H_i^e + g)| = \begin{cases} 1, & \text{if } k = i + e/2 \\ 0, & \text{otherwise.} \end{cases}$$

When f is even,

$$|\{0\} \cap (H_i^e + g)| = \begin{cases} 1, & \text{if } k = i \\ 0, & \text{otherwise.} \end{cases}$$

Lemma 2.2. [8] *Some properties of cyclotomic number are as follows.*
(1) $(i', j')_e = (i, j)_e$, where $i' \equiv i \pmod e$, $j' \equiv j \pmod e$;
(2) $(i, j)_e = (e - i, j - i)_e$, where $e - i$ and $j - i$ take modulo e.

Lemma 2.3. [8] *The cyclotomic numbers of order 2 over Z_v are as follows.*
(1) If $v \equiv 1 \pmod 4$ then

$$(0,1)_2 = (1,1)_2 = (1,0)_2 = (v-1)/4,$$
$$(0,0)_2 = (v-5)/4.$$

(2) If $v \equiv 3 \pmod 4$ then

$$(0,0)_2 = (1,1)_2 = (1,0)_2 = (v-3)/4,$$
$$(0,1)_2 = (v+1)/4.$$

2.3. **Generalized cyclotomic class of order 2 over Z_{2v}.** For simplicity, let H_0 and H_1 denote cyclotomic classes of order 2 here and here after.

Definition 2.5. *Let $v = 2f+1$ be an odd prime, and H_j denote the cyclotomic classes of order 2 over Z_v and Z_{2v} be a residue class ring. According to the Chinese Remainder Theorem, $Z_{2v} \cong Z_2 \times Z_v$ under the isomorphism $f(\theta) = (\theta_1, \theta_2)$, where $\theta_1 = \theta \bmod 2$, $\theta_2 = \theta \bmod v$. Let $D_{i,j} = f^{-1}(\{i\} \times H_j)$, $i, j = 0, 1$, and then $D_{i,j}$ is called a generalized cyclotomic class of order 2 over Z_{2v}.*

Obviously, $|D_{i,j}| = |H_j| = f$ if $j = 0, 1$. Set $H_2 = \{0\}$, and then we can get

$$D_{0,2} = f^{-1}(\{0\} \times H_2) = f^{-1}(0,0) = \{0\},$$
$$D_{1,2} = f^{-1}(\{1\} \times H_2) = f^{-1}(1,0) = \{v\}.$$

3. New Construction Methods for ADSP.

3.1. Construction methods.

Theorem 3.1. *For an odd prime $v = 2f+1$, let B and B' be two subsets of the residue class ring Z_{2v}. If $B = D_{1,2}$, $B' \in \{D_{0,h} \cup D_{1,h} \cup D_{1,2}, D_{0,h} \cup D_{1,h+1} \cup D_{1,2}, D_{k,0} \cup D_{k,1} \cup D_{1,2}\}$, then (B, B') is a $(2v, 1, v, 1, 0, n)$ADSP, where $k, h \in Z_2 = \{0, 1\}$ and $h + 1 = (h + 1) \bmod 2$.*

Proof: Firstly, we consider $B' = D_{0,h} \cup D_{1,h} \cup D_{1,2}$. According to Definition 2.5, we can calculate the parameters of ADSP as follows.

$$k = |B| = |D_{1,2}| = |\{v\}| = 1,$$
$$k' = |B'| = 2f + 1 = v,$$
$$c = |B \cap B'| = |D_{1,2}| = 1.$$

When $h = 0$, $B' = D_{0,0} \cup D_{1,0} \cup D_{1,2}$. Let $f(\theta) = (\theta_1, \theta_2)$, where $\theta_1 \in \{0, 1\}$, $\theta_2 \in H_j$, $j \in \{0, 1, 2\}$. By Definition 2.1, the difference function $d_{B,B'}(\theta)$ is

$$\begin{aligned} d_{B,B'}(\theta) &= |B' \cap (B + \theta)| \\ &= |(D_{0,0} \cup D_{1,0} \cup D_{1,2}) \cap (D_{1,2} + \theta)| \\ &= |D_{0,0} \cap (D_{1,2} + \theta)| + |D_{1,0} \cap (D_{1,2} + \theta)| + |D_{1,2} \cap (D_{1,2} + \theta)| \\ &= |(\{0\} \times H_0) \cap ((\{1\} \times \{0\}) + (\theta_1, \theta_2))| \\ &\quad + |(\{1\} \times H_0) \cap ((\{1\} \times \{0\}) + (\theta_1, \theta_2))| \\ &\quad + |(\{1\} \times \{0\}) \cap ((\{1\} \times \{0\}) + (\theta_1, \theta_2))| \\ &= |\{0\} \cap (\{1\} + \theta_1)||H_0 \cap \{\theta_2\}| + |\{1\} \cap (\{1\} + \theta_1)||H_0 \cap \{\theta_2\}| \\ &\quad + |\{1\} \cap (\{1\} + \theta_1)||\{0\} \cap \{\theta_2\}|. \end{aligned} \tag{2}$$

For $\theta \neq 0$, consider several cases.
Case 1: $\theta \in D_{0,0}$, then $\theta_1 = 0$ and $\theta_2 \in H_0$.
By Lemma 2.1, Equation (2) becomes

$$d_{B,B'}(\theta) = 1.$$

Case 2: $\theta \in D_{0,1}$, then $\theta_1 = 0$ and $\theta_2 \in H_1$.

By Lemma 2.1, Equation (2) becomes

$$d_{B,B'}(\theta) = 0.$$

Case 3: $\theta \in D_{1,0}$, then $\theta_1 = 1$ and $\theta_2 \in H_0$.

By Lemma 2.1, Equation (2) becomes

$$d_{B,B'}(\theta) = 1.$$

Case 4: $\theta \in D_{1,1}$, then $\theta_1 = 1$ and $\theta_2 \in H_1$.

By Lemma 2.1, Equation (2) becomes

$$d_{B,B'}(\theta) = 0.$$

Case 5: $\theta \in D_{1,2}$, then $\theta_1 = 1$ and $\theta_2 \in H_2 = \{0\}$.

By Lemma 2.1, Equation (2) becomes

$$d_{B,B'}(\theta) = 0.$$

Concluding from Cases 1-5, we can obtain

$$d_{B,B'}(\theta) = \begin{cases} 0, & \text{if } \theta \in D_{0,1} \cup D_{1,1} \cup D_{1,2} \\ 1, & \text{if } \theta \in D_{0,0} \cup D_{1,0}. \end{cases}$$

According to Definition 2.2, (B, B') is a $(2v, 1, v, 1, 0, v)ADSP$.

When $h = 1$ and B' takes the other two sets, they can also be proved by the same method.

Theorem 3.2. *For an odd prime $v = 2f + 1$, let B and B' be two subsets of the residue class ring Z_{2v}. (B, B') is a $(2v, v, v, (v+1)/2, (v-1)/2, v)ADSP$ if the values of B and B' satisfy one of the following three conditions:*

(1) $B = D_{k,0} \cup D_{k,1} \cup D_{x,2}$, $B' = D_{0,h} \cup D_{1,h+1} \cup D_{x,2}$,

(2) $B = D_{k,0} \cup D_{k,1} \cup D_{x,2}$, $B' = D_{0,h} \cup D_{1,h} \cup D_{x,2}$,

(3) $B = D_{0,k} \cup D_{1,k} \cup D_{k,2}$, $B' = D_{0,h} \cup D_{1,h+1} \cup D_{k,2}$,

where $k, h, x \in Z_2 = \{0, 1\}$ and $h + 1 = (h + 1) \bmod 2$.

Proof: Firstly, consider condition (1), that is, $B = D_{k,0} \cup D_{k,1} \cup D_{x,2}$, $B' = D_{0,h} \cup D_{1,h+1} \cup D_{x,2}$.

Let $k = h = x = 0$, then $B = D_{0,0} \cup D_{0,1} \cup D_{0,2}$, $B' = D_{0,0} \cup D_{1,1} \cup D_{0,2}$. According to Definition 2.2, we can calculate the parameters of ADSP as follows.

$$\begin{aligned} k &= |B| = 2f + 1 = v, \\ k' &= |B'| = 2f + 1 = v, \\ c &= |B \cap B'| = |D_{0,0} \cup D_{0,2}| = (v+1)/2. \end{aligned}$$

Let $f(\theta) = (\theta_1, \theta_2)$, where $\theta_1 \in \{0, 1\}$, $\theta_2 \in H_j$, $j \in \{0, 1, 2\}$. By Definition 2.1, the difference function $d_{B,B'}(\theta)$ is

$$\begin{aligned} d_{B,B'}(\theta) &= |B' \cap (B + \theta)| \\ &= |(D_{0,0} \cup D_{1,1} \cup D_{0,2}) \cap (D_{0,0} \cup D_{0,1} \cup D_{0,2} + \theta)| \\ &= |D_{0,0} \cap (D_{0,0} + \theta)| + |D_{1,1} \cap (D_{0,0} + \theta)| + |D_{1,2} \cap (D_{0,0} + \theta)| \\ &\quad + |D_{0,0} \cap (D_{0,1} + \theta)| + |D_{1,1} \cap (D_{0,1} + \theta)| + |D_{1,2} \cap (D_{0,1} + \theta)| \\ &\quad + |D_{0,0} \cap (D_{0,2} + \theta)| + |D_{1,1} \cap (D_{0,2} + \theta)| + |D_{1,2} \cap (D_{0,2} + \theta)| \\ &= |(\{0\} \times H_0) \cap ((\{0\} \times H_0) + (\theta_1, \theta_2))| \\ &\quad + |(\{1\} \times H_1) \cap ((\{0\} \times H_0) + (\theta_1, \theta_2))| \\ &\quad + |(\{1\} \times \{0\}) \cap ((\{0\} \times H_0) + (\theta_1, \theta_2))| \\ &\quad + |(\{0\} \times H_0) \cap ((\{0\} \times H_1) + (\theta_1, \theta_2))| \end{aligned}$$

$$
\begin{aligned}
&+|(\{1\}\times H_1)\cap((\{0\}\times H_1)+(\theta_1,\theta_2))|\\
&+|(\{1\}\times \{0\})\cap((\{0\}\times H_1)+(\theta_1,\theta_2))|\\
&+|(\{0\}\times H_0)\cap((\{0\}\times \{0\})+(\theta_1,\theta_2))|\\
&+|(\{1\}\times H_1)\cap((\{0\}\times \{0\})+(\theta_1,\theta_2))|\\
&+|(\{1\}\times \{0\})\cap((\{0\}\times \{0\})+(\theta_1,\theta_2))|\\
=\ &|\{0\}\cap\{\theta_1\}||H_0\cap(H_0+\theta_2)|+|\{1\}\cap\{\theta_1\}||H_1\cap(H_0+\theta_2)|\\
&+|\{0\}\cap\{\theta_1\}||\{0\}\cap(H_0+\theta_2)|+|\{0\}\cap\{\theta_1\}||H_0\cap(H_1+\theta_2)|\\
&+|\{1\}\cap\{\theta_1\}||H_0\cap(H_1+\theta_2)|+|\{0\}\cap\{\theta_1\}||\{0\}\cap(H_1+\theta_2)|\\
&+|\{0\}\cap\{\theta_1\}||H_0\cap\{\theta_2\}|+|\{1\}\cap\{\theta_1\}||H_1\cap\{\theta_2\}|\\
&+|\{0\}\cap\{\theta_1\}||\{0\}\cap\{\theta_1\}|. \qquad (3)
\end{aligned}
$$

For $\theta\neq 0$, consider several cases.

Case 1: $\theta\in D_{0,0}$, then $\theta_1=0$ and $\theta_2\in H_0$.

By Lemma 2.1 and Lemma 2.2, Equation (3) can be reduced to

$$d_{B,B'}(\theta)=(0,0)_2+|\{0\}\cap(H_0+\theta_2)|+(1,0)_2+|\{0\}\cap(H_1+\theta_2)|+1. \qquad (4)$$

The following two situations are discussed:

(1) When f is even, by Lemma 2.1 and Lemma 2.3, Equation (4) can be further reduced to

$$
\begin{aligned}
d_{B,B'}(\theta)&=(0,0)_2+1+(1,0)_2+1\\
&=(v-5)/4+1+(v-1)/4+1\\
&=(v-1)/2+1.
\end{aligned}
$$

(2) When f is odd, by Lemma 2.1 and Lemma 2.3, Equation (4) can be further reduced to

$$
\begin{aligned}
d_{B,B'}(\theta)&=(0,0)_2+1+(1,0)_2+1\\
&=(v-3)/4+1+(v-3)/4+1\\
&=(v-1)/2+1.
\end{aligned}
$$

In a word, when $\theta\in D_{0,0}$, $d_{B,B'}(\theta)=(v-1)/2+1$.

Case 2: $\theta\in D_{0,1}$, then $\theta_1=0$ and $\theta_2\in H_1$.

Similar to Case 1, by Lemma 2.1, Lemma 2.2, Lemma 2.3 and Equation (3), we can get

$$
\begin{aligned}
d_{B,B'}(\theta)&=(1,1)_2+|\{0\}\cap(H_0+\theta_2)|+(0,1)_2+|\{0\}\cap(H_1+\theta_2)|+1\\
&=(1,1)_2+1+(0,1)_2+1\\
&=(v-1)/2+1.
\end{aligned}
$$

So, when $\theta\in D_{0,1}$, $d_{B,B'}(\theta)=(v-1)/2+1$.

Case 3: $\theta\in D_{1,0}$, then $\theta_1=1$ and $\theta_2\in H_0$.

Similar to Case 1, by Lemmas 2.1-2.3 and Equation (3), we can get

$$d_{B,B'}(\theta)=(0,1)_2+(1,1)_2=(v-1)/2.$$

So, when $\theta\in D_{1,0}$, $d_{B,B'}(\theta)=(v-1)/2$.

Case 4: $\theta\in D_{1,1}$, then $\theta_1=1$ and $\theta_2\in H_1$.

Similar to Case 1, by Lemmas 2.1-2.3 and Equation (3), we can get

$$d_{B,B'}(\theta)=(1,0)_2+(0,0)_2+1=(v-1)/2.$$

So, when $\theta\in D_{1,1}$, $d_{B,B'}(\theta)=(v-1)/2$.

Case 5: $\theta\in D_{1,2}$, then $\theta_1=1$ and $\theta_2\in H_2=\{0\}$.

Equation (3) becomes

$$d_{B,B'}(\theta) = |H_1| = (v-1)/2.$$

So, when $\theta \in D_{1,2}$, $d_{B,B'}(\theta) = (v-1)/2$.
Concluding from Cases 1-5, we can obtain

$$d_{B,B'}(\theta) = \begin{cases} (v-1)/2, & \text{if } \theta \in D_{1,0} \cup D_{1,1} \cup D_{1,2}, \\ (v-1)/2+1, & \text{if } \theta \in D_{0,0} \cup D_{0,1}. \end{cases}$$

According to Definition 2.2, (B, B') is a $(2v, v, v, (v+1)/2, (v-1)/2, v)ADSP$.

When h, k, x take other values, they can also be proved by the same method.

In summary, when $B = D_{k,0} \cup D_{k,1} \cup D_{x,2}$, $B' = D_{0,h} \cup D_{1,h+1} \cup D_{x,2}$, (B, B') is a $(2v, v, v, (v+1)/2, (v-1)/2, v)ADSP$.

(B, B') in condition (2) and condition (3) can be proved to be a $(2v, v, v, (v+1)/2, (v-1)/2, v)ADSP$ in a similar manner.

3.2. **Examples.**

Example 3.1. *Let $v = 11$, and then the cyclotomic classes of order 2 over Z_{11} are as follows.*

$$H_0 = \{1, 3, 4, 5, 9\}, \quad H_1 = \{2, 6, 7, 8, 10\}.$$

Let $H_2 = \{0\}$, and then the generalized cyclotomic classes of order 2 over Z_{22} are as follows by Definition 2.5.

$$\begin{aligned} D_{0,0} &= f^{-1}(\{0\} \times H_0) = \{12, 4, 16, 20, 14\}, \\ D_{0,1} &= f^{-1}(\{0\} \times H_1) = \{2, 8, 10, 18, 6\}, \\ D_{1,0} &= f^{-1}(\{1\} \times H_0) = \{1, 15, 5, 9, 3\}, \\ D_{1,1} &= f^{-1}(\{1\} \times H_1) = \{13, 19, 21, 7, 17\}, \\ D_{0,2} &= f^{-1}(\{0\} \times H_2) = \{0\}, \\ D_{1,2} &= f^{-1}(\{1\} \times H_2) = \{1\}. \end{aligned}$$

According to Theorem 3.1, let $B = D_{1,2}$, $B' = D_{0,h} \cup D_{1,h} \cup D_{1,2}$. When $h = 0$, we can get

$$\begin{aligned} B &= D_{1,2} = \{11\}, \\ B' &= D_{0,0} \cup D_{1,0} \cup D_{1,2} = \{12, 4, 16, 20, 14, 1, 15, 5, 9, 3, 11\}. \end{aligned}$$

Then (B, B') is a $(22, 1, 11, 1, 0, 11)ADSP$.

When $h = 1$, we can get

$$\begin{aligned} B &= D_{1,2} = \{11\}, \\ B' &= D_{0,1} \cup D_{1,1} \cup D_{1,2} = \{2, 8, 10, 18, 6, 13, 19, 21, 7, 17, 11\}. \end{aligned}$$

(B, B') is also a $(22, 1, 11, 1, 0, 11)ADSP$.

Example 3.2. *Let $v = 19$, and then the cyclotomic classes of order 2 over Z_{19} are as follows.*

$$\begin{aligned} H_0 &= \{1, 4, 16, 7, 9, 17, 11, 6, 5\}, \\ H_1 &= \{2, 8, 13, 14, 18, 15, 3, 12, 10\}. \end{aligned}$$

Let $H_2 = \{0\}$, and then the generalized cyclotomic classes of order 2 over Z_{38} are as follows by Definition 2.5.

$$\begin{aligned} D_{0,0} &= f^{-1}(\{0\} \times H_0) = \{20, 4, 16, 26, 28, 36, 30, 6, 24\}, \\ D_{0,1} &= f^{-1}(\{0\} \times H_1) = \{2, 8, 32, 14, 18, 34, 22, 12, 10\}, \\ D_{1,0} &= f^{-1}(\{1\} \times H_0) = \{1, 23, 35, 7, 9, 17, 11, 25, 5\}, \\ D_{1,1} &= f^{-1}(\{1\} \times H_1) = \{21, 27, 13, 33, 37, 15, 3, 31, 29\}, \end{aligned}$$

$$D_{0,2} = f^{-1}(\{0\} \times H_2) = \{0\},$$
$$D_{1,2} = f^{-1}(\{1\} \times H_2) = \{1\}.$$

According to Theorem 3.2, let $B = D_{0,0} \cup D_{0,1} \cup D_{0,2}$, $B' = D_{0,0} \cup D_{1,0} \cup D_{0,2}$. *When* $h = 0$, *we can get*

$$\begin{aligned} B &= D_{0,0} \cup D_{0,1} \cup D_{0,2} \\ &= \{20, 4, 16, 26, 28, 36, 30, 6, 24, 2, 8, 32, 14, 18, 34, 22, 12, 10, 0\}, \\ B' &= D_{0,0} \cup D_{1,0} \cup D_{0,2} \\ &= \{20, 4, 16, 26, 28, 36, 30, 6, 24, 21, 27, 13, 33, 37, 15, 3, 31, 29, 0\}. \end{aligned}$$

Then (B, B') *is a* $(38, 19, 19, 10, 9, 19)$*ADSP.*

4. **Conclusions.** In this letter, two new construction methods for almost different set pairs are proposed by using generalized cyclotomic classes of order 2 over residual classes ring Z_{2v}. These new construction methods not only expanded the number of almost different set pairs, but also enriched the combination design theory. The constructions of ADSPs with period $4v$ based on Chinese Remainder Theorem and cyclotomic will be our further research point.

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REFERENCES

[1] K. T. Arasu, D. Arya and A. Bakshi, Constructions of punctured difference set pairs and their corresponding punctured binary array pairs, *IEEE Trans. Information Theory*, vol.61, no.4, pp.2191-2199, 2015.

[2] J. Z. Li and P. H. Ke, Study on the almost difference set pairs and almost perfect autocorrelation binary sequence pairs, *Journal of Wuyi University*, vol.27, no.2, pp.10-14, 2008.

[3] X. Peng, C. Xu and K. T. Arasu, New families of binary sequence pairs with two-level and three-level correlation, *IEEE Trans. Information Theory*, vol.58, no.11, pp.6968-6978, 2012.

[4] X. Liu, J. Wang and D. Wang, Two new classes of binary sequence pairs with three-level cross-correlation, *Advances in Mathematics of Communications*, vol.9, no.1, pp.117-128, 2015.

[5] Y. Jia, Y. Shen, X. Shen and J. Wang, Some new constructions of the almost difference set pairs based on generalized cyclotomic classes, *International Journal of Innovative Computing, Information and Control*, vol.12, no.2, pp.467-475, 2016.

[6] Y. Mu, J. Wang, X. Song and Y. Jia, Some new constructions of the almost difference set pairs based on Whiteman generalized cyclotomic, *ICIC Express Letters*, vol.11, no.7, pp.1245-1251, 2017.

[7] X. Shen, Y. Jia, X. Song and Y. Li, New construction methods for binary sequence pairs of period pq with ideal two-level correlation, *IEICE Trans. Fundamentals of Electronics, Communications and Computer Sciences*, vol.E101-A, no.4, pp.704-712, 2018.

[8] T. W. Cusick, C. Ding and A. Renvall, *Stream Ciphers and Number Theory*, Elsevier, Amsterdam, 1998.