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STABILIZATION DESIGN OF A CLASS OF SWITCHED POSITIVE LINEAR SYSTEMS WITH LINEAR PROGRAMMING

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Abstract. *This paper investigates the stabilization design of a class of switched positive linear systems under two classes of the dwell time switching signals. One class of dwell time switching signals is confined by a certain pair of upper and lower bounds, and the other is the minimum dwell time. The distinguishing feature of the proposed method is the application of a type of multiple time-varying linear copositive Lyapunov functions. A more efficient controller design method is also established. The computable sufficient conditions on the stabilization are obtained in the framework of dwell-time switching and the controller gains are solved by linear programming technique. Finally, a numerical example is provided to verify the validity of the proposed design.*
Keywords: Switched positive linear systems, Stabilization, Dwell time, Linear programming

1. **Introduction.** Switched positive linear systems are a class of switched systems composed of a family of positive linear subsystems. Switched positive linear systems arise in a variety of applications [1, 2]. In the last few years, a considerable effort has been devoted to the development of both theories and applications of switched positive linear systems [3, 4, 5]. The main concern in the study of switched positive linear systems is the issue of stability. Due to the fact that the state variables are confined to the positive orthant, it is very difficult to study the stability of switched positive linear systems. Thus, it is impossible to solve the stability of switched positive linear systems by using well developed methods for general switched systems [6, 7, 8]. Therefore, various methods have been proposed to study the stability under arbitrary switchings of switched positive linear systems by using the common vector-parameterized copositive Lyapunov function [9], the common quadratic copositive Lyapunov function [3], and the switched linear copositive Lyapunov function [10]. Most switched systems in practice, however, are not stability under arbitrary switchings, yet they still may be asymptotically stable under some properly chosen switching law based on the multiple linear copositive Lyapunov function [11, 12]. Many results in this direction have been available (see, for example, [13, 14]). For switched positive linear systems under time-constrained switching, it has been shown that multiple linear copositive Lyapunov functions have the advantage of extra flexibilities in system analysis and synthesis [15, 16]. It is known that a dwell time of active subsystem can subside for potentially possible large state transients [7, 8]. [17] studied the issues of robust stability analysis for class switched positive linear systems with interval uncertainties by constructing a class of multiple time-varying linear copositive Lyapunov functions.

On the other hand, stabilization is another of the basic fundamental issues in switched positive linear systems [18]. For the stabilization of switched positive systems, the controller designed not only guarantees the stability, but also ensures the positive of the

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closed-loop switched systems. Using multiple linear copositive Lyapunov functions associated with linear programming, [19, 20] have investigated the stabilization of continuous-time and discrete-time switched positive linear systems, respectively. To remove some restrictions on the heavy computational burden, [21] proposed the controller design approach to switched positive systems. It is worth noting that there is still a lot of room for improvements in the aforementioned results since there exist some restrictions in the controller design. It is known that, for switched positive systems, the multiple time-varying linear copositive Lyapunov functions may reduce some conservatism of stability analysis [8, 17, 22, 23], and it may also be an effective tool for stabilization design.

In this paper, we will investigate the stabilization of a class of switched positive systems under two classes of the dwell time switching signals. We apply a type of multiple time-varying linear copositive Lyapunov functions to obtain the computable sufficient conditions on the stabilization of such switched systems in the framework of dwell-time switching. The controller gains can be solved by linear programming technique. Lastly, an example is provided to demonstrate the effectiveness of the proposed result. The main contributions of this manuscript are summarized as follows. 1) A type of multiple time-varying linear copositive Lyapunov functions is utilized to achieve the stabilization of switched positive linear systems. 2) A dwell time is pregiven and then check if there exists a feasible solution to the linear programming conditions for such a dwell time, which may reduce much conservatism.

This paper is organized as follows. Section 2 presents the problem formulation. Section 3 shows the main results. In Section 4, an illustrative example is presented to demonstrate the effectiveness of the proposed method. Finally, Section 5 concludes the paper.

Notations: The notations used in this paper are standard. N is the set of nonnegative and positive integer; $\Re$, $\Re^n$, and $\Re^{n\times n}$ denote the fields of real numbers, n-tuples of real numbers, and the space of $n \times n$ real matrices, respectively. Define $\|x\|_1$ and $\|x\|_2$ as 1-norm and Euclidean norm of $x \in \Re^n$, respectively; $\Re^+$ represents the set of positive real numbers. A^T is the transpose of matrix A. I_n is the $n \times n$ identity matrix. a_{ij} stands for the element in the ith row and the jth column of matrix A, and $A \succ 0$ ($A \succeq 0$) means $a_{ij} > 0$ ($a_{ij} \geq 0$) for $i, j = 1, 2, \ldots, n$. $A \succ B$ ($A \succeq B$) means $a_{ij} > b_{ij}$ ($a_{ij} \geq b_{ij}$) for $i, j = 1, 2, \ldots, n$.

2. **Problem Formulation.**

Consider the following switched positive linear systems:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x(t) \in \Re^n$ is the state; $u(t) \in \Re^r$ is the control input; $\sigma(t) : [0, \infty) \to S = \{1, 2, \ldots, M\}$ represents a switching signal, which is assumed to be a piecewise constant or piecewise continuous (from the right) function depending on time; $M \geq 2$ is the number of models (called subsystems) of the switched system. Moreover, $A_p = [a_{pij}] \in \Re^{n\times n}$, $B_p \in \Re^{n\times r}$, $B_p^T = \left[b^{(b1)T} \ldots b^{(bn)T}\right]$, $\forall p \in S$ are unknown real constant matrices of appropriate dimensions. When $t \in [t_k, t_{k+1})$, $k \in N$, we say that the $\sigma(t_k)$th subsystem is active. We assume that the state of the switched system (1) does not jump at the switching instants, i.e., the trajectory $x(t)$ is everywhere continuous.

Our goal is to design controllers to stabilize the switched positive systems (1) under the following minimum dwell time signals, which satisfy the switching time sequence $\{t_k\}$:

$$S_1(\delta_1, \delta_2) = \{\{t_k\}; \delta_1 \leq t_{k+1} - t_k \leq \delta_2, k \in N\}, \tag{2}$$

$$S_2(\delta_1, \infty) = \{\{t_k\}; t_{k+1} - t_k \geq \delta_1, k \in N\}, \tag{3}$$

where the known constants δ_1 and δ_2 satisfy $\delta_2 \geq \delta_1 > 0$.

To study the stabilization of switched positive systems (1), we will recall some definitions and lemmas.

Definition 2.1. [21] *System*

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{4}$$

is positive if its states are non-negative for all time t whenever the initial condition $x(t_0)$ and control input $u(t)$ are non-negative, where t_0 is the initial time instant.

Lemma 2.1. [16] *System (4) is positive if and only if A is a Metzler matrix, and $B \succeq 0$.*

3. **Main Results.** In this section, to ensure the stabilization of switched system (1), the controller gains are designed and certain classes of switching signals are identified.

First, we define a class of multiple time-varying linear copositive Lyapunov candidate functions as the following.

For any given switching times sequence $\{t_k\} \in S_1(\delta_1, \delta_2)$, $\mu_p^{(l)} > 1$, $p \in S$, we define two piecewise linear functions ρ, $\rho_1 : [t_0, \infty) \to \Re^+$ and function $\varphi_{\sigma(t)}(t)$:

$$\rho(t) = \frac{t - t_k}{t_{k+1} - t_k}, \quad \rho_1(t) = \frac{1}{t_{k+1} - t_k}, \quad t \in [t_k, t_{k+1}), \quad k \in N. \tag{5}$$

$$\varphi_{\sigma(t)}(t) = \left(\mu_{\sigma(t)}^{(1)} \mu_{\sigma(t)}^{(2)}\right)^{\rho(t)-1}. \tag{6}$$

It is easy to check that

$$\rho(t) \in [0, 1), \quad \rho(t_k^+) = 0, \quad \rho\left(t_{k+1}^-\right) = 1. \tag{7}$$

$$\left(\mu_{\sigma(t)}^{(1)} \mu_{\sigma(t)}^{(2)}\right)^{-1} \le \varphi_{\sigma(t)}(t) \le 1, \quad t \in [0, \infty). \tag{8}$$

Using $\rho(t)$ and $\varphi(t)$ above, we define the following class of multiple time-varying linear copositive Lyapunov candidate functions:

$$V(t) = \varphi_{\sigma(t)}(t) x^T(t) \left(\rho(t) v_{\sigma(t)}^{(1)} + \tilde{\rho}(t) v_{\sigma(t)}^{(2)}\right) \triangleq \varphi_{\sigma(t)}(t) x^T(t) v_{\sigma(t)}(t), \tag{9}$$

where $v_{\sigma(t)}^{(1)} \succ 0$, $v_{\sigma(t)}^{(2)} \succ 0$, $\tilde{\rho}(t) = 1 - \rho(t)$.

We now state our main results.

3.1. Case 1: $\boldsymbol{\sigma(t) \in S_1(\delta_1, \delta_2)}$.

Theorem 3.1. *Consider the switched system (1). For given positive scalars $\gamma > 0$, $\mu_p^{(l)} > 1$, $0 < \eta < 1$, $\varsigma_p = (\varsigma_{p1}, \varsigma_{p2})^T \succ 0$, and vectors $\tilde{v}_p \in \Re^n$, $\tilde{v}_p \succ 0$, if there exist vectors $z_p = (z_{p1}, z_{p2}, \ldots, z_{pn})^T \prec 0$, $v_p^{(l)} = \left(v_p^{(l1)}, v_p^{(l2)}, \ldots, v_p^{(ln)}\right)^T \succ 0$, $p \in S$ and $l = 1, 2$, such that*

$$\varsigma_{p1} \prec \tilde{v}_p^T B_p^T v_p^{(l)} \prec \varsigma_{p2}, \tag{10}$$

$$\varsigma_p^2 a_{pij} + b^{(pi)} \tilde{v}_p z_{pj} \ge 0, \ 1 \le i, j \le n, \ i \ne j, \tag{11}$$

$$e^{-\gamma\delta_1} v_p^{(2)} \preceq \eta \mu_p^{(1)} \mu_p^{(2)} v_{p'}^{(1)}, \tag{12}$$

$$\Theta_{plq} \preceq 0, \ p \in S, \ l, q = 1, 2, \tag{13}$$

where $\Theta_{plq} = \vartheta_{plq} + \frac{z_p}{\varsigma_{p2}}$, with $\vartheta_{plq} = \left(\frac{1}{\delta_1} \ln\left(\mu_p^{(1)} \mu_p^{(2)}\right) I + A_p^T + \gamma I\right) v_p^{(l)} + \frac{1}{\delta_q}\left(v_p^{(1)} - v_p^{(2)}\right)$, $q = 1, 2$, then, the switched system (1) with the controller

$$u(t) = K_p x(t) = \frac{\tilde{v}_p z_p^T}{\tilde{v}_p^T B_p^T v_p^{(2)} \tilde{v}_p^T B_p^T v_p^{(1)}} x(t) \tag{14}$$

is positive and exponential stable under the dwell time switching signal $\sigma(t) \in S_1(\delta_1, \delta_2)$.

Proof: According to (10), (11) and (14), we have

$$\varsigma_p^2 a_{pij} + b^{(pi)}\tilde{v}_p z_{pj} = [A_p + B_p K_p]_{ij} \geq 0, \quad i \neq j. \tag{15}$$

Therefore, $A_p + B_p K_p$ is a Metzler matrix, $\forall p \in S$. Then, the positive property of the closed-loop switched system (1) can be obtained by Lemma 2.1.

Define $W(t) = e^{\gamma t}V(t)$. For $\sigma(t) = p$, $p \in S$, when $t \in [t_k, t_{k+1})$, the time derivative of $W(t)$ is

$$\begin{aligned} \dot{W}(t) &= e^{\gamma t}\Big(\dot{\varphi}_p(t)x^T v_p(t) + \varphi_p(t)(\dot{x})^T v_p(t) + \varphi_p(t)x^T \rho_1(t)\left(v_p^{(1)} - v_p^{(2)}\right) + \gamma\varphi_p(t)x^T v_p(t)\Big) \\ &\leq e^{\gamma t}\left(\varphi_p(t)x^T \frac{1}{\delta_1}\ln\left(\mu_p^{(1)}\mu_p^{(2)}\right)v_p(t) + \varphi_p(t)x^T\left(A_p + B_p K_p\right)^T v_p(t)\right. \\ &\quad \left. + \varphi_p(t)x^T \rho_1(t)\left(v_p^{(1)} - v_p^{(2)}\right) + \gamma\varphi_p(t)x^T v_p(t)\right). \end{aligned} \tag{16}$$

Now, define a function $\rho_2(t) \in [0, 1]$, such that

$$\rho_1(t) = \frac{1}{\delta_1}\tilde{\rho}_2(t) + \frac{1}{\delta_2}\rho_2(t), \tag{17}$$

where $\tilde{\rho}_2(t) = 1 - \rho_2(t)$. Therefore, by (10), (14) and (17), one can get that

$$\begin{aligned} &\dot{V}(t) + \gamma V(t) \\ &\leq \varphi_p(t)x^T \frac{1}{\delta_1}\ln\left(\mu_p^{(1)}\mu_p^{(2)}\right)\left(\rho(t)v_p^{(1)} + \tilde{\rho}(t)v_p^{(2)}\right) \\ &\quad + \varphi_p(t)x^T\left(A_p + B_p K_p\right)^T\left(\rho(t)v_p^{(1)} + \tilde{\rho}(t)v_p^{(2)}\right) \\ &\quad + \gamma\varphi_p(t)x^T v_p(t)\left(\rho(t)v_p^{(1)} + \tilde{\rho}(t)v_p^{(2)}\right) + \varphi_p(t)x^T\left(\frac{1}{\delta_1}\tilde{\rho}_2(t) + \frac{1}{\delta_2}\rho_2(t)\right)\left(v_p^{(1)} - v_p^{(2)}\right) \\ &= \varphi_p(t)x^T\rho(t)\left(\left(\frac{1}{\delta_1}\ln\left(\mu_p^{(1)}\mu_p^{(2)}\right)I + A_p^T + \gamma I\right)v_p^{(1)} + \frac{z_p}{\tilde{v}_p^T B_p^T v_p^{(2)}}\right) \\ &\quad + \varphi_p(t)x^T\tilde{\rho}(t)\left(\left(\frac{1}{\delta_1}\ln\left(\mu_p^{(1)}\mu_p^{(2)}\right)I + A_p^T + \gamma I\right)v_p^{(2)} + \frac{z_p}{\tilde{v}_p^T B_p^T v_p^{(1)}}\right) \\ &\quad + \varphi_p(t)x^T\left(\rho(t) + \tilde{\rho}(t)\right)\left(\frac{1}{\delta_1}\tilde{\rho}_2(t) + \frac{1}{\delta_2}\rho_2(t)\right)\left(v_p^{(1)} - v_p^{(2)}\right) \\ &= \varphi_p(t)x^T\rho(t)\rho_2(t)\left(\vartheta_{p12} + \frac{z_p}{\tilde{v}_p^T B_p^T v_p^{(2)}}\right) + \varphi_p(t)x^T\rho(t)\tilde{\rho}_2(t)\left(\vartheta_{p11} + \frac{z_p}{\tilde{v}_p^T B_p^T v_p^{(2)}}\right) \\ &\quad + \varphi_p(t)x^T\tilde{\rho}(t)\rho_2(t)\left(\vartheta_{p22} + \frac{z_p}{\tilde{v}_p^T B_p^T v_p^{(1)}}\right) + \varphi_p(t)x^T\tilde{\rho}(t)\tilde{\rho}_2(t)\left(\vartheta_{p21} + \frac{z_p}{\tilde{v}_p^T B_p^T v_p^{(1)}}\right) \\ &\leq \varphi_p(t)x^T\rho(t)\rho_2(t)\left(\vartheta_{p12} + \frac{z_p}{\varsigma_{p2}}\right) + \varphi_p(t)x^T\rho(t)\tilde{\rho}_2(t)\left(\vartheta_{p11} + \frac{z_p}{\varsigma_{p2}}\right) \\ &\quad + \varphi_p(t)x^T\tilde{\rho}(t)\rho_2(t)\left(\vartheta_{p22} + \frac{z_p}{\varsigma_{p2}}\right) + \varphi_p(t)x^T\tilde{\rho}(t)\tilde{\rho}_2(t)\left(\vartheta_{p21} + \frac{z_p}{\varsigma_{p2}}\right). \end{aligned} \tag{18}$$

For $t \in [t_k, t_{k+1})$, one has that

$$\dot{V}(t) + \gamma V(t) \leq 0, \tag{19}$$

which describes the exponential decay of the Lyapunov function of the active subsystem.

On the other hand, according to (19), when $t \in [t_k, t_{k+1})$, we have

$$V\left(t_{k+1}^-\right) \leq e^{-\gamma\delta_1}V\left(t_k\right). \tag{20}$$

We assume that the switched system (1) switches from subsystem p' to p at switching instant t_{k+1}. Then, by using the fact that $\varphi_p\left(t_{k+1}^+\right) = \left(\mu_{\sigma(t)}^{(1)}\mu_{\sigma(t)}^{(2)}\right)^{-1}$, $\varphi_{p'}\left(t_{k+1}^-\right) = 1$, and applying (9), (12) and (20), we obtain that

$$\begin{aligned} V_p\left(t_{k+1}\right) &= \varphi_p\left(t_{k+1}^+\right) x^T\left(\rho\left(t_{k+1}^+\right) v_p^{(1)} + \tilde{\rho}\left(t_{k+1}^+\right) v_p^{(2)}\right) = \left(\mu_p^{(1)}\mu_p^{(2)}\right)^{-1} x^T v_p^{(2)} \\ &\leq \eta e^{\gamma\delta_1} x^T v_{p'}^{(1)} = \eta e^{\gamma\delta_1}\varphi_{p'}\left(t_{k+1}^-\right) x^T\left(\rho\left(t_{k+1}^-\right) v_{p'}^{(1)} + \tilde{\rho}\left(t_{k+1}^-\right) v_{p'}^{(2)}\right) \\ &= \eta e^{\gamma\delta_1} V_{p'}\left(t_{k+1}^-\right) \leq \eta V_{p'}\left(t_k\right). \end{aligned} \tag{21}$$

For any given $t > t_0$, there exists a positive integer $k_0 \in N$, such that $t \in [t_{k_0}, t_{k_0+1})$. Combining (20) and (21), we have

$$\begin{aligned} V(t) &\leq e^{-\gamma\delta_1} V\left(t_{k_0}\right) \leq \eta e^{-\gamma\delta_1} V\left(t_{k_0-1}\right) \leq \eta^2 e^{-\gamma\delta_1} V\left(t_{k_0-2}\right) \leq \cdots \leq \eta^{k_0} e^{-\gamma\delta_1} V\left(t_0\right) \\ &\leq \eta^{t-t_0} e^{-\gamma\delta_1} V\left(t_0\right). \end{aligned} \tag{22}$$

According to (9), we obtain that

$$x^T v_{\sigma(t)}(t) = \sum_{i=1}^n x_i\left(\rho(t) v_{\sigma(t)}^{(1i)} + \tilde{\rho}(t) v_{\sigma(t)}^{(2i)}\right) \geq \omega_1 \sum_{i=1}^n x_i \geq \omega_1 \|x\|_2,\ l = 1, 2.$$

On the other hand, we have

$$x^T v_{\sigma(t)}(t) = \sum_{i=1}^n x_i\left(\rho(t) v_{\sigma(t)}^{(1i)} + \tilde{\rho}(t) v_{\sigma(t)}^{(2i)}\right) \leq \omega_2 \sum_{i=1}^n x_i(t) \leq \sqrt{n}\omega_2 \|x(t)\|_2,\ l = 1, 2, \tag{23}$$

where $\omega_1 = \min\limits_{1\leq i\leq n, 1\leq p\leq N}\left\{v_p^{(1i)}, v_p^{(2i)}\right\}$, $\omega_2 = \max\limits_{1\leq i\leq n, 1\leq p\leq N}\left\{v_p^{(1i)}, v_p^{(2i)}\right\}$.

With the help of (22), (23), (24), and $\|x\|_2 \leq \|x\|_1 \leq \sqrt{n}\|x\|_2$, we have

$$\begin{aligned} \omega_1\varphi_{\sigma(t)}(t)\|x(t)\|_2 &\leq \varphi_{\sigma(t)}(t) x^T(t) v_{\sigma(t)} \leq \varphi_{\sigma(t_0)}(t_0) x^T(t_0) v_{\sigma(t)} e^{-\gamma(t-t_0)} \\ &\leq \sqrt{n}\omega_2\varphi_{\sigma(t_0)}(t_0)\|x(t_0)\|_2 e^{-\gamma(t-t_0)}. \end{aligned} \tag{24}$$

Then, $\|x(t)\|_2 \leq \alpha e^{-\gamma(t-t_0)}\|x(t_0)\|_2$, where $\alpha = \sqrt{n}\frac{\omega_2}{\omega_1}\max\limits_{1\leq p\leq N}\left\{\mu_p^1\mu_p^2\right\}$. We obtain that $\lim\limits_{t\to\infty}\|x(t)\|_2 = 0$.

In summary, the closed-loop switched system (1) is positive and exponential stable under switching signal $\sigma(t) \in S_1(\delta_1, \delta_2)$. This completes the proof.

The condition on the upper bound of the dwell time in Theorem 3.1 can be removed if some addition conditions are added. In the following, sufficient conditions for the stabilization of the switched systems (1) will be under the switching signal $\sigma(t) \in S_2(\delta_1, \infty)$.

3.2. Case 2: $\boldsymbol{\sigma(t) \in S_2(\delta_1, \infty)}$.

Theorem 3.2. *Consider the switched system (1). For given positive scalars* $\gamma > 0$, $\mu_p^{(l)} > 1$, $0 < \eta < 1$, $\varsigma_p = (\varsigma_{p1}, \varsigma_{p2})^T \succ 0$, *and vectors* $\tilde{v}_p \in \Re^n$, $\tilde{v}_p \succ 0$, *if there exist vectors* $z_p = (z_{p1}, z_{p2}, \ldots, z_{pn})^T \prec 0$, $v_p^{(l)} = \left(v_p^{(l1)}, v_p^{(l2)}, \ldots, v_p^{(ln)}\right)^T \succ 0$, $p \in S$ *and* $l = 1, 2$, *such that*

$$\varsigma_{p1} \prec \tilde{v}_p^T B_p^T v_p^{(l)} \prec \varsigma_{p2}, \tag{25}$$

$$\varsigma_p^2 a_{pij} + b^{(pi)}\tilde{v}_p z_{pj} \geq 0,\ 1 \leq i, j \leq n,\ i \neq j, \tag{26}$$

$$e^{-\gamma\delta_1} v_p^{(2)} \preceq \eta\mu_p^{(1)}\mu_p^{(2)} v_{p'}^{(1)}, \tag{27}$$

$$\Theta_{pl} \preceq 0,\ p \in S,\ l = 1, 2, \tag{28}$$

$$v_p^{(1)} \succeq v_p^{(2)}, \tag{29}$$

where $\Theta_{pl} = \vartheta_{pl} + \frac{z_p}{\varsigma_{p2}}$, *with* $\vartheta_{pl} = \left(\frac{1}{\delta_1}\ln\left(\mu_p^{(1)}\mu_p^{(2)}\right)I + A_p^T + \gamma I\right)\upsilon_p^{(l)} + \frac{1}{\delta_1}\left(\upsilon_p^{(1)} - \upsilon_p^{(2)}\right)$, *then, the switched system (1) with the controller*

$$u(t) = K_p x(t) = \frac{\tilde{\upsilon}_p z_p^T}{\tilde{\upsilon}_p^T B_p^T \upsilon_p^{(2)} \tilde{\upsilon}_p^T B_p^T \upsilon_p^{(1)}} x(t)$$

is positive and stable under the dwell time switching signal $\sigma(t) \in S_2(\delta_1, \infty)$.

Proof: The proof of Theorem 3.2 is very similar to the one of Theorem 3.1, and it can be easily derived by the methodology as above. Therefore, the proof of Theorem 3.2 is omitted.

4. **Example.** This section provides an illustrative example to show the effectiveness of the obtained result in this paper. Consider the switched positive linear system with two subsystems:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u_{\sigma(t)},\ x(0) = x_0,\ \sigma(t) : [0, \infty) \to S = \{1, 2\}, \tag{30}$$

$$A_1 = \begin{bmatrix} -0.15 & 0.18 \\ 0.4 & -0.4 \end{bmatrix},\ A_2 = \begin{bmatrix} -0.6 & 2 \\ 0.8 & -0.5 \end{bmatrix},\ B_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.3 & 0.3 \end{bmatrix},\ B_2 = \begin{bmatrix} 0.2 & 0.3 \\ 0.4 & 0.4 \end{bmatrix}.$$

In this case, we can apply Theorem 3.2 to designing a controller to stabilize the switched positive linear system. Let $\gamma = 0.01$, $\eta = 0.99$, $\varsigma_1 = \begin{bmatrix} 10 \\ 30 \end{bmatrix}$, $\varsigma_2 = \begin{bmatrix} 10 \\ 30 \end{bmatrix}$, $\tilde{\upsilon}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\tilde{\upsilon}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then, by means of Theorem 3.2, when $\delta_1 = 0.1$, we obtain:

$$\upsilon_1^{(1)} = \begin{pmatrix} 8.0853 \\ 7.5647 \end{pmatrix},\quad \upsilon_1^{(2)} = \begin{pmatrix} 8.0831 \\ 7.4297 \end{pmatrix},\quad \upsilon_2^{(1)} = \begin{pmatrix} 8.1745 \\ 7.5122 \end{pmatrix},$$

$$\upsilon_2^{(2)} = \begin{pmatrix} 8.0093 \\ 7.4864 \end{pmatrix},\quad z_1 = \begin{pmatrix} -64.6756 \\ -15.4971 \end{pmatrix},\quad z_2 = \begin{pmatrix} -96.1484 \\ -387.6896 \end{pmatrix},$$

$$K_1 = \begin{pmatrix} -0.4665 & -0.1118 \\ -0.4665 & -0.1118 \end{pmatrix},\quad K_2 = \begin{pmatrix} -0.9528 & -3.9411 \\ -0.9528 & -3.9411 \end{pmatrix}.$$

For the dwell time switching signal $\sigma(t) \in S_2(0.1, \infty)$, we randomly choose a switching signal shown in Figure 1. Let the initial state $x(0) = [2, 1]^T$, Figure 2 shows the state responses of the closed-loop switched system (30), and the corresponding state trajectories of the closed-loop switched system (30) are shown in Figure 3. The example demonstrates

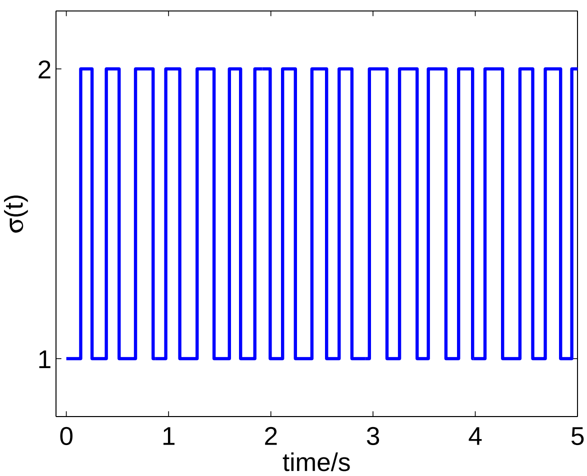

FIGURE 1. Switching signal $\sigma(t) \in S_2(0.1, \infty)$

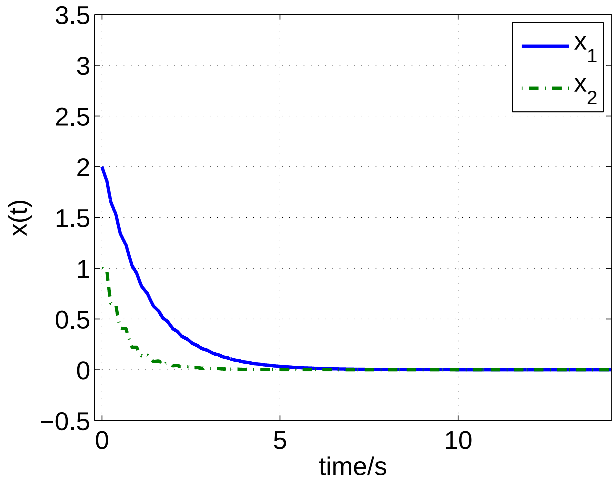

FIGURE 2. State responses of the closed-loop switched system (30)

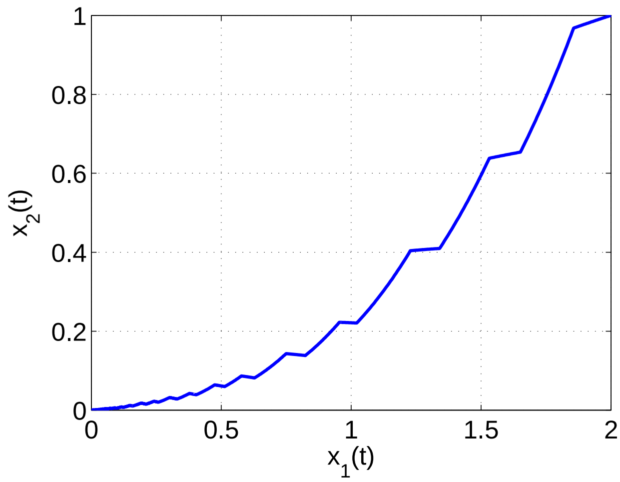

FIGURE 3. State trajectories of the closed-loop switched system (30)

the effectiveness of our obtained results. It is worth noting that the dwell time $\delta_1 = 0.1$s in our paper is a pre-specification constant, which is different from the dwell time in [18] by calculation. In addition, in this example, our specified dwell time 0.1s is less than the dwell time 0.3256s obtained in [18].

5. **Conclusions.** The stabilization problem for a class of switched positive linear systems via linear programming has been investigated based on multiple time-varying linear copositive Lyapunov functions. Sufficient conditions on stabilization have been presented under two classes of dwell time switching signals, and the stabilization controller has been obtained by solving the linear programming. The correctness and effectiveness of the proposed approach are illustrated by a numerical example. The stabilization problem for a class of discrete-time switched positive systems will be addressed in the future research.

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