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COMPOSITE OBSERVER BASED ROBUST ATTITUDE CONTROL FOR FLEXIBLE SATELLITE

Xiaofeng Xu, Mou Chen, Qingxian Wu and Rong Mei
College of Automation Engineering
Nanjing University of Aeronautics and Astronautics
No. 29, Yudao Street, Qinhuai District, Nanjing 210016, P. R. China
572600562@qq.com

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Abstract. *This paper investigates the flexible satellite attitude control system (ACS) with time delay, actuator fault and disturbance. Based on robust control technique, a control strategy is proposed to improve the performance of ACS; meanwhile, the disturbance and actuator fault are estimated accurately by composite observer. Specially, the correlative factor of time delay is built to decrease the impact of time delay on flexible satellite ACS. And lastly, the effectiveness of the presented method is illustrated by simulations.*
Keywords: Flexible satellite, Attitude control system, Disturbance observer, Actuator fault, Robust control, Time delay

1. **Introduction.** The flexible satellite is one of the most important devices for human to explore the space. To work persistently in space, the flexible satellite must carry one pair or more solar panels to provide sustainable energy [1]. The solar panels are flexible appendages, which may cause time-varying disturbance to affect the stability of flexible satellite ACS [2,3]. Therefore, the disturbance must be considered in the ACS design of flexible satellite.

Due to the high stability demands, many sophisticated control methods have been applied to handling the disturbance of flexible satellite [4-6]. In [7], disturbance observer based control (DOBC) and feedback controller were proposed to estimate disturbance and obtain desirable performance. A novel control scheme was proposed to further improve the accuracy of flexible satellite based on DOBC and the state observer in [8]. However, the control and feedback information are transmitted over the network, and the network communication delay often exists in the ACS of flexible satellite. Thus, the time delay cannot be ignored in flexible satellite ACS.

To achieve the high precision of flexible satellite ACS, the input delay was considered to enhance the reliability in [9]. In [10], the technique of time delay compensation was introduced to weaken the impact of time delay on flexible satellite ACS. In addition to time delay and disturbances, the actuator fault is also one of the factors, which affects the performance of flexible satellite ACS. To handle the actuator fault, the fault-tolerant control (FTC) and Chebyshev neural network were combined to improve the accuracy of flexible satellite ACS in [11]. In [12], a reliable controller was designed based on FTC and sliding mode control (SMC), and the stability of flexible satellite ACS was improved under actuator fault. However, in order to guarantee the attitude control performance of flexible satellite, the disturbance, time delay and actuator fault need to be further studied together for flexible satellite ACS.

Motivated by the above, disturbance, time delay and actuator fault are taken into account for flexible satellite ACS in this paper. Based on composite observer and robust

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control technique, a feasible control strategy is proposed to improve the performance of flexible satellite ACS, and the time delay correlative decomposition factor is introduced to lower the impact of time delay on flexible satellite ACS. Finally, the effectiveness of the presented strategy is proved by simulations.

2. **Model Description of Flexible Satellite.** The dynamic equation of a single axis rotational flexible satellite [13,14] is described as follows:

$$\begin{cases} J\ddot{\alpha}(t) + G\ddot{\mu}(t) = u(t) + F(t) \\ \ddot{\mu}(t) + C_m\dot{\mu}(t) + \Lambda\mu(t) + G^T\ddot{\alpha}(t) = 0 \end{cases} \tag{1}$$

where J denotes the rotational inertia of the satellite, $\alpha(t)$ represents the attitude angle, G is the rigid-elastic coupling coefficient, $\mu(t)$ denotes the flexible modal coordinate, $u(t)$ is the control torque, $F(t)$ represents the bounded fault of reaction wheel and is supposed to satisfy $F(t) \in l_2[0, +\infty)$, Λ denotes a known stiffness matrix with $\Lambda = [diag(\omega_i^2), i = 1, 2, \ldots, n]$, ω_i is the modal frequency, n represents number of the modes, C_m denotes a known modal damping matrix with $C_m = [diag(2\xi_i\omega_i), i = 1, 2, \ldots, n]$, and ξ_i is the damping ratio. Because of the low frequency modes with concentrative vibration energy in a flexible structure, the first two bending modes are taken into account in this paper. Therefore, the system (1) can be further described as the following state-space form:

$$\dot{x}_a(t) = A_a x_a(t) + B_a u(t) + B_a F(t) + B_a D(t) \tag{2}$$

where $D(t) = G\left[C_m\dot{\mu}(t) + \Lambda\mu(t)\right]$, $A_a = \begin{bmatrix} O & I \\ O & O \end{bmatrix}$, $B_a = \begin{bmatrix} O \\ \left(J - GG^T\right)^{-1} \end{bmatrix}$, $x_a(t) = \begin{bmatrix} \alpha(t) \\ \dot{\alpha}(t) \end{bmatrix}$.

In this paper, $D(t)$ is a bounded time-varying disturbance caused by flexible appendages. The state variable $x_a(t)$ consists of the attitude angle $\alpha(t)$ and attitude angular velocity $\dot{\alpha}(t)$.

Remark 2.1. [4]. *Considering the physical characteristics of the flexible satellite, $J - GG^T$ is nonsingular and $\left(I - G^TJ^{-1}G\right)^{-1}$ exists.*

Assumption 2.1. [15]. *The pair (A_a, B_a) is completely controllable.*

Assumption 2.2. [3]. *This paper considers the additive actuator fault $F(t)$. It is reasonable to assume that the derivative of $F(t)$ is bounded in practice.*

3. **Composite Observer and Controller Design.** In this subsection, a composite system will be designed by disturbance observer, fault estimation observer and robust controller. Based on (1), then we have

$$\ddot{\mu}(t) = -R_\mu C_m\dot{\mu}(t) - R_\mu\Lambda\mu(t) - R_\mu G^TJ^{-1}u(t) - R_\mu G^TJ^{-1}F(t) \tag{3}$$

where $R_\mu = \left(I - G^TJ^{-1}G\right)^{-1}$, then the following system is used to represent the modeled bounded time-varying disturbance:

$$\begin{cases} \dot{\omega}(t) = H\omega(t) - H_\mu u(t) - H_\mu F(t) \\ D(t) = L\omega(t) \end{cases} \tag{4}$$

where $\omega(t)$ denotes the state variable of the $D(t)$. H, H_μ and L are expressed as $\omega(t) = \begin{bmatrix} \mu(t) \\ \dot{\mu}(t) \end{bmatrix}$, $L = \begin{bmatrix} G\Lambda & GC_m \end{bmatrix}$, $H = \begin{bmatrix} O & I \\ -R_\mu\Lambda & -R_\mu C_m \end{bmatrix}$, $H_\mu = \begin{bmatrix} O \\ R_\mu G^TJ^{-1} \end{bmatrix}$.

For $D(t)$, the disturbance observer is designed as

$$\begin{cases} \hat{D}(t) = L\hat{\omega}(t) \\ \hat{\omega}(t) = Q(t) - N_0 x_a(t) \\ \dot{Q}(t) = (H + N_0 B_a L)(Q(t) - N_0 x_a(t)) + N_0 A_a x_a(t) \\ \qquad + (N_0 B_a - H_\mu)\left(u(t) + \hat{F}(t)\right) \end{cases} \tag{5}$$

where $\hat{D}(t)$, $\hat{\omega}(t)$ and $\hat{F}(t)$ are the estimations of $D(t)$, $\omega(t)$ and $F(t)$ respectively. N_0 is the observer gain, and $Q(t)$ represents an auxiliary variable of disturbance observer. For $F(t)$, the fault estimation observer is given by

$$\begin{cases} \hat{F}(t) = Q_0(t) - N_1 x_a(t) \\ \dot{Q}_0(t) = N_1\left(A_a x_a(t) + B_a u(t) + B_a\hat{D}(t)\right) + N_1 B_a(Q_0(t) - N_1 x_a(t)) \end{cases} \tag{6}$$

where $Q_0(t)$ denotes an auxiliary variable of fault estimation observer, and N_1 is the observer gain. The estimation errors of $F(t)$ and $D(t)$ are defined respectively as $e_F(t) = F(t) - \hat{F}(t)$ and $e_\omega(t) = \omega(t) - \hat{\omega}(t)$. Then, we have

$$\dot{e}_\omega(t) = \dot{\omega}(t) - \dot{\hat{\omega}}(t) = (H + N_0 B_a L)e_\omega(t) + (N_0 B_a - H_\mu)e_F(t) \tag{7}$$

$$\dot{e}_F(t) = \dot{F}(t) - \dot{\hat{F}}(t) = \dot{F}(t) + N_1 B_a e_F(t) + N_1 B_a L e_\omega(t) \tag{8}$$

Since network transmission time delay exists from the controller to actuator, the robust controller can be designed as follows:

$$u(t) = Kx_a(t - \tau(t)) - \hat{D}(t) - \hat{F}(t) \tag{9}$$

where K is the controller gain to be determined later. It is assumed that $\tau(t)$ is unknown bounded delay with $0 < \tau(t) \le \tilde{\tau}$, and delay-rate satisfies $0 < \dot{\tau}(t) \le \varepsilon < 1$, $\tilde{\tau}$ and ε denote the upper bounds of $\tau(t)$ and $\dot{\tau}(t)$ respectively. From (2), (7), (8) and (9), the composite system can be obtained

$$\dot{x}(t) = Ax(t) + A_\tau x(t - \tau(t)) + B\dot{F}(t) \tag{10}$$

where

$$x(t) = \begin{bmatrix} x_a^T(t) & e_\omega^T(t) & e_F^T(t) \end{bmatrix}^T,$$

$$x(t-\tau(t)) = \begin{bmatrix} x_a^T(t-\tau(t)) & e_\omega^T(t-\tau(t)) & e_F^T(t-\tau(t)) \end{bmatrix}^T,$$

$$A = \begin{bmatrix} A_a & B_a L & B_a \\ O & H + N_0 B_a L & N_0 B_a - H_\mu \\ O & N_1 B_a L & N_1 B_a \end{bmatrix}, \quad A_\tau = \begin{bmatrix} B_a K & O & O \\ O & O & O \\ O & O & O \end{bmatrix}, \quad B = \begin{bmatrix} O \\ O \\ I \end{bmatrix}.$$

The H_∞ performance reference control output is given by

$$y(t) = Cx(t) + C_\tau x(t - \tau(t)) \tag{11}$$

where $C = \begin{bmatrix} C_1 & C_2 & C_3 \end{bmatrix}$ and $C_\tau = \begin{bmatrix} C_{\tau 1} & C_{\tau 2} & C_{\tau 3} \end{bmatrix}$ are the known parameter matrices. On the basis of (10) and (11), it yields

$$\begin{cases} \dot{x}(t) = Ax(t) + A_\tau x(t - \tau(t)) + B\dot{F}(t) \\ y(t) = Cx(t) + C_\tau x(t - \tau(t)) \end{cases} \tag{12}$$

In this paper, two following objectives for the system (12) need to be achieved:

• When $\dot{F}(t) = 0$, the system (12) is asymptotically stable (AS) by designing the gains K, N_0 and N_1.

• The system (12) with any nonzero $\dot{F}(t) \in [0, \infty)$ is AS, and H_∞ performance is satisfied with $||y(t)||_2^2 < \gamma^2||\dot{F}(t)||_2^2$, where $\gamma > 0$ is prescribed scalar.

To reach the main results, the following lemma is given.

Lemma 3.1. [13]. *For any matrix $\Xi_a > O$, scalars $\beta_1 > \beta_2 > 0$, if a Lebseque vector $\varphi(s)$ exists, then the following inequality holds:*

$$-\int_{\beta_2}^{\beta_1} \varphi^T(s)\Xi_a\varphi(s)ds \le -\frac{1}{\beta_1-\beta_2}\int_{\beta_2}^{\beta_1}\varphi^T(s)ds\Xi_a\int_{\beta_2}^{\beta_1}\varphi(s)ds \tag{13}$$

4. **Main Results.** In this section, the controller gain K, observer gain N_0 and observer gain N_1 will be computed simultaneously by LMI. To simplify the description, defining $x_\tau(t) = x(t-\tau(t))$, symbol $sym()$ denotes $sym(Z) = Z + Z^T$ and Z is a matrix.

Theorem 4.1. *The scalars $\gamma > 0$, $0 < \delta < 1$, $0 \le c \le 1$ and $\tilde{\tau} > 0$ are given, if matrices S_1, S_2, S_3, S_4, $M > O$, $W > O$ and $R > O$ exist, the following LMI holds:*

$$\Theta_5 < O \tag{14}$$

then, the system (12) is AS, and the H_∞ performance is satisfied with $||y(t)||_2^2 < \gamma^2\left\|\dot{F}(t)\right\|_2^2$. Here

$$\Theta_5 = \begin{bmatrix} \Omega_1 + C^TC & \Omega_2 + C^TC_\tau & \Omega_4 & \Omega_6 & S_1B \\ * & \Omega_3 + C_\tau^TC_\tau & \Omega_5 & \Omega_7 & S_2B \\ * & * & \sigma_aW & -S_3 & S_3B \\ * & * & * & \Omega_8 & S_4B \\ * & * & * & * & -\gamma^2 I \end{bmatrix},$$

$\Omega_1 = M + \sigma_aW + sym(S_1A)$, $\Omega_2 = -\sigma_bW + S_1A_\tau + A^TS_2^T$, $\Omega_4 = \sigma_cW + A^TS_3^T$,
$\sigma_a = -\frac{1}{\tilde{\tau}}$, $\sigma_b = -\frac{1-c}{\tilde{\tau}}$, $\sigma_c = \frac{c}{\tilde{\tau}}$, $\Omega_3 = -\delta M + 2\sigma_bW + sym(S_2A_\tau)$,
$\Omega_5 = -\sigma_bW + A_\tau^TS_3^T$, $\Omega_6 = R - S_1 + A^TS_4^T$, $\Omega_7 = A_\tau^TS_4^T - S_2$,
$\Omega_8 = sym(-S_4) + \tilde{\tau}W$.

Proof: The following Lyapunov-Krasovskii functional is chosen:

$$V_x(x(t),t) = x^T(t)Rx(t) + \int_{t-\tau(t)}^t x^T(s)Mx(s)ds + \int_{-\tilde{\tau}}^0\int_{t+\sigma}^t \dot{x}^T(s)W\dot{x}(s)dsd\sigma \tag{15}$$

and then, the time derivative of $V_x(x(t),t)$ is provided by

$$\begin{aligned}\dot{V}_x(x(t),t) =\ & \dot{x}^T(t)Rx(t) + x^T(t)R\dot{x}(t) + x^T(t)Mx(t) - (1-\dot{\tau}(t))x_\tau^T(t)Mx_\tau(t) \\ & + \tilde{\tau}\dot{x}^T(t)W\dot{x}(t) - \int_{t-\tilde{\tau}}^t \dot{x}^T(s)W\dot{x}(s)ds\end{aligned} \tag{16}$$

Then, the equivalent decomposition of $\int_{t-\tilde{\tau}}^t \dot{x}^T(s)W\dot{x}(s)ds$ is described as follows:

$$-\int_{t-\tilde{\tau}}^t \dot{x}^T(s)W\dot{x}(s)ds = -c\int_{t-\tilde{\tau}}^t \dot{x}^T(s)W\dot{x}(s)ds - (1-c)\int_{t-\tilde{\tau}}^t \dot{x}^T(s)W\dot{x}(s)ds \tag{17}$$

where c is known as the time delay correlative decomposition factor, and satisfies $0 \le c \le 1$. Combining (16) with Lemma 3.1, then we have

$$\begin{aligned}&\dot{V}_x(x(t),t) \\ &\le \dot{x}^T(t)Rx(t) + x^T(t)R\dot{x}(t) + x^T(t)Mx(t) - \delta x_\tau^T(t)Mx_\tau(t) + \tilde{\tau}\dot{x}^T(t)W\dot{x}(t) + \Theta_a\end{aligned} \tag{18}$$

where $\delta = 1-\varepsilon$, $\Theta_a = \Theta^T(t)\Theta_1\Theta(t)$, $\Theta(t) = \begin{bmatrix} x^T(t) & x_\tau^T(t) & x^T(t-\tilde{\tau}) \end{bmatrix}^T$, $\Theta_1 = \begin{bmatrix} \sigma_a W & -\sigma_b W & \sigma_c W \\ * & 2\sigma_b W & -\sigma_b W \\ * & * & \sigma_a W \end{bmatrix}$.

First, $\dot{F}(t) = 0$ is considered to analyze the stability of system (12). Note that

$$2\left(x^T(t)S_1 + x_\tau^T(t)S_2 + x^T(t-\tilde{\tau})S_3 + \dot{x}^T(t)S_4\right)\left(-\dot{x}(t) + Ax(t) + A_\tau x_\tau(t)\right) = 0 \tag{19}$$

where S_1, S_2, S_3 and S_4 are the dimensional matching arbitrary matrices. On the basis of (18) and (19), one has

$$\dot{V}_x(x(t),t) \le \Theta_2^T(t)\Theta_3\Theta_2(t) \tag{20}$$

where $\Theta_2(t) = \begin{bmatrix} x^T(t) & x_\tau^T(t) & x^T(t-\tilde{\tau}) & \dot{x}^T(t) \end{bmatrix}^T$, $\Theta_3 = \begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_4 & \Omega_6 \\ * & \Omega_3 & \Omega_5 & \Omega_7 \\ * & * & \sigma_a W & -S_3 \\ * & * & * & \Omega_8 \end{bmatrix}$.

By employing the Schur complement lemma for (14), then we obtain $\Theta_3 < O$, which means system (12) is AS. When $\dot{F}(t) \neq 0$, similar to (19), the following equality is established:

$$2\left(x^T(t)S_1 + x_\tau^T(t)S_2 + x^T(t-\tilde{\tau})S_3 + \dot{x}^T(t)S_4\right)\left(-\dot{x}(t) + Ax(t) + A_\tau x_\tau(t) + B\dot{F}(t)\right) = 0 \tag{21}$$

Next, the H_∞ performance will be verified based on the following auxiliary function:

$$\Psi(x(t)) = \int_0^t \left(\|y(s)\|_2^2 - \gamma^2\left\|\dot{F}(s)\right\|_2^2\right)ds \tag{22}$$

Furthermore, we obtain

$$\Psi(x(t)) \le \int_0^t \left(\|y(s)\|_2^2 - \gamma^2\left\|\dot{F}(s)\right\|_2^2 + \dot{V}_x(x(s),s)\right)ds \tag{23}$$

Combining (12), (16), (21) and (23), we have

$$\|y(s)\|_2^2 - \gamma^2\left\|\dot{F}(s)\right\|_2^2 + \dot{V}_x(x(s),s) \le \Theta_4^T(s)\Theta_5\Theta_4(s) \tag{24}$$

where $\Theta_4(s) = \begin{bmatrix} x^T(s) & x_\tau^T(s) & x^T(s-\tilde{\tau}) & \dot{x}^T(s) & \dot{F}^T(s) \end{bmatrix}^T$.

Based on (14), we have known that $\Theta_5 < O$, which means $\|y(t)\|_2^2 < \gamma^2\left\|\dot{F}(t)\right\|_2^2$. Therefore, the H_∞ performance is satisfied with $\|y(t)\|_2^2 < \gamma^2\left\|\dot{F}(t)\right\|_2^2$, and the system (12) is AS. This concludes the proof. Because the gains K, N_0 and N_1 are unable to be solved by Theorem 4.1, on the basis of Theorem 4.1, the following theorem is further achieved.

Theorem 4.2. *Given scalars $\gamma > 0$, $0 < \delta < 1$, $0 \le c \le 1$, $\tilde{\tau} > 0$, σ_2, σ_3, σ_4, if there exist matrices $M_{a1} > O$, $M_2 > O$, $M_3 > O$, $W_{a1} > O$, $W_2 > O$, $W_3 > O$, $R_{a1} > O$, $R_2 > O$, $R_3 > O$, S_{N_0}, S_{N_1}, S_k and nonsingular matrices X, S_{22}, S_{33}, the following LMI holds:*

$$\Theta_6 < O \tag{25}$$

when the observer gains $N_0 = S_{22}^{-1}S_{N_0}$, $N_1 = S_{33}^{-1}S_{N_1}$, controller gain $K = S_k X^{-T}$, the system (12) is AS, and the H_∞ performance is satisfied with $\|y(t)\|_2^2 < \gamma^2\left\|\dot{F}(t)\right\|_2^2$. Here

$$\Theta_6 = \begin{bmatrix} \theta'_{11} & \theta_c^T & B_a & \theta'_{14} & O & O & \theta'_{17} & O & O & \theta'_{1a} & O & O & O & \theta'_{1e} \\ * & \theta'_{22} & \theta'_{23} & \theta'_{24} & \theta'_{25} & \theta'_{26} & \theta'_{27} & \theta'_{28} & \theta'_{29} & \theta'_{2a} & \theta'_{2b} & \theta'_{2c} & O & C_2^T \\ * & * & \theta'_{33} & \theta'_{34} & \theta'_{35} & \theta'_{36} & \theta'_{37} & \theta'_{38} & \theta'_{39} & \theta'_{3a} & \theta'_{3b} & \theta'_{3c} & S_{33} & C_3^T \\ * & * & * & \theta'_{41} & O & O & \theta'_{47} & O & O & \theta'_{4a} & O & O & O & \theta'_{4e} \\ * & * & * & * & \theta'_{55} & O & O & \theta'_{58} & O & O & \theta'_{5b} & O & O & C_{\tau 2}^T \\ * & * & * & * & * & \theta'_{66} & O & O & \theta'_{69} & O & O & \theta'_{6c} & \theta'_{6d} & C_{\tau 3}^T \\ * & * & * & * & * & * & \theta'_{77} & O & O & \theta'_{7a} & O & O & O & O \\ * & * & * & * & * & * & * & \theta'_{88} & O & O & \theta'_{8b} & O & O & O \\ * & * & * & * & * & * & * & * & \theta'_{99} & O & O & \theta'_{9c} & \theta'_{9d} & O \\ * & * & * & * & * & * & * & * & * & \theta'_{aa} & O & O & O & O \\ * & * & * & * & * & * & * & * & * & * & \theta'_{bb} & O & O & O \\ * & * & * & * & * & * & * & * & * & * & * & \theta'_{cc} & \theta'_{cd} & O \\ * & * & * & * & * & * & * & * & * & * & * & * & -\gamma^2 I & O \\ * & * & * & * & * & * & * & * & * & * & * & * & * & -I \end{bmatrix},$$

$$\theta'_{11} = sym(A_a X^T) + \theta'_{77} + M_{a1}, \quad \theta'_{14} = -\sigma_b W_{a1} + \theta_d^T + \sigma_2 X A_a^T,$$
$$\theta'_{17} = \sigma_3 X A_a^T + \sigma_c W_{a1}, \quad \theta'_{1a} = \sigma_4 X A_a^T + R_{a1} - X^T, \quad \theta'_{1e} = X C_1^T,$$
$$\theta'_{22} = sym\left(\theta_b^T + S_{22} H\right) + \sigma_a W_2 + M_2, \quad \theta'_{23} = \theta_e^T + \theta_a - \theta_g^T, \quad \theta'_{24} = \sigma_2 \theta_c,$$
$$\theta'_{25} = -\sigma_b W_2 + \sigma_2 H^T S_{22}^T + \sigma_2 \theta_b, \quad \theta'_{26} = \sigma_2 \theta_a, \quad \theta'_{27} = \sigma_3 \theta_c, \quad \theta'_{29} = \sigma_3 \theta_a,$$
$$\theta'_{2a} = \sigma_4 \theta_c, \quad \theta'_{28} = \sigma_c W_2 + \sigma_3 H^T S_{22}^T + \sigma_3 \theta_b, \quad \theta'_{2b} = R_2 - S_{22} + \sigma_4 H^T S_{22}^T + \sigma_4 \theta_b,$$
$$\theta'_{2c} = \sigma_4 \theta_a, \quad \theta'_{33} = sym\left(\theta_f^T\right) + \theta'_{99} + M_3, \quad \theta'_{34} = \sigma_2 B_a^T, \quad \theta'_{35} = \sigma_2 \theta_e - \sigma_2 \theta_g,$$
$$\theta'_{36} = \sigma_2 \theta_f + \theta'_{69}, \quad \theta'_{37} = \sigma_3 B_a^T, \quad \theta'_{38} = \sigma_3 \theta_e - \sigma_3 \theta_g, \quad \theta'_{39} = \sigma_3 \theta_f + \sigma_c W_3,$$
$$\theta'_{3a} = \sigma_4 B_a^T, \quad \theta'_{3b} = \sigma_4 \theta_e - \sigma_4 \theta_g, \quad \theta'_{3c} = R_3 - S_{33} + \sigma_4 \theta_f,$$
$$\theta'_{41} = -\delta M_{a1} + 2\sigma_b W_{a1} + sym\left(\sigma_2 \theta_d^T\right), \quad \theta'_{47} = -\sigma_b W_{a1} + \sigma_3 \theta_d,$$
$$\theta'_{4a} = -\sigma_2 X^T + \sigma_4 \theta_d, \quad \theta'_{4e} = X C_{\tau 1}^T, \quad \theta'_{55} = -\delta M_2 - 2\theta'_{58},$$
$$\theta'_{58} = -\sigma_b W_2, \quad \theta'_{5b} = -\sigma_2 S_{22}, \quad \theta'_{66} = -\delta M_3 - 2\theta'_{69}, \quad \theta'_{69} = -\sigma_b W_3,$$
$$\theta'_{6c} = -\sigma_2 S_{33}, \quad \theta'_{6d} = -\theta'_{6c}, \quad \theta'_{77} = \sigma_a W_{a1}, \quad \theta'_{7a} = -\sigma_3 X^T, \quad \theta'_{88} = \sigma_a W_2,$$
$$\theta'_{8b} = -\sigma_3 S_{22}, \quad \theta'_{99} = \sigma_a W_3, \quad \theta'_{9c} = -\sigma_3 S_{33}, \quad \theta'_{9d} = -\theta'_{9c}, \quad \theta_g = H_\mu^T S_{22}^T,$$
$$\theta'_{aa} = sym\left(-\sigma_2 X^T\right) + \tilde{\tau} W_{a1}, \quad \theta'_{bb} = sym(-\sigma_4 S_{22}) + \tilde{\tau} W_2, \quad \theta'_{cd} = \sigma_4 S_{33},$$
$$\theta_c = L^T B_a^T, \quad \theta'_{cc} = sym\left(-\sigma_4 S_{33}\right) + \tilde{\tau} W_3, \quad \theta_a = \theta_c S_{N_1}^T, \quad \theta_b = \theta_c S_{N_0}^T,$$
$$\theta_d = S_k^T B_a^T, \quad \theta_e = B_a^T S_{N_0}^T, \quad \theta_f = B_a^T S_{N_1}^T.$$

Proof: Because X, S_{22} and S_{33} are nonsingular matrices, pre-multiplying and post-multiplying both sides of (25) with $diag\left\{X^{-1}, I, I, X^{-1}, I, I, X^{-1}, I, I, X^{-1}, I, I, I, I\right\}$ and its transpose, some matrices are defined as follows:

$$M = diag\{M_1, M_2, M_3\}, \quad W = diag\{W_1, W_2, W_3\}, \quad R = diag\{R_1, R_2, R_3\},$$
$$S_1 = diag\{S_{11}, S_{22}, S_{33}\}, \quad S_2 = \sigma_2 S_1, \quad S_3 = \sigma_3 S_1, \quad S_4 = \sigma_4 S_1, \quad W_{a1} = X W_1 X^T,$$
$$M_{a1} = X M_1 X^T, \quad R_{a1} = X R_1 X^T, \quad X = S_{11}^{-1}, \quad K = S_k X^{-T}, \quad S_{N_0} = S_{22} N_0,$$
$$S_{N_1} = S_{33} N_1.$$

Then, we can arrive at (14), the system (12) is AS, and the H_∞ performance is satisfied with $||y(t)||_2^2 < \gamma^2 \left\|\dot{F}(t)\right\|_2^2$ based on Throrem 4.1. The proof is thus completed.

5. **Numerical Simulation.** This section assumes that the flexible satellite only runs in an altitude of 500 km with a circular orbit, the orbit rate $n_a = 0.0011$ rad/s. A ramp fault with slope 0.015 is assumed to take place from 20 s to 40 s. The model parameters are given in [14]

$$J = 35.72 \text{ kg}\cdot\text{m}^2, \quad \varepsilon = 0.1, \quad \omega_1 = 3.17 \text{ rad/s}, \quad \omega_2 = 7.38 \text{ rad/s}, \quad \xi_1 = 0.0001,$$
$$\xi_2 = 0.00015, \quad G = [\ 1.27814 \quad 0.91756\ ], \quad \dot{\alpha}(0) = 0.001 \text{ rad/s}, \quad \alpha(0) = 0.08 \text{ rad},$$
$$C_1 = [\ 1 \quad 0\ ], \quad C_{\tau 1} = [\ 0 \quad 0\ ], \quad C_2 = C_{\tau 2} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, \quad C_3 = C_{\tau 3} = 0,$$
$$\gamma = 1.67, \quad \sigma_2 = \sigma_3 = \sigma_4 = 1.$$

When $c = 0.9$, $\tilde{\tau} = 2.5$ ms, simulation results are based on Theorem 4.2 under Matlab environment, and then the controller gain and observer gains can be obtained as follows:

$$N_1 = [\ 0 \quad -62.1871\ ], \quad K = [\ -19.0634 \quad -102.6237\ ], \quad N_0 = \begin{bmatrix} 0 & -1.3678 \\ 0 & -1.8598 \\ 0 & 1.2782 \\ 0 & 0.9210 \end{bmatrix}.$$

Based on Figure 1 and Figure 2, we can see that the disturbance $D(t)$ and actuator fault $F(t)$ are estimated effectively. Figure 3 and Figure 4 show $\alpha(t)$ and $\dot{\alpha}(t)$ under different c respectively, and it can be seen that $\alpha(t)$ and $\dot{\alpha}(t)$ tend to zero gradually, which means the system (12) is AS. From Figure 3 and Figure 4, we can observe that the case $c = 0.1$ is the best case and case $c = 0.9$ is the worst case, but case $c = 0.7$ is better than case $c = 0.4$ and case $c = 0.9$, which means c is useful to reduce design conservatism.

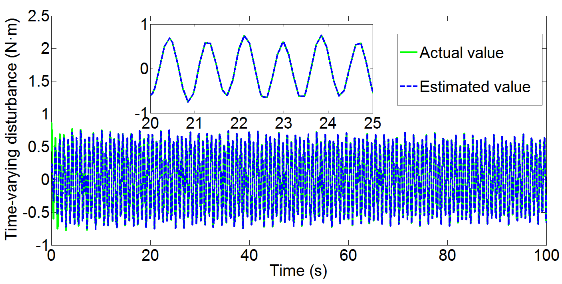

FIGURE 1. $D(t)$ and its estimation

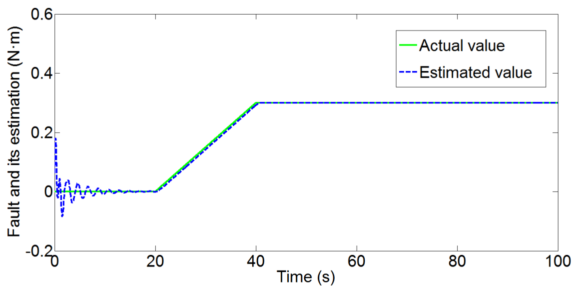

FIGURE 2. $F(t)$ and its estimation

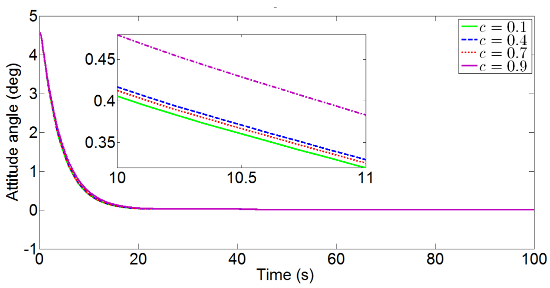

FIGURE 3. $\alpha(t)$ under different c

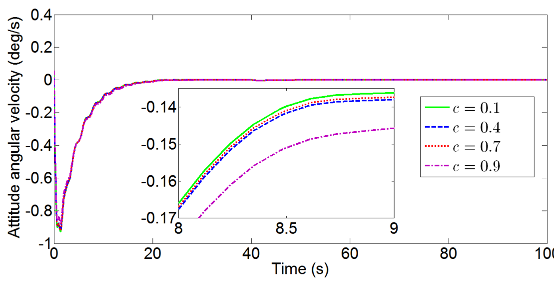

FIGURE 4. $\dot{\alpha}(t)$ under different c

6. **Conclusions.** In this paper, a composite observer based robust control approach has been introduced for flexible satellite ACS to achieve a stable attitude control performance. Specially, the time delay correlative decomposition factor has been introduced to reduce the effects of time delay on ACS. The simulations further show that the proposed strategy can guarantee the stability of ACS, and time-varying disturbance and actuator fault can be estimated effectively. However, this paper does not consider the input delay in the flexible satellite ACS, and it can be further studied in the future work.

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REFERENCES

[1] Y. C. Xie, Y. J. Lei and J. X. Guo, *Spacecraft Dynamics and Control*, Beijing Institute of Technology Press, Beijing, China, 2018.
[2] H. H. Zhang, *The High Performance Attitude Control for Complex Spacecraft*, National Defense Industry Press, Beijing, China, 2018.
[3] S. Y. Cao and Y. F. Zhao, Anti-disturbance fault-tolerant attitude control for satellites subject to multipe disturbances and actuator saturation, *Nonlinear Dynamics*, vol.89, no.4, pp.2657-2667, 2017.
[4] P. X. Zhang, J. Z. Qiao, L. Guo and W. S. Li, Sliding mode friction observer based control for flexible spacecraft with reaction wheel, *IET Control Theory & Applications*, vol.11, no.8, pp.1274-1281, 2017.
[5] S. J. Zhang and C. Z. Yu, Controller design for flexible spacecraft attitude tracking with actuator saturation, *The 10th International Conference on Measuring Technology and Mechatronics Automation*, Changsha, China, pp.10-14, 2018.
[6] C. B. Zhou, L. Hou, Y. F. Li and W. G. Yang, Robust adaptive nonlinear attitude tracking control of flexible spacecraft, *Proc. of the 37th Chinese Control Conference*, Wuhan, China, pp.10082-10087, 2018.
[7] Y. K. Zhu, L. Guo, J. Z. Qiao and W. S. Li, An enhanced anti-disturbance attitude control law for flexible spacecrafts subject to multiple disturbances, *Control Engineering Practice*, vol.84, pp.274-283, 2018.
[8] Z. Wang, Z. Wu, L. J. Li and J. Yuan, A composite anti-disturbance control scheme for attitude stabilization and vibration suppression of flexible spacecrafts, *Journal of Vibration and Control*, vol.23, no.15, pp.2470-2477, 2017.
[9] F. R. Lei, B. Zhang and T. Li, Multi-bounded-dependent fault-tolerant control for flexible spacecraft against partial actuator failures, *Proc. of the 35th Chinese Control Conference*, Chengdu, China, pp.2934-2939, 2016.
[10] C. Dong, L. Xu, Y. Chen and Q. Wang, Network flexible spacecraft attitude maneuver based on adaptive fuzzy sliding mode control, *Acta Astronautica*, vol.65, nos.11-12, pp.1561-1570, 2009.
[11] K. F. Lu, T. Y. Li and L. J. Zhang, Active attitude fault-tolerant tracking control of flexible spacecraft via the Chebyshev neural network, *Transactions of the Institute of Measurement and Control*, vol.41, no.4, pp.925-933, 2019.
[12] Q. L. Hu and B. Xiao, Fault-tolerant sliding mode attitude control for flexible spacecraft under loss of actuator effectiveness, *Nonlinear Dynamics*, vol.64, nos.1-2, pp.13-23, 2011.
[13] X. F. Xu, B. Zhang, T. Li and B. C. Zheng, Anti-disturbance control for flexible spacecraft with time-varying input delay, *2016 Chinese Control and Decision Conference*, pp.3006-3011, 2016.
[14] X. F. Li, L. Guo and Y. M. Zhang, A composite disturbance observer and H_∞ control scheme for flexible spacecraft with time-varying input delay, *The 31st Chinese Control Conference*, pp. 2824-2829, 2012.
[15] Q. Zhou, G. Zhang and S. Yan, Disturbance-observer-based reliable output control for time-delay systems with actuator faults, *International Journal of Innovative Computing, Information and Control*, vol.15, no.2, pp.629-640, 2019.