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THE PARAMETERIZATION OF ALL STABILIZING MINIMUM-PHASE CONTROLLERS FOR MINIMUM-PHASE PLANTS

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Abstract. *In the present paper, we examine the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants. The parameterization problem is the problem in which all stabilizing controllers for plants are sought. Since this parameterization can successfully search for all stabilizing minimum-phase controllers, it is used as a tool for many control problems. Glaria and Goodwin gave a simple parameterization of stabilizing controllers for linear minimum-phase plants. However, the difficulty remains that the parameterization of all stabilizing controllers given by Glaria and Goodwin generally includes improper controllers. In practical applications, the controller is required to be proper. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for linear minimum-phase plants. There exist many design methods of proper stabilizing controller for minimum-phase plants, but most of the proposed design methods do not consider the stabilizing minimum-phase controllers. Using the minimum-phase controllers can make the sensitivity function lower and the lower values of sensitivity function suggest further attenuation of the external disturbance. From this viewpoint, it is desirable to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants. The purpose of this paper is to propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants.*
Keywords: Parameterization, Minimum-phase, Stabilization

1. **Introduction.** In this paper, we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants. That is, we consider a design method for minimum-phase plants using the parameterization of minimum-phase controllers. The parameterization problem is to seek all stabilizing controllers for a plant [1, 2, 3, 4, 5, 6, 7] and to obtain plants that can be stabilized [8, 9]. Since this parameterization of all stabilizing minimum-phase controllers for minimum-phase plants can be successfully searched, it is used as a tool for many control problems.

For minimum-phase system, Glaria and Goodwin [4] gave a simple parameterization of controllers for linear minimum-phase plants. However, two difficulties remain. One is that the parameterization of all stabilizing controllers given by Glaria and Goodwin generally includes improper controllers. In practical applications, the controller is required to be proper. The other is that they do not give the parameterization of all internally stabilizing controllers. Yamada overcame these problems and proposed the parameterization of all proper internally stabilizing controllers for linear minimum-phase plants [5]. The parameterization of all stabilizing controllers in [5] is applied to many control problems such as the parameterization of all stabilizing modified repetitive controllers for minimum-phase

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plants [10], adaptive control systems [11, 12], model feedback control systems [13], parallel compensation technique [14], PI control [15] and PID control [16]. Chen et al. proposed the parameterization of all proper stabilizing internal model controllers for minimum-phase unstable plants [17]. Expanded from the result in [17], the parameterization of all strongly stabilizable plants is clarified in [9, 18].

However, there exists a question whether or not, stabilizing controllers for minimum-phase plants can be of minimum-phase that has advantages, for example, the inverse system of the minimum-phase system is still stable. The inverse system of the minimum-phase system is stable, because the stable poles of the inverse system of the minimum phase system are the stable zeros of the original system. If we use a minimum-phase controller to stabilize a minimum-phase plant, it would make the sensitivity function of this system lower. And lower values of sensitivity function suggest further attenuation of the external disturbance. The minimum-phase system is widely used in signal processing and other related fields, such as state system, design of causal stable digital filter, and calculation and processing of cepstrum and inverse filtering. If the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants is clarified, we will obtain a new control method for minimum-phase system. From this viewpoint, it is desirable to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants.

In this paper, we propose the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants, that is, we consider the parameterization that the stabilizing controller makes minimum-phase plant stable, which the stabilizing controller is of minimum-phase. This paper is organized as follows. In Section 2, we show the problem considered in this paper. In Section 3, we clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants. In Section 4, we show a numerical example. In Section 5 we give concluding remarks.

2. **Problem Formulation.** Consider the control system in

$$\begin{cases} y(s) = G(s)u(s) \\ u(s) = C(s)(r(s) - y(s)) \end{cases}, \tag{1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $y(s) \in R(s)$ is the output, $u(s) \in R(s)$ is the control input and $r(s) \in R(s)$ is the reference input. $R(s)$ denotes the set of real rational functions with s. $G(s)$ and $C(s)$ are assumed to be of minimum-phase, that is, $G(s)$ and $C(s)$ have no zero in the closed right half plane. And $G(s)$ is assumed to be biproper. The minimum-phase controller controls the minimum-phase plant to make the closed-loop system in (1) stable.

According to [5], if $G(s)$ is of minimum-phase and biproper, then the proper controller $C(s)$ stabilizes the feedback control system in (1) if and only if $C(s)$ is parametrized as

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G(s)}, \tag{2}$$

where $1/Q(s)$ is any minimum-phase and biproper rational function. And $C(s)$ is not necessarily of minimum-phase.

The problem considered in this paper is to clarify the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants.

3. **The Parameterization of All Stabilizing Minimum-Phase Controllers for Minimum-Phase Plants.** In this section, we clarify the parameterization of all stabilizing minimum-phase controllers $C(s)$ for minimum-phase plants $G(s)$.

This parameterization is summarized in the following theorem.

Theorem 3.1. *$G(s)$ is assumed to be of minimum-phase and to be biproper. Then the minimum-phase controller $C(s)$ stabilizes the feedback control system in (1) if and only if $C(s)$ is written by the form of*

$$C(s) = \frac{Q(s)}{(1 - Q(s))G(s)}, \tag{3}$$

where $Q(s) \in RH_\infty$ is any minimum-phase function to make $(1 - Q(s))G(s) \in RH_\infty$.

Proof: First, the necessity is shown. That is, we show that if the minimum-phase controller $C(s)$ makes minimum-phase plant $G(s)$ stable, then $C(s)$ takes the form of (3). From the assumption that $C(s)$ in (3) makes $G(s)$ in (1) stable, $1/(1 + C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are all included in RH_∞. RH_∞ denotes the set of stable proper real rational functions.

From the assumption that $G(s)$ and $C(s)$ are both assumed to be of minimum-phase,

$$\frac{1 + C(s)G(s)}{C(s)G(s)} = 1 + \frac{1}{C(s)G(s)} \tag{4}$$

is stable. Using $Q(s) \in RH_\infty$, $C(s)G(s)/(1 + C(s)G(s)) \in RH_\infty$ can be rewritten as

$$\frac{C(s)G(s)}{1 + C(s)G(s)} = Q(s). \tag{5}$$

From (4) and (5), $Q(s)$ must be of minimum-phase. From simple manipulation, (5) is rewritten as

$$C(s) = \frac{Q(s)}{(1 - Q(s))G(s)}. \tag{6}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $C(s)$ takes the form of (3), then the minimum-phase controller $C(s)$ stabilizes the minimum-phase plant $G(s)$ to make the control system stable. Then we set $C(s)$ as

$$C(s) = \frac{Q(s)}{(1 - Q(s))G(s)}, \tag{7}$$

where $Q(s) \in RH_\infty$ is any minimum-phase function and $(1 - Q(s))G(s) \in RH_\infty$ is any function. If the controller $C(s)$ makes $G(s)$ stable, according to definition of internal stability, the transfer functions $1/(1+C(s)G(s))$, $C(s)/(1+C(s)G(s))$, $G(s)/(1+C(s)G(s))$ and $C(s)G(s)/(1+C(s)G(s))$ are stable. After simple manipulation, the transfer functions are rewritten as

$$\frac{1}{1 + C(s)G(s)} = 1 - Q(s), \tag{8}$$

$$\frac{C(s)}{1 + C(s)G(s)} = \frac{Q(s)}{G(s)}, \tag{9}$$

$$\frac{G(s)}{1 + C(s)G(s)} = (1 - Q(s))G(s) \tag{10}$$

and

$$\frac{C(s)G(s)}{1 + C(s)G(s)} = Q(s). \tag{11}$$

Because $Q(s) \in RH_\infty$ and $G(s)$ is of minimum-phase, transfer functions in (8), (9) and (11) are stable. If the transfer function in (10) is unstable, unstable poles of the transfer function in (10) are unstable poles of $G(s)$. From the assumption that $(1 - Q(s))G(s) \in RH_\infty$, unstable poles of $G(s)$ are not poles of $(1 - Q(s))G(s)$. Therefore, the transfer function in (10) is stable. Thus, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

4. **Numerical Example.** In this section, a numerical example is illustrated to show that a stabilizing minimum-phase controller written by the form of (3) can stabilize the minimum-phase plant.

Consider the problem to make the control system in (1) stable using stabilizing minimum-phase controller, where the minimum-phase and biproper plant $G(s)$ is given by

$$G(s) = \frac{(s+7)(s+5)}{(s+1)(s+3)}. \tag{12}$$

According to the internal model principle [19], the output $y(s)$ will track the output $r(s)$ asymptotically and the tracking is robust if $C(s)G(s)$ contains the unstable modes of $r(s)$. Therefore,

$$C(0) = \infty \tag{13}$$

and

$$Q(0) = 1. \tag{14}$$

$Q(s) \in RH_\infty$ is any minimum-phase function and is designed as

$$Q(s) = \frac{0.01(s+200)(s+2)}{(s+4)(s+1)}. \tag{15}$$

From the assumption that $Q(s)$ in (3) makes $(1 - Q(s))G(s) \in RH_\infty$,

$$(1 - Q(s))G(s) = \frac{0.99s(s+3.01)(s+5)(s+7)}{(s+4)(s+3)(s+1)^2}. \tag{16}$$

From (12) and (15), a stabilizing controller $C(s)$ written by the form of (3) to make the control system in (1) stable is written as

$$C(s) = \frac{0.010101(s+200)(s+3)(s+2)(s+1)}{s(s+3.01)(s+5)(s+7)}. \tag{17}$$

$C(s)$ in (17) is obviously of minimum-phase. Therefore, if $C(s)$ in (17) makes $G(s)$ in (12) stable, then $C(s)$ in (17) is a stabilizing minimum-phase controller for the minimum-phase plant $G(s)$ in (12).

Using the stabilizing minimum-phase controller $C(s)$ in (17), the response of the output $y(t)$ of the control system in (1) for the step reference input $r(t) = 1$ is shown in Figure 1. Figure 1 shows that the control system in (1) is stabilized by using a stabilizing minimum-phase controller $C(s)$ in (17).

In this way, we find that if the stabilizing controller $C(s)$ is written by the form of (3), the minimum-phase plant is stabilized.

In order to check the robustness of this example, we consider the situation that the minimum-phase controller $C(s)$ in (17) stabilizes the plant

$$G_1(s) = \frac{(s+7)(s+15)}{(s+1)(s+3)}, \tag{18}$$

which is obtained by perturbing the plant $G(s)$. In this situation, the response of the output $y(t)$ for the step reference input $r(t) = 1$ is shown in Figure 2. Figure 2 shows that the control system in (1) is robust by using a stabilizing minimum-phase controller $C(s)$ in (17).

Furthermore, we compare the proposed design method with that of [5] where $C(s)$ is parametrized as

$$C(s) = \frac{1}{Q(s)} - \frac{1}{G(s)}. \tag{19}$$

By using the same $Q(s)$ in (15), we obtain another controller written as

$$C(s) = \frac{99(s+1)(s+3.602)\,(s^2+10.49s+35.9)}{(s+200)(s+7)(s+5)(s+2)}. \tag{20}$$

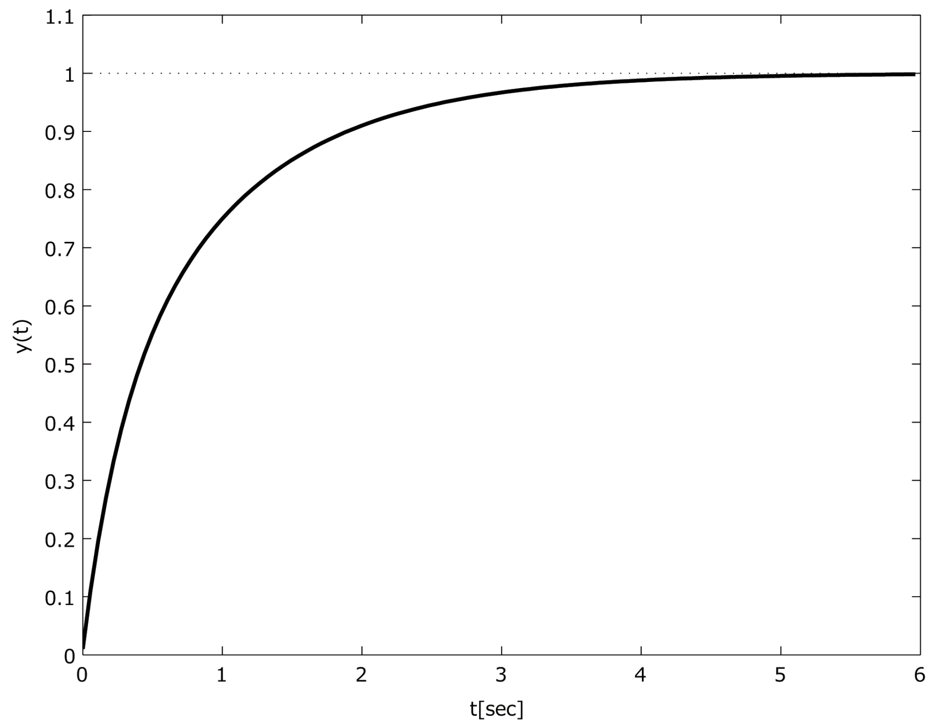

FIGURE 1. Response of the output $y(t)$ of the control system in (1) for the step reference input $r(t) = 1$

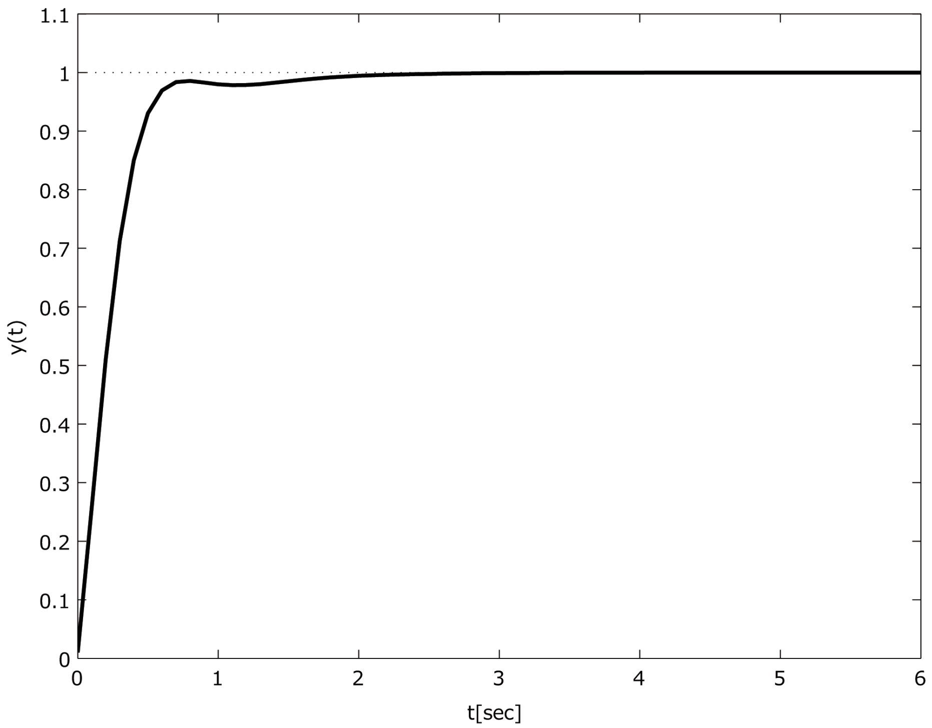

FIGURE 2. Response of the output $y(t)$ of the control system of $G_1(s)$ for the step reference input $r(t) = 1$

Here, the zeros of $C(s)$ are -1, -3.602 and $5.245 \pm 2.967j$, and $C(s)$ in (20) is obviously of nonminimum-phase. Therefore, by using the method of [5], the controllers that we obtain are not necessarily of minimum-phase.

5. **Conclusion.** In this paper, we clarified the parameterization of all stabilizing minimum-phase controllers for minimum-phase plants. That is, we showed that if the stabilizing controller $C(s)$ is written by the form of (3), the minimum-phase plant is stabilized. In addition, we showed a numerical example to illustrate that a stabilizing minimum-phase controller written by the form of (3) can stabilize the minimum-phase plant. We will present the parameterization of all stabilizing minimum-phase controllers for minimum-phase strictly proper plants.

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