ICIC Express Letters
Volume **14**, Number **10**, October **2020**
ICIC International ©2020 ISSN 1881-803X
pp. 1009–1015

A STOCHASTIC MODEL FOR DAIRY COW BODY CONDITION SCORES CHANGES BETWEEN TWO SUCCESSIVE CALVING EVENTS

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Received March 2020; accepted June 2020

Abstract. *In this paper, we shall propose a stochastic model to investigate and analyze the patterns of dairy cow body condition scores between two successive calving events. Also, a robust Markov chain was introduced and used for stochastic evaluation of body condition score fluctuations from time to time. The study confirmed greater beneficial with increased healthy performance, but a great variation among farms needs to be taken into account. The stochastic model can fully describe the pattern and quantify its characteristics composed of the sum of random variables derived from milk yields, feeding intakes and transition periods in the body energy reserves changes. For this purpose, mathematical modeling techniques can be used to develop decision making systems, in order to achieve optimality of dairy farm management systems. In this aspect, the body condition score plays a key role to make the system successfully carried out. That is to achieve maintaining target score in corresponding periods such as a few weeks after calving, early lactation, mid lactation and dry periods. This concept leads to looking into the dairy cow energy reserves problem of within-the-two successive calving events since the body condition score fluctuation is critical especially at the time of calving, with improvements in production. However, a little has known the statistical and probabilistic tools for relating the body condition score pattern change and milk production, feeding management and animal health during the inter-calving periods. Therefore, we shall formulate the problem of energy reserves in dairy cow body, as a stochastic model of special in which inputs (feed intakes), outputs (mike produced) and the body condition score (energy research storage) are used as random variables. Utilizing a generalized gamma distribution and the univariate normal distribution functions for the marginal and joint distributions of the inputs and outputs in the model, the expected change patterns in body condition scores with respect to time are derived and analyzed. In order to confirm the validity of the proposed method, some simulation results are obtained by using the estimated parameters for inputs and outputs derived from real life dataset. These results show that the proposed approach is well suited to analyze the behaviors of dairy cows associations with body condition scores changing patterns.*
Keywords: Markov chain model, Cow Body Condition Score (BCS), Dairy farm management systems, Generalized gamma distribution, Univariate normal distribution

1. **Introduction.** Nowadays, the shape of a dairy farm is transformed into a modern precision context which is based on the utilization of information and technologies such as Artificial Intelligence (AI), deep learning and Internet of Things (IoT). A dairy farm is a complex and fine system with many interacting factors (e.g., cow welfare and health

DOI: 10.24507/icicel.14.10.1009

care, environments, market conditions, and farm management strategies) that determine the profitability of the farm system and the amount of the system production. The dairy farmers or their consultants (decision makers) need to make informed and robust decisions continuously (day-to-day) to maintain a sustainable business in such a challenging environment. Thus, the ability of decision makers to make the right decisions at the right times is an important factor that influences the performance of a dairy herd.

In this aspect, most of dairy farm managers and technical experts well recognized the importance of Cow Body Condition Score (BCS) with respect to milk production, health problems, feeding according to the needs of dairy cows, reproduction and optimal calving cycles. In particular, the body condition score plays a significant role in measuring the probability of a dairy farm. On the other hand, health problem may occur when the cow is too fat (mostly happen at the end of lactation) and too thin (particularly at the beginning of lactation). Thus, the absolute value of BCS at calving and change in BCS during the periods of lactation (early or mid) influences overall profitability of the dairy farms.

Generally, an energy reserves management system controls and regulates the BCS to serve a wide variety of indicators. For example, the regulation of the BCS can make optimal milk production, used as an indicator of cow's health, artificial insemination, and smooth or difficult calving and so on. One of the most important aspects of the energy reserves management systems is the BCS regulation. It basically represents man's interference with the calving cycle in an attempt to "balance" inputs and outputs. In other words, one often needs to smooth out the peaks and lows of feed intakes so that we can obtain a greater optimality of the BCS fits. Thus, the fluctuations of the BCS during the time interval between the two successive calving can be formulated as a partial sum of two or more random variables represented by inputs and outputs in storage, queuing, inventory control theory and many other stochastic processes.

Nowadays, dairy farmers have access to large amounts of data that could be used to guide on-farm decisions. However, this historical data could not be used efficiently without further transformations and projections. The raw data need to be processed to obtain useful information and knowledge, which could be used for important on-farms decisions. This step of processing raw data is to generate valuable information and knowledge using mathematical models.

In this paper, we will propose a stochastic model of special type known as Markov chain for studying dairy cow body condition score change patterns from time to time. To investigate the problems in detail, we organize the rest of the paper as follows. In Section 2, we describe some related works followed by in Section 3 the overview of the major contributions. Section 4 shows some experimental results. Conclusions and future works are presented in Section 5.

2. **Some Related Works.** The application and popularity of stochastic analysis have been on the rise in these days [1]. Since stochastic process is the process of some values changing randomly over time, a stochastic model is well suited to apply in studying the dairy cow body condition score changing patterns over the lactation period, dry period, and calving period for dairy farm healthier and wealthier. Moreover, the system analysis can give the better understanding of dairy farm status. One of the stochastic processes attractable to the dairy cow researchers is the simple type of Markov chain which is related to random walk models.

The probabilistic nature of the Markov chain model made it suitable for many problems facing the dairy farm management systems. These types of models could usually be used for projection, cow reproduction, finding distribution of the farm size, and calving time process analysis [2]. Markov chain application in dairy industry ranges from health and disease and estimating the herd structure at the steady-state which could be used to explore managerial changes in the herd economics, dynamics, and its environmental impacts [3]. The random walk models are one of the most important classes of the Markov chains which appear in many real world problems such as a queuing and water storage problem, a study of fluctuation problems in economic market, the motion of a particle in space and time, and a problem of an animal movement [4-6].

Several researchers describe some significant characteristics of the random walk models in [7,8]. In this aspect, in [9], the authors illustrate the benefits of using a correlated random walk to investigate the relationship between the genetic nature of dairy cows and production of maximum milk yield. Their study showed that the milk yield patterns seem to follow a probability distribution of generalized gamma function in line with Wood function with some modification [10]. Similarly, there are significant associations between the body condition scores and increase of milk production during post-calving and at the beginning of lactation [12-14].

Moreover, the degree of loss of the BCS during early lactation is directly related to the condition of the cow in the last stage of lactation and dry period [15]. In general, the body condition score is an energy balance defined by the difference between energy intake from feed intake and energy required for production [7]. Thus, the BCS problem falls into a category of stochastic process in which a partial sum of random variables is to be investigated. This partial sum is a random walk having wide areas of application such as queuing theory, theory of storage, inventory controls and study of particle motions [16-18].

In this paper, we shall explore the potentials of random walk model to make dairy cow body conditioning patterns analysis by considering the energy reserves stored from feed intakes along with the energy reserves utilized for milk production as random variables.

3. **Markov Chain Model for Dairy Cow Body Condition Scoring.** In the dairy cow behavior analysis, we shall formulate the changes in body condition scores as a stochastic difference equation. In this concern, we shall use a 1-5-point body condition scale with the increment of 0.25. In this system, 1 represents thin cow and 5 represents too fat. In terms of random walk terminology these two can be considered as impenetrable barriers. Let Z_{t+1} be defined as the BCS score for discrete t ($t = 0$, 1, 2, days or weeks or months) just after an instantaneous energy release (produce milk) at t and just before an input X_t (energy feed intakes) over the time-interval $(t, t + 1)$. The model is subject to condition that

(i) the inputs X_t during the intervals $(t, t + 1)$ are independently and identically distributed;

(ii) there is an overfat score $\max(Z_t + X_t - 5, 0)$ during the interval $(t, t+1)$, a quantity $\min(5, Z_t + X_t)$ being recorded just before the release of score 0.25 occurs;

(iii) the amount of energy loss due to milk production at time $t + 1$ is $(0.25, Z_t + X_t)$.

Then the stochastic difference equation for BCS becomes:

$$Z_{t+1} = \min(5, Z_t + X_t) - (0.25, Z_t + X_t) \tag{1}$$

The model in Equation (1) is a type of a stochastic model in which the stochastic processes $\{Z_t\}$ and $\{Z_t + X_t\}$ are both Markov chains. Without loss of generality we can assume that the possible values of Z_t and X_t are discrete values. For Z_t, the values range from 0 to 5 with increment 0.25. In order to make the problem more tractable in analytical concept, we shall denote $m = 0.25$ and $K = 5$, and then the possible values of Z_t can be written as $0, m, 2m, \ldots, 20m = K$.

In new notation, the stochastic Equation (1) can be written as in Markov chain. In order to do so let us define following notations:

$$\overrightarrow{Z_t} = [Z_0, Z_1, \ldots, Z_n];\ \overrightarrow{X_t} - [X_0, X_1, \ldots, X_n];\ \overrightarrow{1 = [1, 1, \ldots, 1]}$$

where n stands for the number of weeks between two successive calving intervals.

From Equation (1) we then have the stochastic vector equation

$$\overrightarrow{Z_{t+1}} = Q\overrightarrow{Z_t} + \overrightarrow{RX_t} - \overrightarrow{1} \tag{2}$$

where Q and R are the transition matrices for the Markov chain and the body condition increments changes ranging between -0.25 and $+0.25$ including 0.

The model described in Equation (2) has been introduced a variety of applications in queuing theory, theory of water storage, traffic congestion, and inventory control in economic and so on since 1950. In queuing theory, the inputs $\{X_t\}$ represent new arrivals at a queue, new water inflows into a reservoir, or new vehicles preparing to cross a minor road in traffic; outputs denote a serviced customer in queueing, a released unit of water from a reservoir, or a vehicle which has crossed the minor road in traffic.

3.1. **Independent distributed inputs.** In animal behavior analysis, we interpret the inputs as the amount of energy intakes necessary for producing milk and the outputs as energy loss after production. Thus, the random variable X_t can be thought of mixture of two probability distributions. In literature, the lactation curve (milk producing patterns) generally follows a gamma type ditribution and the patterns of energy intakes are assumed to have normal or exponential distributions [19-21].

In this paper we assume the random variable X_t has an independent and identically distribution. Let us denote

$$\Pr(X_t = j \times m) = g_i \text{ for } j = 0, 1, 2, \ldots \tag{3}$$

where $m = 0.25$.

From the pratical point of views in dairy farm management, the quantities of particular interest are (i) time dependent and (ii) stationary distributions of the sequence $\{Z_t\}$ defined as:

$$p_j(t) = \Pr(Z_t = j \times m) \text{ and } \pi_j = \lim_{t \to \infty} \Pr(Z_t = j \times m) \tag{4}$$

for $t = 0, 1, 2, \ldots, n$ weeks, and $j = 0, 1, 2, \ldots, K - 1$.

Also, the probabilities of reaching the critical BCS scores of 5 and score of 0 points and time taken to reach such critical values are equally important.

3.2. **Derivation of probability distribution for cow body condition scores.** In order to derive the probability distribution of BCS scores, we first consider the finite difference relations between $p_j(t) = \Pr(Z_t = j)$ and $p_j(t+1) = \Pr(Z_{t:1} = j)$ by using Equations (2) and (4).

$$\begin{aligned}
p_0(t+1) &= \Pr(Z_{t:1} = 0) = \Pr(Z_t + X_t \leq m) \\
&= \Pr(Z_t = 0)[\Pr(X_t = 0) + \Pr(X_t = m)] + \Pr(Z_t = m)\Pr(X_t = 0) \\
&= (g_0 + g_1)p_0(t) + g_0 p_1(t) \\
p_j(t+1) &= g_{j+1}p_0(t) + g_j p_1(t) + \cdots + g_0 p_{j+1} \text{ for } j = 1, \ldots, K-2
\end{aligned}$$

$$p_{K-1}(t+1) = (g_K + g_{K-1} + \cdots)p_0(t) + (g_{K-1} + g_K + \cdots)p_1(t) + \cdots + (g_1 + g_2 + \cdots)p_{K-1}(t) \tag{5}$$

Define $\overrightarrow{P(t)} = [p_0(t), p_1(t), \ldots, p_{K-1}(t)]^T$ as column vector and

$$Q = \begin{bmatrix}
g_0 + g_1 & g_0 & 0 & 0 & 0 & \ldots & \ldots & \ldots & 0 & 0 \\
g_2 & g_1 & g_0 & 0 & \ldots & 0 & 0 & \ldots & \ldots & 0 \\
\vdots & & & & & & & & & \\
g_{j+1} & g_j & \ldots & \ldots & \ldots & g_0 & 0 & 0 & 0 & 0 \\
\vdots & & & & & & & & & \\
h_K & h_{K-1} & h_{K-2} & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & h_1
\end{bmatrix}$$

We can then have

$$\overrightarrow{P(t+1)} = Q\overrightarrow{P(t)} = Q^2\overrightarrow{P(t-1)} = \cdots = Q^{(t+1)}\overrightarrow{P(0)} \tag{6}$$

From Equation (6) the body condition scores are obtained for $t = 1, 2, \ldots, K-1$. Similarly, we can derive the stationary distrbution (long term prediction) for body condition scores.

3.3. **Stationary distribution for BCS.** By taking limit $t \to \infty$, in Equations (4) and (5), we obtain

$$\left.\begin{array}{l} \pi_0 = (g_0 + g_1)\pi_0 + g_0\pi_1 \\ \pi_1 = g_2\pi_0 + g_1\pi_1 + g_0\pi_2 \\ \pi_2 = g_3\pi_0 + g_2\pi_1 + g_1\pi_2 + g_0\pi_3 \\ \vdots \\ \pi_{K-2} = g_{K-1}\pi_0 + \cdots + g_0\pi_{K-1} \\ \pi_{K-1} = h_K\pi_0 + h_{K-1}\pi_1 + \cdots + h_1\pi_{K-1} \end{array}\right\} \tag{7}$$

$$\prod = Q\prod \tag{8}$$

where $\prod = [\pi_0, \pi_1, \ldots, \pi_{K-1}]$. And $\sum_{j=0}^{K-1} \pi_j = 1$, and now define as

$$\pi(s) = \sum_{j=0}^{K-1} \pi_j s^j \text{ and } \sum_{j=0}^{\infty} g_j s^j \text{ for } |s| \le 1$$

Multiply each equation in (7) by $s, s^2, s^3, \ldots$ and adding altogether, we have

$$s\pi(s) = \pi_0(g_0 s - g_0) + \pi(s)G(s) + \left(\textit{terms containing } s^K\right)$$

$$(s - G(s))\pi(s) = \pi_0 g_0(s-1) + \left(\textit{terms containing } s^K\right)$$

$$\pi(s) = \frac{\pi_0 g_0(s-1)}{(s - G(s))} + \left(\textit{terms containing } s^K\right) \tag{9}$$

Since π_j is the coefficient of s^j for $j = 0, 1, 2, \ldots, K-1$, it is not necessary to consider the second part of Equation (9).

That is π_j are just the coefficients of s^j in the power series expansion of $\frac{\pi_0 g_0(s-1)}{s-G(s)} = \pi_0 g_0 \left[c_0 + c_1 s + c_2 s^2 + \cdots + c_{K-1}s^{K-1} + \left(\textit{terms with power } s^K\right)\right]$. Thus we have $\pi_j = \pi_0 g_0 c_j$ for $j = 0, 1, 2, \ldots, K-1$. By normalization, we obtain

$$\sum_{j=0}^{K-1} \pi_j = 1$$

$$\pi_0 g_0 \sum_{j=0}^{K-1} c_j = 1$$

$$\pi_0 = \frac{1}{\left(g_0 \sum_{j=0}^{K-1} c_j\right)}$$

Therefore, the stationary distribution for body condition score is given by

$$\pi_j = \frac{c_j}{\sum_{j=0}^{K-1} c_j} \text{ for } j = 0, 1, 2, \ldots, K-1 \tag{10}$$

In order to investigate the gradual changes in body condition score with respect to time, for example weekly, we just take $t = 1$ week, 2 weeks and so on up to dry period and claving week.

4. **Illustrative Simulation Results.** In order to illustrate the method developed in Section 3, we will use the geometric distribution for the input X_t with parameters p and q.

This will take the form

$$g_j = pq^j \text{ for } j = 0, 1, 2, \ldots$$

where $0 < p < 1$ and $q = 1 - p$. Then the probability generating function is given by

$$G(x) = \frac{p}{1 - qx} \tag{11}$$

From Equation (10), we have

$$\pi(x) = \frac{1 - qx}{1 - \rho x}, \text{ where } \rho = p/q \text{ is the mean input.}$$

Hence we obtain the stationary distribution π_j

$$\pi_0 = \frac{1 - \rho}{p\left(1 - \rho^{K+1}\right)}, \quad \pi_j = \frac{\rho^{j+1}(1 - \rho)}{1 - \rho^{K+1}} \text{ for } j = 1, 2, \ldots, K - 1$$

For simulation purposes, we truncate the normal distribution to stay on the function of positive real line. Also, without loss of generality, we can take the suitable numerical values of parameters, p and q. By using these stationary ditribution for the time of body condition reach below acceptable level of 1 and above the over fat level. These probabilities in Markov chain terminologies are known as the absorbing states.

For example, the probability of reaching very thin body contion score with geometric input distribution is given by

$$\textit{Probability of reaching thin level} = \frac{1 - \rho^{K-j}}{1 - \rho^K}, \textit{ where } j \textit{ is starting state and } \rho \neq 1$$

$$\textit{Probability of reaching thin level} = 1 - \frac{j}{K}, \textit{ where } j \textit{ is starting state and } \rho = 1$$

for $j = 1, 2, \ldots, K - 2$.

In particular, we take $p = 0.4$. Then, the patterns of body condition scores are obtained and presented in Figure 1 and Table 1.

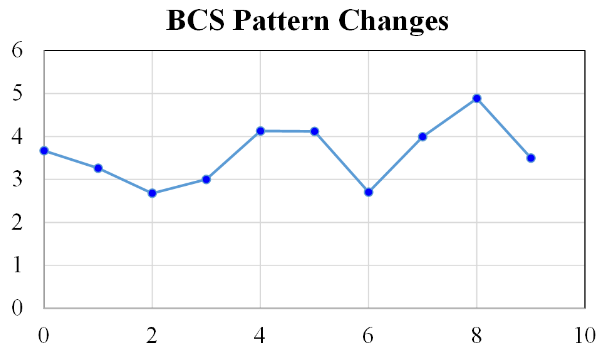

FIGURE 1. Patterns of body condition scores

TABLE 1. Patterns of body condition scores

Time	0	1	2	3	4	5	6	7	8	9
BCS	3.671	3.261	2.678	3.002	4.127	4.12	2.704	3.993	4.884	3.498

5. **Conclusions.** In this paper, we have investigated the changes in the pattern of body condition scores using a simple Markov chain model in which the input distribution is assumed an independent and identically distributed random variable. For illustration, we utilized a particular distribution of geometric type. However, the results seem to be realistic. In future we would study using the real life data collected from the dairy farms.

Acknowledgment. This work was supported in part by SCOPE: Strategic Information and Communications R&D Promotion Program (Grant No. 172310006).

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