DISTRIBUTED FAULT DIAGNOSIS OF MULTI-AGENT SYSTEMS
WITH TIME-VARYING SENSOR FAULTS

KE ZHANG¹, BIN JIANG¹, XING-GANG YAN² AND JINGPING XIA¹

¹College of Automation Engineering
Nanjing University of Aeronautics and Astronautics
No. 29, Jiangjun Avenue, Jiangning District, Nanjing 211106, P. R. China
{kezhang; binjiang; xiajingping}@nuaa.edu.cn

²School of Engineering and Digital Arts
University of Kent, Canterbury
Kent CT2 7NT, United Kingdom
x.yan@kent.ac.uk

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ABSTRACT. In this paper, a distributed fault estimation observer design is proposed for a class of multi-agent systems with sensor faults. Through the equivalent transformation, an augmented multi-agent system is constructed. Then a distributed fault estimation observer is proposed to achieve asymptotic estimation for sensor faults. Furthermore, the observer gain matrix is calculated based on $L_2 - L_\infty$ performance index. Finally, simulation results are presented to show effectiveness of the proposed techniques.

Keywords: Fault diagnosis, Fault estimation, Sensor faults, Multi-agent systems

1. Introduction. Over the past three decades, due to the critical role of fault diagnosis and fault-tolerant control technology in improving the safety and reliability of control systems, they have achieved unprecedented rapid development and a large number of research results have emerged. Once the system has actuator, sensor or component faults, it will change the behavior of the system and even lead to system instability. The fault diagnosis module can detect the time when the fault occurs and determine the damage level of the fault, while fault-tolerant control can compensate for the impact of the fault and recover the performance of the system [1-5].

Due to the strong application background of multi-agent technology, it has been rapidly developed, such as formation flying of unmanned aerial vehicles, and multi-robot collaborative control [6-8]. Due to the information interaction between each subsystem, once a certain system fails, it will inevitably affect the performance of other subsystems through the topology [9-12]. In [13], the problem of fault detection and isolation was dealt with for discrete-time Markovian Jump Linear Systems and a geometric property related to the unobservable subspace of Markovian Jump Systems was considered. In [14], a robust fault estimation method using sliding mode observers was proposed for a class of multi-agents systems with actuator faults. In [15], the distributed fault detection and isolation problem for a class of second-order discrete-time multiagent systems was studied based on an optimal robust observer approach. In [16], the issue of fault detection was presented for high-order multi-agent systems with disturbances based on the unknown input observer design. Although many research results have been achieved on the fault diagnosis of multi-agent systems, the robust asymptotic estimation of time-varying sensors has not been studied in depth.

In this work, a distributed fault estimation observer was proposed for linear multi-agent systems with sensor faults. Main contributions of this manuscript are as follows: 1)
for each subsystem with sensor faults, an augmented system is constructed; 2) a robust asymptotic estimation method of sensor faults is proposed for multi-agents.

The rest of this paper is organized as follows. System description and problem statement are given in Section 2. In Section 3, distributed fault estimation observer and fault estimator are constructed. Simulation results are presented in Section 4 to illustrate the effectiveness of the distributed design strategy. Section 5 concludes this manuscript.

2. Preliminaries and System Description.

2.1. Preliminaries. Consider a group of $N$ agents. Denote a directed graph $G = (\mathcal{V}, \varepsilon, \mathcal{A})$ to be a communication graph among $N$ agents, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ represents the set of agents, $\varepsilon = \mathcal{V} \times \mathcal{V}$ is the set of edges or arcs, and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the associated adjacency matrix. In this paper, the graph is assumed to be time-invariant, i.e., $\mathcal{A}$ is constant. An edge rooted at agent $v_j$ and ended at agent $v_i$ is denoted by $(v_j, v_i)$, which means information can flow from agent $v_j$ to agent $v_i$. $a_{ij}$ is the weight of edge $(v_j, v_i)$ and $a_{ij} = 1$ if $(v_j, v_i) \in \varepsilon$, otherwise $a_{ij} = 0$. Agent $v_j$ is called a neighbor of agent $v_i$ if $(v_j, v_i) \in \varepsilon$. The set of neighbors of agent $v_i$ is denoted as $N_i = \{j | (v_j, v_i) \in \varepsilon\}$. Define the in-degree matrix as $\mathcal{D} = \text{diag}\{d_i\} \in \mathbb{R}^{N \times N}$ with $d_i = \sum_{j \in N_i} a_{ij}$ and the Laplacian matrix as $L = \mathcal{D} - \mathcal{A}$.

The edges in the form of $(v_i, v_i)$ are called loops. $G = \text{diag}\{g_i\} \in \mathbb{R}^{N \times N}$ is denoted as a loop matrix and has at least one diagonal item being 1.

2.2. System description. Consider the following multi-agent with sensor faults

$$
\begin{align*}
\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + D\omega_i(t) \\
y_i(t) &= Cx_i(t) + Ef_i(t)
\end{align*}
$$

(1)

where $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ and $y_i(t) \in \mathbb{R}^p$ are the state, the input and the output of the $i$th agent respectively. $\omega_i(t) \in \mathbb{R}^d$ is the external disturbance and $f_i(t) \in \mathbb{R}^r$ represents the sensor fault. The sensor fault is bounded. $A$, $B$, $C$, $D$ and $E$ are constant real matrices of appropriate dimensions. It is supposed that matrix $E$ is of full rank and the pair $(A, C)$ is observable.

From (1), the following augmented system description can be obtained with the purpose of asymptotic sensor fault estimation:

$$
\begin{align*}
\begin{bmatrix}
I_n & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x}_i(t) \\
\dot{f}_i(t)
\end{bmatrix} &= 
\begin{bmatrix}
A & 0 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
x_i(t) \\
f_i(t)
\end{bmatrix} + 
\begin{bmatrix}
B & 0 \\
0 & 0
\end{bmatrix}
u_i(t) + 
\begin{bmatrix}
D & 0 \\
0 & 0
\end{bmatrix}\omega_i(t)
\end{align*}
$$

(2)

The augmented variable and matrices are denoted as below:

$$
\begin{align*}
\tilde{x}_i(t) &= \begin{bmatrix} x_i(t) \\ f_i(t) \end{bmatrix},
S &= \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix},
\tilde{A} &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix},
\tilde{B} &= \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix},
\tilde{D} &= \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix},
\end{align*}
$$

$$
\tilde{C} = \begin{bmatrix} C & E \end{bmatrix}.
$$

It follows:

$$
\begin{align*}
S\dot{\tilde{x}}_i(t) &= \tilde{A}\tilde{x}_i(t) + \tilde{B}u_i(t) + \tilde{D}\omega_i(t) \\
y_i(t) &= \tilde{C}\tilde{x}_i(t)
\end{align*}
$$

(3)

**Remark 2.1.** The augmented system (3) is equivalent to the original system (1). For system (3), if $\tilde{x}_i(t)$ can be estimated robustly asymptotically, accurate estimation of sensor fault can be obtained.
3. Main Results. In order to estimate sensor faults, the following observer is constructed:
\[
\begin{align*}
S\ddot{x}_i(t) &= \bar{A}\dot{x}_i(t) + \bar{B}u_i(t) - \bar{R}\xi_i(t) - \bar{F} \left( \dot{\hat{y}}_i(t) - \hat{y}_i(t) \right) \\
\hat{y}_i(t) &= \bar{C}\dot{x}_i(t)
\end{align*}
\]
(4)
where \( \hat{x}_i(t) \in \mathbb{R}^{n+r} \) and \( \hat{y}_i(t) \in \mathbb{R}^p \) are the state and output of the observer, respectively. \( \bar{R} \in \mathbb{R}^{(n+r) \times p} \) and \( \bar{F} \in \mathbb{R}^{(n+r) \times p} \) are observer gain matrices to be designed, and \( \bar{F} = \begin{bmatrix} 0 & F \end{bmatrix} \) with \( F \in \mathbb{R}^{r \times p} \). \( \xi_i(t) \in \mathbb{R}^p \) is the relative output estimation error of the \( i \)th node and is defined as follows
\[
\xi_i(t) = \sum_{j \in N_i} a_{ij} ((\hat{y}_j(t) - y_i(t)) - (\hat{y}_j(t) - y_j(t))) + g_i (\hat{y}_i(t) - y_i(t))
\]

**Remark 3.1.** Since matrix \( S \) in (4) is singular, it is not convenient to analyze and design the fault estimation observer. So the term \( \bar{F} \left( \dot{\hat{y}}_i(t) - \hat{y}_i(t) \right) \) is added in (4) to handle this problem, which is helpful for analysis and design for the proposed design method.

Denote the error vector:
\[
\bar{e}_i(t) = \hat{x}_i(t) - \bar{x}_i(t)
\]
then it derives
\[
(S + \bar{F}C) \bar{e}_i(t) = \bar{A}\bar{e}_i(t) - \bar{R}\xi_i(t) - \bar{D}\omega_i(t)
\]
(5)
According to
\[
S + \bar{F}C = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & F \end{bmatrix} \left[ \begin{array}{cc} C & E \end{array} \right] = \begin{bmatrix} I_n & 0 \\ FC & FE \end{bmatrix}
\]
it is seen that since matrix \( E \) is of full-column rank, we can choose a suitable matrix \( F \) such that \( FE \) is nonsingular square matrix. It is concluded that \( S \) is a full rank matrix.

In the view of the global system, we denote global vectors:
\[
\begin{align*}
\bar{e}(t) &= \begin{bmatrix} \bar{e}_1^T(t) & \bar{e}_2^T(t) & \cdots & \bar{e}_N^T(t) \end{bmatrix}^T, \\
e_f(t) &= \begin{bmatrix} e_{f1}^T(t) & e_{f2}^T(t) & \cdots & e_{fN}^T(t) \end{bmatrix}^T, \\
\omega(t) &= \begin{bmatrix} \omega_1^T(t) & \omega_2^T(t) & \cdots & \omega_N^T(t) \end{bmatrix}^T,
\end{align*}
\]
where \( e_{f_i}(t) = \dot{\hat{f}}_i(t) - f_i(t) \), then the global error dynamics is
\[
\begin{align*}
(I_N \otimes (S + \bar{F}C)) \bar{e}(t) &= (I_N \otimes A) \bar{e}(t) - ((L + G) \otimes \bar{R}C) \bar{e}(t) - (I_N \otimes \bar{D}) \omega(t) \\
&= (I_N \otimes A - (L + G) \otimes \bar{R}C) \bar{e}(t) - (I_N \otimes \bar{D}) \omega(t)
\end{align*}
\]
(6)
where \( \otimes \) is kroneck product.

Further, we denote:
\[
\bar{M} = I_N \otimes (S + \bar{F}C)
\]
and the error dynamic equation becomes
\[
\begin{align*}
\begin{cases}
\dot{\bar{e}}(t) &= \bar{M}^{-1} \left( I_N \otimes \bar{A} - (L + G) \otimes \bar{R}C \right) \bar{e}(t) - \bar{M}^{-1} \left( I_N \otimes \bar{D} \right) \omega(t) \\
e_f(t) &= (I_N \otimes \bar{A}) \bar{e}(t)
\end{cases}
\end{align*}
\]
(7)
where \( \bar{I}_r = \begin{bmatrix} 0 & I_r \end{bmatrix} \).

**Theorem 3.1.** Given a circular region \( D(\alpha, \tau) \) with center \( \alpha + 0j \) and radius \( \tau \), and an \( L_2 - L_\infty \) performance index \( \gamma \). If there exist a symmetric positive definite matrix \( \bar{P} \in \mathbb{R}^{(n+r) \times (n+r)} \) and a matrix \( \bar{Y} \in \mathbb{R}^{(n+r) \times p} \) such that the following conditions are satisfied:
\[
\begin{bmatrix}
I_N \otimes (\bar{P}) & I_N \otimes (\bar{P}\bar{A}) - (L + G) \otimes (\bar{Y}C) - \alpha (I_N \otimes \bar{P}) \bar{M} \\
* & -\tau^2 \bar{M}^T (I_N \otimes \bar{P}) \bar{M}
\end{bmatrix} < 0
\]
(8)
\[
\begin{bmatrix}
\varphi & -\tilde{M}^T (I_N \otimes \tilde{P} \tilde{D}) \\
* & -\gamma I
\end{bmatrix} < 0
\]  
(9)

\[
\begin{bmatrix}
-\tilde{M}^T (I_N \otimes \tilde{P}) M & I_N \otimes \tilde{I}_r \\
* & -\gamma I
\end{bmatrix} < 0
\]  
(10)

where \( \varphi = \tilde{M}^T (I_N \otimes (\tilde{P} \tilde{A}) - (L + G) \otimes (\tilde{Y} \tilde{C})) + (I_N \otimes (\tilde{P} \tilde{A}) - (L + G) \otimes (\tilde{Y} \tilde{C}))^T M \), then the eigenvalues of \( \tilde{M}^{-1} (I_N \otimes \tilde{A} - (L + G) \otimes \tilde{R} \tilde{C}) \) belong to \( D(\alpha, \tau) \) and the error dynamic equation satisfies \( L_2 - L_\infty \) performance \( \| \epsilon_f(t) \|_\infty < \gamma \| \omega(t) \|_2 \).

**Proof:** Condition (8): For the given circular region \( D(\alpha, \tau) \) and Lyapunov matrix \( \tilde{M}^T (I_N \otimes \tilde{P}) M \), condition (8) can be obtained directly according to the regional pole replacement lemma [5].

Conditions (9)-(10): Consider the following Lyapunov function
\[
V(t) = \tilde{e}^T(t) \tilde{M}^T (I_N \otimes \tilde{P}) \tilde{M} \tilde{e}(t)
\]  
(11)

The following performance index \( J_1(t) \) is denoted as:
\[
J_1(t) = V(t) - \gamma \int_0^t \omega^T(s) \omega(s) ds
\]  
(12)

Under zero initial condition, one gets
\[
J_1(t) = \int_0^t \dot{V}(s) - \gamma \omega^T(s) \omega(s) ds
\]  
(13)

Further,
\[
\begin{align*}
\dot{V}(t) - \gamma \omega^T(t) \omega(t) & \\
& \leq \tilde{e}^T(t) \tilde{M}^T (I_N \otimes (\tilde{P} \tilde{A}) - (L + G) \otimes (\tilde{Y} \tilde{C})) \tilde{e}(t) \\
& + \tilde{e}^T(t) (I_N \otimes (\tilde{P} \tilde{A}) - (L + G) \otimes (\tilde{Y} \tilde{C}))^T \tilde{M} \tilde{e}(t) \\
& - 2\tilde{e}^T(t) \tilde{M}^T (I_N \otimes (\tilde{P} \tilde{D})) \omega(t) - \gamma \omega^T(t) \omega(t)
\end{align*}
\]  
(14)

If condition (9) holds, then \( J_1(t) < 0 \) and \( V(t) < \gamma \| \omega(t) \|^2 \).

We denote the performance index \( J_2(t) \)
\[
J_2(t) = \tilde{e}^T(t) \epsilon_f(t) - \gamma V(t)
\]
\[
= \tilde{e}^T(t) (I_N \otimes \tilde{I}_r I^T_{\tau}) \tilde{e}(t) - \gamma \tilde{e}^T(t) \tilde{M}^T (I_N \otimes \tilde{P}) \tilde{M} \tilde{e}(t)
\]
\[
= \tilde{e}^T(t) (I_N \otimes \tilde{I}_r I^T_{\tau} - \gamma \tilde{M}^T (I_N \otimes \tilde{P}) \tilde{M}) \tilde{e}(t)
\]  
(15)

Based on the Schur complement lemma, \( (I_N \otimes \tilde{I}_r I^T_{\tau}) - \gamma \tilde{M}^T (I_N \otimes \tilde{P}) \tilde{M} < 0 \) is equivalent to condition (10). If condition (10) is satisfied, then \( \epsilon_f(t) \epsilon_f(t) < \gamma V(t) \).

Therefore, if conditions (9) and (10) hold, the error dynamic equation (7) satisfies \( L_2 - L_\infty \) performance \( \| \epsilon_f(t) \|_\infty < \gamma \| \omega(t) \|_2 \). □

The observer gain has been calculated from Theorem 3.1. Since matrix \( S \) is singular in fault estimation observer (4), the following equivalent form can be derived:
\[
\dot{\hat{x}}_i(t) = \tilde{A} \hat{x}_i(t) + \tilde{B} u_i(t) - \tilde{R} \xi_i(t) - \tilde{F} \tilde{C} \hat{x}_i(t) + \tilde{F} \hat{y}_i(t)
\]  
(16)

Further, one obtains
\[
(S + \tilde{F} \tilde{C}) \dot{\hat{x}}_i(t) = \tilde{A} \hat{x}_i(t) + \tilde{B} u_i(t) - \tilde{R} \xi_i(t) + \tilde{F} \hat{y}_i(t)
\]  
(17)

It follows from the fact \( S + \tilde{F} \tilde{C} \) is nonsingular
\[
\left\{ \begin{array}{l}
\dot{\hat{x}}_i(t) = (S + \tilde{F} \tilde{C})^{-1} (\tilde{A} \hat{x}_i(t) + \tilde{B} u_i(t) - \tilde{R} \xi_i(t) + \tilde{F} \hat{y}_i(t)) \\
\dot{f}_i(t) = I^T_r \hat{x}_i(t)
\end{array} \right.
\]  
(18)
Therefore, the online sensor fault estimation can be obtained from observer (18).

4. Simulation Results. Consider the following multi-agent systems with sensor faults

\[
\begin{align*}
\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + D\omega_i(t) \\
y_i(t) &= Cx_i(t) + Ef_i(t)
\end{align*}
\]

where the state \(x_i(t)\) contains horizontal velocity, vertical velocity, pitch rate, and pitch angle. The control \(u_i(t)\) is collective pitch control and longitudinal cyclic pitch control. And

\[
A = \begin{bmatrix}
-9.9477 & -0.7476 & 0.2632 & 5.0337 \\
52.1659 & 2.7452 & 5.5532 & -24.4221 \\
0 & 0 & 1 & 0
\end{bmatrix},
B = \begin{bmatrix}
0.4422 & 0.1761 \\
3.5446 & -7.5922 \\
-5.5200 & 4.49 \\
0 & 0
\end{bmatrix},
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},
E = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix},
D = \begin{bmatrix}
0.1 \\
0.1 \\
0.1 \\
0.1
\end{bmatrix}.
\]

We consider the following directed graph illustrated in Figure 1.

\[
\begin{align*}
L &= \begin{bmatrix}
1 & -1 & 0 \\
0 & 1 & -1 \\
-1 & 0 & 1
\end{bmatrix},
G = \begin{bmatrix}
1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\end{align*}
\]

Moreover, the first node contains a loop. For such directed communication topology, we can get the matrices

\[
F = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0
\end{bmatrix}
\]

such that we obtain \(FE = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}\) and the augmented matrix \((S + \tilde{F}C)\) is nonsingular.

Under the regional pole constraint \(D(-5,5)\) and solving conditions of Theorem 3.1, one derives \(\gamma = 0.5093\) and observer gain matrix:

\[
\tilde{R} = \begin{bmatrix}
0.0011 & -0.4040 & 1.0120 \\
-0.0009 & 1.8717 & 10.8967 \\
-0.0163 & 2.7899 & -1.0042 \\
-0.0005 & -0.2065 & 3.3341 \\
2.8330 & 0.0037 & 0.0000 \\
0.0050 & 2.9231 & -0.0002
\end{bmatrix}
\]

In simulation, it is assumed that there are sensor faults in the first and second agents, that is

\[
f_{11}(t) = \begin{cases}
0, & 0s \leq t \leq 20s \\
0.5 \left(1 - e^{-0.3(t-20)}\right), & 20s < t \leq 50s
\end{cases}, \quad f_{12}(t) = 0.
\]
\[
f_{21}(t) = \begin{cases} 
0, & 0 \leq t \leq 25s \\
0.5 \sin(t - 25), & 25s < t \leq 50s 
\end{cases}, \quad f_{22}(t) = 0 \\
f_{31}(t) = 0, \quad f_{32}(t) = 0
\]

Simulation results are illustrated in Figure 2 and Figure 3. From simulation results, we can see that the proposed design method can achieve an accurate estimation of sensor faults, which has verified the effectiveness of the presented estimation techniques.

Figure 2. Fault estimation of \( f_{11}(t) \)

Figure 3. Fault estimation of \( f_{21}(t) \)

5. **Conclusions.** In this work, a distributed fault estimation observer design has been proposed to estimate sensor faults asymptotically. Using \( L_2 - L_\infty \) performance index, observer gain matrix is calculated in terms of linear matrix inequalities. Finally, simulation results are given to illustrate effectiveness of the proposed stagey. The issue of fault diagnosis for multi-agent systems with switching topologies will be studied in our future work.

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