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ADAPTIVE H_∞ SLIDING MODE CONTROL FOR A CLASS OF UNCERTAIN MARKOVIAN JUMP SYSTEMS WITH TIME-DELAY

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Abstract. *In this note, robust sliding mode control problem for a class of uncertain Markovian jump systems (MJSs) with time-delay and external disturbance is considered. A new linear sliding surface design is provided for such systems, from which a new stochastic stability condition of the system dynamics during the sliding surface is presented. And an adaptive reaching motion controller is designed to guarantee the arrival of the predesigned switching surface. An illustrative example is shown to finish the effectiveness of the proposed scheme.*
Keywords: Sliding mode control, Markovian jump systems, Adaptive controller, Time-delay

1. **Introduction.** In the past decades, Markovian jump systems (MJSs) have been greatly studied in system control field [1]. It is a fact that control systems such as networked systems, power systems, may subject to random abrupt variations, MJSs can be employed for the description of practical systems, thus plenty of important results have been obtained on this subject, which contain stability and stabilization, filtering and H_∞ control. Time-delay often happens in many plants, e.g., lossless transmission lines, which is one key factor of poor performance and instability [2]. Therefore, it is of great importance to investigate uncertain MJSs with time-delay, see [3-6].

Sliding mode control (SMC), has been commonly a classical control method in view of distinguished merits, e.g., simplicity in algorithm, robustness in parametric uncertainties and external disturbance during the sliding mode [7]. At this point, in the light of strong background and ability to suppress or offset the modelling uncertainties, SMC approach has been proposed to tackle the matched nonlinearities for various systems [8-16]. For MJSs with time-delay, in [11], a SMC design for a class of MJSs was first studied, where the model transformation method was carried out. By the integral-type sliding surface design, uncertain stochastic MJSs were considered via SMC method in [12-15]. For instance, certain restriction is given for the SMC of MJSs in [14] and the case the state variables may not be available is also not considered, and the model reduction process is needed for the SMC design in [15]. Thus, the discussed situation motivates us to probe an issue: how to give a linear sliding surface-based SMC design to deal with the uncertainties in the same channel with control signals without any model reduction becomes meaningful and remains open for the MJSs.

Inspired by the aforementioned problems, robust adaptive control problem via a novel linear sliding surface-based SMC design is considered for a class of MJSs with matched and structural uncertainties in this manuscript. The main features of the paper in comparison

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to the existing ones is that a new linear sliding surface design of the MJSs is proposed via adaptive SMC. For stability analysis of the MJSs, a new sufficient condition for stochastic stability of the sliding mode dynamics of the closed-loop system is presented from the definition of the sliding surface. Finally, an illustrative example is given to justify the effectiveness of the proposed method.

The rest of the note is organized as follows. In Section 2, system description and preliminaries are proposed. In Section 3, main results of the SMC design for the systems are presented. Section 4 gives an example and Section 5 ends the paper.

Notations: $\mathbb{R}^n$ represents the n-dimensional Euclidean space; I and 0 stand for the identity matrix and zero matrix, respectively. $\mathscr{E}(\cdot)$ is the expectation operator. $\|\cdot\|$ denotes the Euclidean norm of a vector or the spectral norm of a matrix. The superscript 'T' denotes the transpose of a matrix or vector, and symmetric elements of a matrix are denoted by '$*$'. $sym\{P\}$ is defined as $P + P^{\mathrm{T}}$, and $\lambda_{\min}(P)$ denotes the minimum eigenvalue of a real matrix P.

2. **Problem Statement and Preliminaries.** For a completed probability space $(\Omega, \mathscr{F}, \mathscr{P})$, where Ω is a sample space, $\mathscr{F}$ is the σ-algebra of subset of the sample space and $\mathscr{P}$ is the probability measure on $\mathscr{F}$. The following uncertain MJS is considered

$$\begin{cases} \dot{x}(t) = [A(r_t) + \Delta A(r_t, t)]x(t) + [A_d(r_t) + \Delta A_d(r_t, t)]x(t-d) \\ \qquad\quad + B(r_t)[u(t) + g(t, x)] + G(r_t)w(t), \\ y(t) = C(r_t)x(t) + D(r_t)w(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ and $y(t) \in \mathbb{R}^p$ are the state vector, control input and measured output, respectively. $A(r_t)$, $A_d(r_t)$, $B(r_t)$, $C(r_t)$, $D(r_t)$ and $G(r_t)$ are known real matrices with appropriate dimensions. $\{r_t, t \geq 0\}$ is a continuous-time Markovian process representing the system mode, which takes discrete values in a finite space $\mathbb{S} = \{1, 2, \ldots, N\}$. Let $\Pi = (\pi_{ij})_{N\times N}$ $(i, j = 1, 2, \ldots, N)$ be transition rate (TR) matrix with the associated transition probabilities (TP) followed by

$$P_{ij} = \Pr(r_{t+\Delta t} = j | r_t = i) = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & \text{if } i = j, \end{cases}$$

where $\Delta t > 0$ and $\lim\limits_{\Delta t\to 0} o(\Delta t)/\Delta t = 0$, π_{ij} satisfies $\pi_{ij} > 0$ with $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$ for each mode r_t.

For notional simplicity, each possible value of r_t is denoted by $r_t = i \in \mathbb{S}$ thereafter, system (1) can be represented by

$$\begin{cases} \dot{x}(t) = [A_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) + B_i[u(t) + g(t, x)] \\ \qquad\quad + G_i w(t), \\ y(t) = C_i x(t) + D_i w(t), \end{cases} \tag{2}$$

with $A(r_t) = A_i$, $A_d(r_t) = A_{di}$, $B(r_t) = B_i$, $C(r_t) = C_i$, $D(r_t) = D_i$ and $G(r_t) = G_i$. $[\Delta A(r_t, t)\ \Delta A_d(r_t, t)] = [\Delta A_i(t)\ \Delta A_{di}(t)]$ denotes the system parametric uncertainty satisfying $[\Delta A_i(t)\ \Delta A_{di}(t)] = M_i F_i(t)[N_i\ N_{di}]$, where M_i, N_i and N_{di} are real matrices, and $F_i(t)$ is an unknown nonlinear matrix function assuming $F_i^{\mathrm{T}}(t)F_i(t) \leq I$, and B_i is of full column rank. The matched uncertainty $g(t, x)$ may be nonlinear and there exist unknown parameters $\alpha > 0$ and $\beta > 0$ assuming $\|g(t, x)\| \leq \beta + \alpha\|x(t)\|$. The external disturbance belongs to $L_2[0\ \infty)$, which may be unknown and bounded by an unknown positive constant ϑ, i.e., $\|w(t)\| \leq \vartheta$.

Lemma 2.1. [8] *Let M, N and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^{\mathrm{T}}(t)F(t) \leq I$. For any scalar $\varepsilon > 0$, it follows that $MF(t)N + N^{\mathrm{T}}F^{\mathrm{T}}(t)M^{\mathrm{T}} \leq \varepsilon N^{\mathrm{T}}N + \varepsilon^{-1}MM^{\mathrm{T}}$.*

3. **Sliding Mode Synthesis.** In this section, a new linear sliding surface-based control design is put forward for the systems.

3.1. **Design of sliding surface.** In this subsection, the following linear switching function is defined for MJS (2):

$$s(t) = B_i^{\mathrm{T}} P_i x(t), \tag{3}$$

where matrix P_i is a positive definite matrix to be designed.

Remark 3.1. *As to the design of sliding surface, it is a linear-type surface, which depends on the current status of the system state only and easy to apply, differing from the results in [11-15] for the MJSs and could be benefit for the deduction of the resultant system during the sliding surface by the new design, see Section 3.2.*

3.2. **Sliding mode dynamics and admissibility analysis.** This part will focus on the expected dynamics on the sliding motion, and a new sufficient condition for stochastic stability of the closed-loop system will be derived. In the position, we will use the following equivalent dynamics to show the stability analysis of the system instead of (2).

$$\begin{cases} \dot{x}(t) = [A_i + B_i K_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) + B_i[u(t) + \tilde{g}(t,x)] \\ \qquad\quad + G_i w(t), \\ y(t) = C_i x(t) + D_i w(t), \end{cases} \tag{4}$$

where $\tilde{g}(t,x) = g(t,x) - B_i K_i x(t)$.

Then, robust H_∞ performance of the considered system can be listed below:

P1) Given a scalar $\gamma > 0$, the H_∞ performance index can be satisfied as follows: for nonzero exogenous input $v(t)$ with zero initial conditions:

$$\mathscr{E}\left\{\int_0^\infty y^{\mathrm{T}}(t)y(t)\mathrm{d}t\right\} \leq \gamma^2 \int_0^\infty w^{\mathrm{T}}(t)w(t)\mathrm{d}t;$$

P2) Stochastic stablity of the system dynamics during the sliding mode can be satisfied when $w(t) = 0$.

Theorem 3.1. *The system (2) (or (4)) restricted on the sliding surface in (3) is stochastically stable, if there exist positive definite matrices P_i, Q_i and positive scalars $\varepsilon_{li} > 0$ $(l = 1, 2)$ for each $i \in \mathbb{S}$, to satisfy the underlying condition:*

$$\begin{bmatrix} \Xi_{11i} & P_i A_{di} & P_i G_i + C_i^{\mathrm{T}} D_i & P_i^{\mathrm{T}} M_i & P_i^{\mathrm{T}} M_i \\ * & \Xi_{22i} & 0 & 0 & 0 \\ * & * & -\gamma^2 I + D_i^{\mathrm{T}} D_i & 0 & 0 \\ * & * & * & -\varepsilon_{1i} I & 0 \\ * & * & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{5}$$

where $\Xi_{11i} = sym\{P_i(A_i + B_i K_i)\} + Q_i + \varepsilon_{1i} N_i^{\mathrm{T}} N_i + \sum_{j=1}^{N} \pi_{ij} P_j + C_i^{\mathrm{T}} C_i$, $\Xi_{22i} = -Q_i + \varepsilon_{2i} N_{di}^{\mathrm{T}} N_{di}$.

Proof: Step 1. The stability of the closed-loop system during the sliding mode is studied. The following Lyapunov function is chosen

$$V(x(t), i) = x^{\mathrm{T}}(t) P_i x(t) + \int_{t-d}^{t} x^{\mathrm{T}}(\theta) Q_i x(\theta) \mathrm{d}\theta.$$

Let (x, i) denote appropriate values of the original state and the associated mode i at time t. Let $\mathcal{L}$ be the infinitesimal operator applying to the function V from the point (x, i) at time t [11], it follows that

$$
\begin{aligned}
\mathcal{L}V(x(t),i) &= 2x^{\mathrm{T}}(t)P_i\dot{x}(t) + x^{\mathrm{T}}(t)Q_ix(t) - x^{\mathrm{T}}(t-d)Q_ix(t-d) + x^{\mathrm{T}}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) \\
&= 2x^{\mathrm{T}}(t)P_i\{[A_i + B_iK_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) \\
&\quad + B_i[u(t) + \tilde{g}(t,x)]\} + x^{\mathrm{T}}(t)Q_ix(t) - x^{\mathrm{T}}(t-d)Q_ix(t-d) \\
&\quad + x^{\mathrm{T}}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) \\
&= 2x^{\mathrm{T}}(t)P_i\{[A_i + B_iK_i]x(t) + A_{di}x(t-d) + B_i[u(t) + \tilde{g}(t,x)]\} \\
&\quad + x^{\mathrm{T}}(t)Q_ix(t) - x^{\mathrm{T}}(t-d)Q_ix(t-d) + x^{\mathrm{T}}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) \\
&\quad + 2x^{\mathrm{T}}(t)P_i\Delta A_i(t)x(t) + 2x^{\mathrm{T}}(t)P_i\Delta A_{di}(t)x(t-d). \qquad (6)
\end{aligned}
$$

The following inequalities hold from Lemma 2.1 that

$$2x^{\mathrm{T}}(t)P_i\Delta A_i(t)x(t) \le \varepsilon_{1i}^{-1}x^{\mathrm{T}}(t)P_iM_iM_i^{\mathrm{T}}P_ix(t) + \varepsilon_{1i}x^{\mathrm{T}}(t)N_i^{\mathrm{T}}N_ix(t), \qquad (7)$$

$$2x^{\mathrm{T}}(t)P_i\Delta A_{di}(t)x(t-d) \le \varepsilon_{2i}^{-1}x^{\mathrm{T}}(t)P_iM_iM_i^{\mathrm{T}}P_ix(t) + \varepsilon_{2i}x^{\mathrm{T}}(t-d)N_{di}^{\mathrm{T}}N_{di}x(t-d). \qquad (8)$$

Besides, considering the arrival of the sliding surface $s(t) = 0$, it follows that $s^{\mathrm{T}}(t) = x^{\mathrm{T}}(t)P_iB_i = 0$. Thus, one can further get from the above discussions that

$$
\begin{aligned}
\mathcal{L}V(x,i) &\le 2x^{\mathrm{T}}(t)P_i\{[A_i + B_iK_i]x(t) + A_{di}x(t-d)\} + x^{\mathrm{T}}(t)Q_ix(t) \\
&\quad - x^{\mathrm{T}}(t-d)Q_ix(t-d) + x^{\mathrm{T}}(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) \\
&\quad + \varepsilon_{1i}^{-1}x^{\mathrm{T}}(t)P_iM_iM_i^{\mathrm{T}}P_ix(t) + \varepsilon_{1i}x^{\mathrm{T}}(t)N_i^{\mathrm{T}}N_ix(t) \\
&\quad + \varepsilon_{2i}^{-1}x^{\mathrm{T}}(t)P_iM_iM_i^{\mathrm{T}}P_ix(t) + \varepsilon_{2i}x^{\mathrm{T}}(t-d)N_{di}^{\mathrm{T}}N_{di}x(t-d) \\
&= \xi^{\mathrm{T}}(t)\Theta_i\xi(t), \qquad (9)
\end{aligned}
$$

where $\xi^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-d)\right]$, $\Theta_i = \begin{bmatrix} \Theta_{11i} & \Theta_{12i} \\ * & \Theta_{22i} \end{bmatrix}$, with $\Theta_{11i} = sym\{P_i(A_i+B_iK_i)\} + Q_i + \varepsilon_{1i}N_i^{\mathrm{T}}N_i + \left(\varepsilon_{1i}^{-1} + \varepsilon_{2i}^{-1}\right)P_i^{\mathrm{T}}M_iM_i^{\mathrm{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j$, $\Theta_{12i} = P_iA_{di}$, $\Theta_{22i} = -Q_i + \varepsilon_{2i}N_{di}^{\mathrm{T}}N_{di}$.

Further by the Schur complement, one gives $\Theta_i < 0$ due to (5), which gives $\mathcal{L}V(x,i) \le -\min_{j\in\mathbb{S}}\{\lambda_{\min}(-\Theta_j)\}\, x^{\mathrm{T}}(t)x(t)$. Via the Dynkin formula with the above description, we have

$$
\begin{aligned}
\mathscr{E}V(x(t),i) - V(x(0),r_0) &= \mathscr{E}\left\{\int_0^t \mathcal{L}V(x(s),r_s)\mathrm{d}s|(x_0,r_0)\right\} \\
&\le -\min_{j\in\mathbb{S}}\{\lambda_{\min}(-\Theta_j)\}\mathscr{E}\left\{\int_0^t x^{\mathrm{T}}(s)x(s)\mathrm{d}s|(x_0,r_0)\right\}, \qquad (10)
\end{aligned}
$$

which results in that for all $t \ge 0$, $\mathscr{E}\left\{\int_0^t x^{\mathrm{T}}(s)x(s)\mathrm{d}s|(x_0,r_0)\right\} \le \frac{V(x(0),r_0)}{\min_{j\in\mathbb{S}}\{\lambda_{\min}(-\Theta_j)\}}$. Therefore, one can get the system in (2) (or (4)) is stochastically stable during the sliding mode.

Step 2. By taking the disturbance $w(t) \ne 0$ and the above deduction into consideration, it follows that

$$\mathcal{L}V(\xi(t),i) + y^{\mathrm{T}}(t)y(t) - \gamma^2w^{\mathrm{T}}(t)w(t) = \zeta^{\mathrm{T}}(t)\Omega_i\zeta(t) \qquad (11)$$

where $\zeta^{\mathrm{T}}(t) = \left[x^{\mathrm{T}}(t)\ x^{\mathrm{T}}(t-d)\ w^{\mathrm{T}}(t)\right]$, $\Omega_i = \begin{bmatrix} \Omega_{11i} & \Omega_{12i} & P_iG_i + C_i^{\mathrm{T}}D_i \\ * & \Omega_{22i} & 0 \\ * & * & -\gamma^2 I + D_i^{\mathrm{T}}D_i \end{bmatrix}$, with $\Omega_{11i} = sym\{P_i(A_i + B_iK_i)\} + Q_i + \varepsilon_{1i}N_i^{\mathrm{T}}N_i + \left(\varepsilon_{1i}^{-1} + \varepsilon_{2i}^{-1}\right)P_i^{\mathrm{T}}M_iM_i^{\mathrm{T}}P_i + \sum_{j=1}^{N}\pi_{ij}P_j + C_i^{\mathrm{T}}C_i$, $\Omega_{12i} = P_iA_{di}$, and $\Omega_{22i} = -Q_i + \varepsilon_{2i}N_{di}^{\mathrm{T}}N_{di}$. Also by the Schur complement, it follows that condition (5) is equivalent to $\Omega < 0$. Then, taking the integral rule and expectation of both sides of (11) with time t, it follows that under zero initial conditions that the H_∞ index is guaranteed, which ends the proof.

Remark 3.2. *For the sliding surface design, the proposed linear switching function in (3) is mode-dependent depending on the designed matrix P_i, which can be solved by condition (5) and indicates the connection between the jumps of system modes and the specified sliding surface, i.e., the effect of Markovian jump may be reflected in the sliding surface. The design in (3) could benefit to deduct the resultant dynamics during the sliding surface from the fact that $s^{\mathrm{T}}(t) = x^{\mathrm{T}}(t)P_iB_i = 0$ above.*

3.3. **Adaptive SMC law synthesis.** In the following discussion, the activity mainly addresses the detailed design of the reaching motion controller, by which the trajectory of the considered system in (1) can reach the predefined sliding surface $s(t) = 0$ in finite-time sense. At this place, the following inequality can be given:

$$\varGamma(t) = \left\|B_i^{\mathrm{T}}P_iB_i\right\| \|g(t,x)\| + \left\|B_i^{\mathrm{T}}P_iG_i\right\| \|w(t)\| \le c_1\|x(t)\| + c_2,\ t \ge 0, \tag{12}$$

where $c_i > 0$ $(i = 1, 2)$ is the estimation bounds and unknown. To benefit the controller design, we utilize $\hat{c}_i(t)$ to estimate it and the error is described by $\tilde{c}_i(t) = \hat{c}_i(t) - c_i$.

Theorem 3.2. *As the adaptive SMC law is established as follows:*

$$\begin{aligned} u(t) = & -\left(B_i^{\mathrm{T}}P_iB_i\right)^{-1}\left[B_i^{\mathrm{T}}P_iA_ix(t) + B_i^{\mathrm{T}}P_iA_{di}x(t-d)\right] \\ & - \left(B_i^{\mathrm{T}}P_iB_i\right)^{-1}\left[\left\|B_i^{\mathrm{T}}P_iM_i\right\| \|N_ix(t)\| + \left\|B_i^{\mathrm{T}}P_iM_i\right\| \|N_{di}x(t-d)\|\right. \\ & \left. + \hat{c}_1(t)\|x(t)\|\|s(t)\| + \hat{c}_2(t)\|s(t)\| + \delta\right]\mathrm{sgn}(s(t)), \end{aligned} \tag{13}$$

where $\delta > 0$, and tracking laws of the estimators $\hat{c}_i(t)$ are taken as $\dot{\hat{c}}_1(t) = l_1\|s(t)\|\|x(t)\|$, $\dot{\hat{c}}_2(t) = l_2\|s(t)\|$, where $l_i > 0$ $(i = 1, 2)$ denote the adaptive gains and then reachability of the designed switching surface $s(t) = 0$ is ensured with probability one.

Proof: Choose the following Lyapunov function

$$V(t) = 0.5\left[s^{\mathrm{T}}(t)s(t) + \sum_{j=1}^{2} l_j\tilde{c}_j^2(t)\right].$$

The infinitesimal generator $\mathcal{L}$ of the function $V(t)$ along the trajectory of system (2) is given below

$$\begin{aligned} \mathcal{L}V(t) &= s^{\mathrm{T}}(t)\dot{s}(t) + \sum_{j=1}^{2} l_j^{-1}\tilde{c}_j(t)\dot{\tilde{c}}_j(t) \\ &= s^{\mathrm{T}}(t)B_i^{\mathrm{T}}P_i\{[A_i + \Delta A_i(t)]x(t) + [A_{di} + \Delta A_{di}(t)]x(t-d) \\ &\quad + B_i[u(t) + g(t,x)] + G_iw(t)\} + \sum_{j=1}^{2} l_j^{-1}\tilde{c}_j(t)\dot{\tilde{c}}_j(t). \end{aligned} \tag{14}$$

Moreover, it is easy to check that $\dot{\hat{c}}_1(t) = \dot{\tilde{c}}_1(t)$, $\dot{\hat{c}}_2(t) = \dot{\tilde{c}}_2(t)$. So it further gives with controller (13)

$$\begin{aligned}\mathcal{L}V(t) &\le \|s(t)\|\left[\left\|B_i^{\mathrm{T}}P_iB_i\right\|\|g(t,x)\| + \left\|B_i^{\mathrm{T}}P_iG_iw(t)\right\|\right] \\ &\qquad -s^{\mathrm{T}}(t)\left[\hat{c}_1(t)\|x(t)\|\|s(t)\| + \hat{c}_2(t)\|s(t)\| + \delta\right]\|s(t)\| + \sum_{j=1}^{2} l_j^{-1}\tilde{c}_j(t)\dot{\tilde{c}}_j(t) \\ &= \|s(t)\|[c_1\|x(t)\| + c_2] - s^{\mathrm{T}}(t)[\hat{c}_1(t)\|x(t)\| + \hat{c}_2(t) + \delta]\mathrm{sgn}(s(t)) + \sum_{j=1}^{2} l_j^{-1}\tilde{c}_j(t)\dot{\tilde{c}}_j(t) \\ &= -\delta\|s(t)\| < 0, \text{ if } \|s(t)\| \neq 0. \end{aligned} \tag{15}$$

Therefore, the arrival of the designed surface will be guaranteed.

4. **Numerical Example.** Consider the following Markovian jump system with time-delay in (1) with two modes, where the system parameters are given by

$$A_1 = \begin{bmatrix} -2.5 & 0.6 \\ -0.5 & -2 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.6 & -0.6 \\ -0.5 & -0.2 \end{bmatrix}, \quad M_1 = [0\ 0]^{\mathrm{T}}, \quad N_1 = N_{d1} = [0\ 0],$$
$$B_1 = [0\ 1]^{\mathrm{T}}, \quad C_1 = [-1\ 1], \quad D_1 = 0.1,$$
$$A_2 = \begin{bmatrix} -2 & 0.5 \\ 1.5 & -1.5 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.4 & 0.1 \\ -0.5 & -0.2 \end{bmatrix}, \quad M_2 = [0\ 0]^{\mathrm{T}}, \quad N_2 = N_{d2} = [0\ 0],$$
$$B_2 = [1\ 0]^{\mathrm{T}}, \quad C_2 = [-0.5\ 1], \quad D_2 = -0.1, \quad G_1 = [0.3\ 0]^{\mathrm{T}}, \quad G_2 = [-0.4\ 0]^{\mathrm{T}}.$$

The parametric matrices K_i, $i = 1, 2$, are chosen by $K_1 = \begin{bmatrix} 2.2514 & -1.1232 \end{bmatrix}$, $K_2 = \begin{bmatrix} 0.3643 & -1.3625 \end{bmatrix}$. The TR matrix is considered as $\Pi = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$. Then, taking $\gamma = 1.2527$ and by virtue of the condition in (5), the feasible solutions used in this method are obtained as

$$P_1 = \begin{bmatrix} 1.1272 & 0.1460 \\ 0.1460 & 0.9528 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.3045 & 0.0619 \\ 0.0619 & 1.3790 \end{bmatrix},$$
$$Q_1 = \begin{bmatrix} 1.9314 & -0.1186 \\ -0.1186 & 1.9827 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 2.0113 & -0.1111 \\ -0.1111 & 1.8981 \end{bmatrix}.$$

Thus, the linear sliding function is designed as

$$s(t) = \begin{cases} [0.1460 \quad 0.9528]x(t), & i = 1; \\ [1.2045 \quad 0.0619]x(t), & i = 2. \end{cases}$$

The matched uncertainty and disturbance input are chosen by $g(t, x) = 0.2\sin(t) * \cos(t)$, and $w(t) = 0.25e^{-2t} * \sin(t)$. Hence, by using the designed adaptive controller in (13) with the data $l_1 = 0.2$, $l_2 = 0.6$, the simulation results are shown in Figures 1-4, which definitely demonstrates the efficiency of the proposed scheme. In detail, Figure 1 shows the system switching mode, and Figure 2 depicts the curve of the system state. The involution of the controller and switching function is given by Figure 3, and the adaptive parameters are plotted by Figure 4.

5. **Conclusions.** In this paper, the problem of adaptive sliding mode control design of uncertain Markovian jump systems with time-delay and external disturbance has been studied. A new linear sliding surface design has been designed, by which a new stochastic stability condition of the system dynamics during the sliding surface has been deducted. Moreover, an adaptive SMC law has been designed to guarantee the arrival of the switching surface. At last, an illustrative example has been given to verify the effectiveness of the proposed scheme. We will consider the systems subject to some stochastic noises via the proposed method in future work.

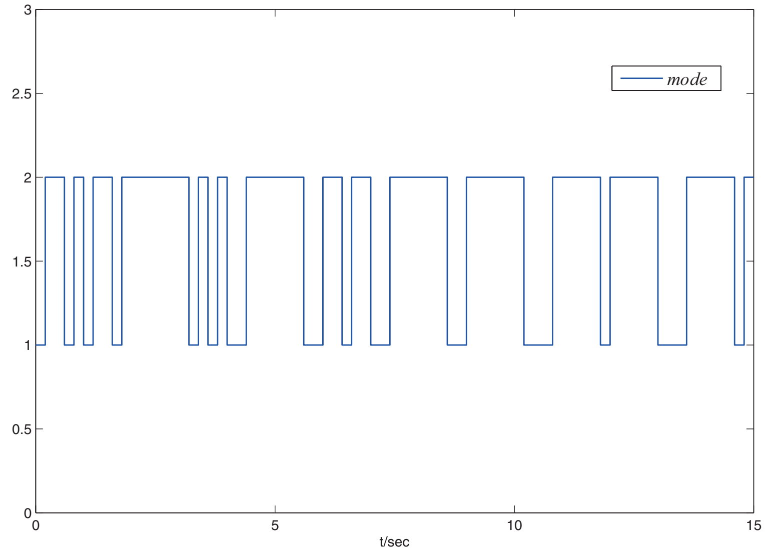

FIGURE 1. System mode

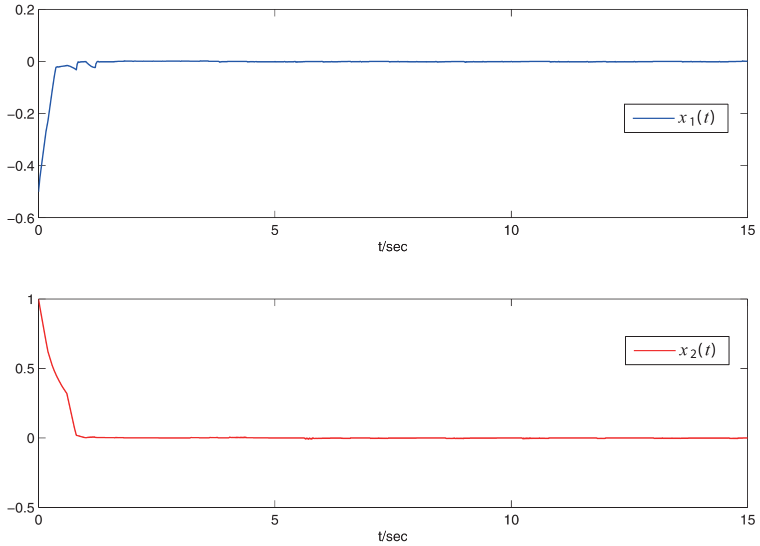

FIGURE 2. Response of system state

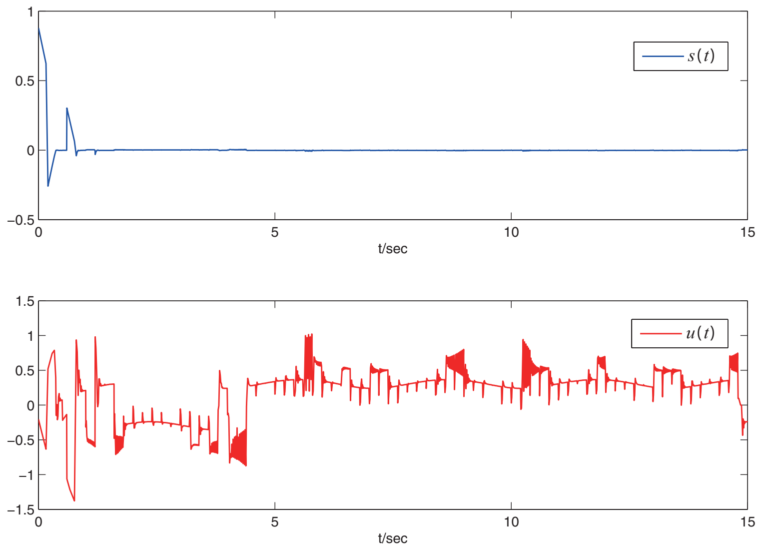

FIGURE 3. Curves of sliding function and controller

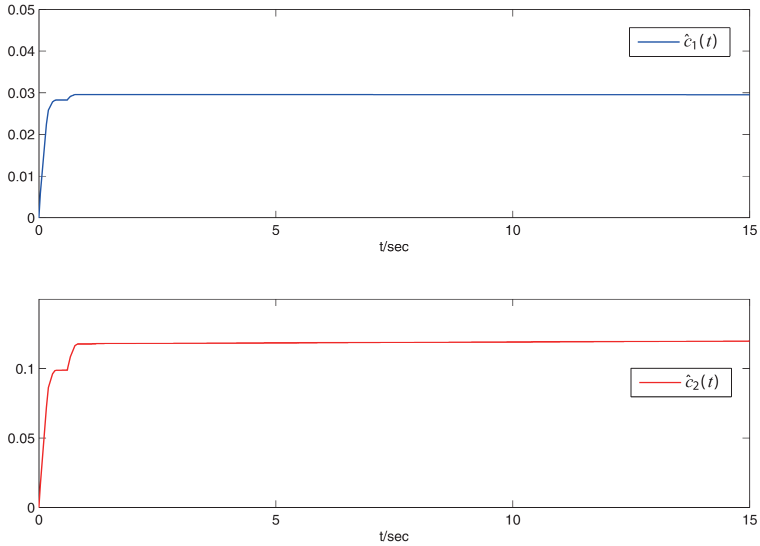

FIGURE 4. Curves of adaptive data

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