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OPTIMAL ORDER QUANTITIES CONSIDERING COMPLEMENTARY ORDERS UNDER TWO SUPPLIERS' DISRUPTION RISKS

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Abstract. *This paper investigates the determination of the optimal order quantity for a vendor due to the disruption risks of the supplier. The optimal order quantity is determined by minimizing the initial supply cost for each supplier, the supply cost for the complementary order, the stock disposal profit, and the total expected cost reflecting the inventory depletion cost. In order to determine the optimal order quantity, the problem is formulated as a mathematical model, and an optimal ordering strategy is derived through mathematical analysis based on the probability of supplier disruption. In addition, the adequacy and validity of the mathematical model are demonstrated through numerical experiments.*

Keywords: Supplier disruption, Optimal order quantity, Complementary order, Procurement policy

1. **Introduction.** On April 14th and 16th in 2016, the automobile parts company Aisin Seiki and its related companies in Kumamoto in Japan were shut down because of an earthquake that hit Kumamoto city. In the aftermath of this earthquake, supplying car doors and engine parts was almost impossible, so Toyota Kyusu, a subsidiary company of Toyota Motors had to stop operations at three Fukuoka factories, causing them to suffer enormous economic damage[1]. In recent years, factory disruption due to natural disasters or social-political instability has become increasingly frequent, and this has become a great threat to supply chain networks among suppliers. Therefore, making optimal decisions while taking into account many uncertain factors related to supply disruption is a crucial issue in business management. This study deals with the problem of determining the optimal order quantity according to the occurrence of supplier vendor disruption.

Procurement policies considering the risk factors of the supply chain can be classified into three phases: single procurement, binary procurement and multiple procurement. First, the single procurement policy [1,2] aims to derive an order policy to cope with supply chain risk by considering joint profit and long-term relationship formation as well as the supplier's reliability level through a partnership between the company and single supplier. Gurler and Palar [3] established a semi-Markov process model considering the supply chain disruption and adopted a continuous review system that statistically assumes demand and lead time to determine the order policy. Second, the dual procurement policy [4,5] aims to derive order policies to deal with supply chain risks by promoting sound competition between the two suppliers in terms of quality, speed, and competitive prices, which could not only mitigate supply chain problems but also maintain high quality. Tomlin [6] considered the capacity constraints of the two suppliers, Chopra et al. [7] determined the optimal order quantity by dividing the risk of one supplier into the two

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[1]S.-H. Kwon, Kumamoto earthquake, 7-days factory shutdown at Toyota, *ETNews*, 2016.

phases (yield uncertainty and supply disruption), and Xanthopoulos et al. [8] applied the news vendor model to determining the tradeoff between inventory policy and discontinued risk. Unlike the preceding studies, he established order policies for each supplier under the assumptions that disruptions can occur and that partial supply may be possible. Finally, Yang and Qian [9] considered a complementary ordering policy [10,11] that uses undisrupted supplier's capacity that holds the undelivered items due to supply chain disruption, while Ray and Jenamani [12] considered a multi-procurement policy that does not take into account fixed costs capacity constraints, or suppliers having different supply and outage probabilities. Kim and Seo [13] considered the number of suppliers, order volume and optimal complementary order quantity against supply chain disruption in the same situation of order quantity, supply cost and disruption probability.

In this study, by considering not only the capacity constraints for the two suppliers evaluated in the binary procurement study but also the complementary ordering policy conducted in previous studies of multiple procurement, we propose the optimal order quantity in the case of two suppliers' disruptions and partial supply. In addition, we have differentiated from previous studies in aspects that capacity constraints, order quantity, supply cost, and disruption probability are set differently for each supplier, and that supplementary order policy and disposal profit are further handled.

The structure of the paper is as follows. In Section 2 mathematical model is formulated, and based on this model the optimal ordering policy is mathematically analyzed in the following Section 3. In Section 4, numerical experiments are conducted to examine the optimal order quantity to minimize the total costs according to the probability of disruption of the supplier. Finally, Section 5 summarizes the conclusions.

2. **Mathematical Model.** In this study, we consider a supply chain structure in which a single vendor trades with two suppliers whose prior knowledge about supply disruption is unknown. The capacity of each supplier is finite and an initial purchasing cost and complementary ordering cost are involved in each supplier transaction. The demand for each supplier's product is assumed to be deterministic for a certain period of time at the purchaser. After the period has expired, the demand will be extinguished and back ordering will not be allowed. In order to prevent excess inventory caused by the consideration of safety stock, it will be disposed to other buyers, at which time disposal profit will be generated. The quality of a single product provided by a supplier is the same, but the supply cost per unit and the probability of each supplier's disruption are different. According to the order quantity not received due to disruption, the purchaser can place a supplemental order within the remaining capacity of the uninterrupted supplier. This means that we have considered the use of supplemental orders to address unsatisfied demand. In this case, assuming that the supply cost of each supplier is different, the supplementary supply cost is also different, and the purchaser must pay a higher cost than the initial supply cost for the complementary order quantity. In addition, if the amount of unsatisfied demand is greater than the total remaining capacity of an uninterrupted supplier, it is treated as a lost sale, resulting in inventory depletion costs.

In this study, we propose an optimal order quantity decision model considering supplier's probability of disruption, initial supply cost, complementary supply cost, supplier capacity, inventory depletion cost, and inventory disposal profit in order to minimize the cost of purchaser. The total expected cost of the purchaser is determined based on the sum of the transaction costs of the two suppliers at the time of the initial order and the supply cost of the quantity supplied from the uninterrupted suppliers.

<**Notations**>

d: decisive demand

p_1: disruption probability of supplier F1

p_2: disruption probability of supplier F2

k_1: capacity of supplier F1
k_2: capacity of supplier F2
q_1: order quantity of supplier F1 ($q_1 < k_1$)
q_2: order quantity of supplier F2 ($q_2 < k_2$)
c_1: supply cost per unit of supplier F1 for initial order quantity
c_2: supply cost per unit of supplier F2 for initial order quantity
e_1: supply cost per unit of supplier F1 for complementary order ($c_1 < e_1$)
e_2: supply cost per unit of supplier F2 for complementary order ($c_2 < e_2$)
β: inventory depletion cost per unit (e_1, $e_2 < \beta$)
m: disposal profit per unit ($m < c_1$, c_2)

This study deals with cases in which the order quantity exceeds the capacity of the supplier ($k_1(k_2) < d \leq q_1 + q_2$), and the minimum expected cost can be expressed as follows.

$$\begin{aligned} E[TC(q_1, q_2)] = {} & d\beta p_1 p_2 + \left(c_2 q_2 + e_2(k_1 - k_2) + \beta(d - k_2)\right) p_1(1 - p_2) \\ & + \left(c_1 q_1 + e_1(k_1 - q_2) + \beta(d - k_1)\right) p_2(1 - p_1) \\ & + \left(c_1 q_1 + c_2 q - m(q_1 + q_2 - d)\right)(1 - p_1)(1 - p_2) \end{aligned} \tag{1}$$

The first term in Equation (1) represents the costs when both suppliers F1 and F2 are interrupted by the probability of disruptions p_1 and p_2. Since the demand is not satisfied, the inventory depletion cost β for d occurs. The second term represents the case where supplier F1 is stopped. Since the initial order has not been satisfied, complementary order quantity $k_2 - q_2$ and unsatisfied demand $d - k_2$ occur. The third term is the case where supplier F2 is stopped. As is the case for the second term, since the initial order is not satisfied, the complementary order quantity $k_1 - q_1$ and the unsatisfied demand $d - k_1$ occur. The last term indicates the case when both suppliers F1 and F2 are not interrupted and they do not have to place any supplementary orders because they fulfill all of the initial demand. However, since the order quantity exceeds the demand quantity, the disposal profit $q_1 + q_2 - d$ will be generated.

Equation (1) can be transformed as follows.

$$\begin{aligned} E[TC(q_1, q_2)] = {} & d\left(\beta\left(p_1(1 - p_2) + p_2(1 - p_1)\right) + m(1 - p_1)(1 - p_2)\right) \\ & + \left(c_1 - e_1 p_2 - m(1 - p_2)\right)(1 - p_1) q_1 \\ & + \left(c_2 - e_2 p_1 - m(1 - p_1)\right)(1 - p_2) q_2 - (\beta - e_2) p_1 (1 - p_2) k_2 \\ & - (\beta - e_1) p_2 (1 - p_1) k_1 \end{aligned} \tag{2}$$

The above Equation (2) shows that the total expected cost is provided as a linear function of the order quantities q_1 and q_2. This means that the optimal order quantity that minimizes the total cost is determined by the coefficients of q_1 and q_2. In other words, the optimal order volume is determined regardless of vendor capacity (k_1, k_2), total demand (d), and inventory depletion cost (β). The purpose of this study is to derive the optimal order quantities q_1^* and q_2^* that minimize the total expected cost of Equation (2). By $< x, y >$, we can denote the optimal order quantity with respect to suppliers F1 and F2, respectively, i.e., $q_1^* = x$ and $q_2^* = y$.

3. **Results Analysis.** In this section, the optimal ordering strategy is mathematically analyzed based on our mathematical model presented in the previous section. For the convenience of expressions, we define the following.

$$p_1^* = (c_2 - m)/(e_2 - m), \quad p_2^* = (c_1 - m)/(e_1 - m), \quad \hat{p} = \frac{(e_2 - c_1)p_1 + (c_1 - c_2)}{(e_2 - e_1)p_1 + (e_1 - c_2)} \tag{3}$$

The following results show that the optimal order quantity depends on the supplier's interruption probability.

Theorem 3.1.
(1) When $p_1 \le p_1^*$ and $p_2 \le p_2^*$, if $p_2 > \hat{p}$ then $< k_1, d - k_1 >$, otherwise $< d - k_2, k_2 >$
(2) If $p_1 \le p_1^*$ and $p_2 > p_2^*$, then $< d - k_2, k_2 >$
(3) If $p_1 > p_1^*$ and $p_2 \le p_2^*$, then $< k_1, d - k_1 >$
(4) If $p_1 > p_1^*$ and $p_2 > p_2^*$, then $< k_1, k_2 >$

Proof: (1) Let $p_1 \le p_1^*$ and $p_2 \le p_2^*$. Then from Equation (3) we have $p_1 \le (c_2 - m)/(e_2 - m)$ and $p_2 \le (c_1 - m)/(e_1 - m)$ which leads to $c_2 - e_2 p_1 - m(1 - p_1) \ge 0$ and $c_1 - e_1 p_2 - m(1 - p_2) \ge 0$. Therefore, the following holds: $(c_2 - e_2 p_1 - m(1 - p_1))(1 - p_2) \ge 0$ and $(c_1 - e_1 p_2 - m(1 - p_2))(1 - p_1) \ge 0$. This means that the coefficients of order q_1 and q_2 are both positive in Equation (2) and that the optimal order quantity can be obtained when the total cost is minimized by maximizing the smaller of the two coefficients. In this case, if $p_2 > \hat{p}$, we have $(c_1 - e_1 p_2 - m(1 - p_2))(1 - p_1) < (c_2 - e_2 p_1 - m(1 - p_1))(1 - p_2)$, implying that $p_2 > \hat{p} = \frac{(e_2 - c_1)p_1 + (c_1 - c_2)}{(e_2 - e_1)p_1 + (e_1 - c_2)}$. That is, the coefficient of q_1 is smaller than the coefficient of q_2. Therefore, the optimal order quantity is the maximum value taken by q_1 and q_2, that is, k_1 and $d - k_1$ (the remaining quantity after excluding the order quantity of q_1); for $\hat{p} \le p_2$, the opposite is true.

(2-4) It can be proven simply by the same process as (1). □

The above result indicates the following. Theorem 3.1(1) is determined by another threshold value $\hat{p}$ when the likelihood of interruption of the two suppliers is sufficiently low to be less than the arbitrary threshold values, p_1^* and p_2^*. That is, if $p_2 > \hat{p}$ ($p_2 \le \hat{p}$), it is optimal to order the maximum capacity ($k_1(d - k_2)$) for the supplier whose order quantity coefficient of the total cost function is smaller than the other's, and to order the other supplier for the unsatisfied demand ($d - k_1(k_2)$). In Theorem 3.1(2), it is optimal to order F2 supplier's maximum capacity (k_2) when the likelihood of interruption of one supplier F2 is sufficiently high to exceed an arbitrary threshold p_2^*; that is, when the coefficient of q_1 is positive and the coefficient of q_2 is negative. Theorem 3.1(3) corresponds to the opposite case to (2). Theorem 3.1(4) shows that it is optimal to order each supplier for maximum when the likelihood of the supplier's interruption is sufficiently larger than the arbitrary threshold values, p_1^* and p_2^*, to prepare for the supplier's interruption. That is, when the coefficients of q_1 and q_2 are negative, the maximum capacities k_1 and k_2 are the optimum order quantities, respectively.

4. **Numerical Experiments.** In this section, numerical experiments are conducted to examine the optimal order quantity to minimize the cost of the vendor according to the probability of disruption of the supplier. The parameters are set as follows for the numerical experiment: the fixed transaction cost of each supplier is 5 million won; the supply cost per unit of supplier (F1), c_1, is 36,000 won/unit for the initial order quantity; the supply cost c_2 per unit of supplier (F2) of the initial order quantity is 38,000 won/unit; the supply cost e_1 is 54,000 won/unit for the supplier and the supplier (F1) for the supplementary order; the supply cost per unit of the supplier (F2) e_2 for the supplementary order is 56,000 won/unit; the inventory depletion cost β is 120,000 won/unit; and the disposal cost m is 30,000 won/unit. The capacities k_1, k_2 of each supplier are 6000, and the demand d is assumed to be 10,000 in a single period, and p_1 is 0.12. Based on the above conditions, the following can be obtained: $p_1^* = 0.307$, $p_2^* = 0.25$, and $\hat{p} = 0.025$.

Table 1 shows the coefficients of q_1 and q_2 according to p_2, as well as the optimal order quantity and total cost for each supplier. This result is valid for the case of $p_1 (= 0.12) < p_1^* (= 0.307)$ and $p_2 (= 0.12) < p_2^* (= 0.25)$ in Theorem 3.1. As shown in Table 1, since the coefficient of q_2 at $p_2 \le \hat{p} (= 0.025)$ is smaller than the coefficient of q_1, it is optimal to order the maximum quantity that can be ordered by supplier F2 and order the rest of the quantity of demand to the other supplier F1. Therefore, the optimal order quantity for each supplier is $q_1^* = 4{,}000 (d - k_2)$ and $q_2^* = 6{,}000 (k_2)$. If $p_2 > \hat{p} (= 0.025)$, the coefficient

TABLE 1. Coefficients of q_1 and q_2 according to p_2 order quantity and total cost

Disruption percentage (p_2)	Coefficient of order quantity q_1	Coefficient of order quantity q_2	Optimal order quantity (q_1^*)	Optimal order quantity (q_2^*)	Total cost (million won)
0.01	5,068.8	4,831.2	4,000$(d-k_2)$	6,000(k_2)	456
0.02	4,857.6	4,782.4	4,000$(d-k_2)$	6,000(k_2)	430
0.03	4,646.4	4,733.6	6,000(k_1)	4,000$(d-k_1)$	433
0.04	4,435.2	4,684.8	6,000(k_1)	4,000$(d-k_1)$	437
0.05	4,224.0	4,636	6,000(k_1)	4,000$(d-k_1)$	440
0.06	4,012.8	4,587.2	6,000(k_1)	4,000$(d-k_1)$	444

of q_1 will be smaller than the factor of q_2, so this time it is optimal to order the largest amount available for supplier F1 and order the remaining demand to another supplier F2. Hence, the optimal order quantity is given by $q_1^* = 6{,}000(k_1)$ and $q_2^* = 4{,}000(d-k_1)$. For the other cases in Theorem 3.1, we can confirm the similar results by the same experiment described above, so we skip the explanation here.

5. **Conclusions.** Our study proposed a complementary order policy for supply chain network with consideration of the suppliers' disruption and a mathematical model that determines the optimal order quantity for purchasing companies that procure parts through outsourcing. From Theorem 3.1, our main results showed the threshold values of the probability of interruption p_1^*, p_2^* and we compared the threshold values with each vendor's probability of interruption. We presented five different ordering policies according to p_1^*, p_2^*. In particular, we suggested that the optimal supply amount is determined according to the threshold value $\hat{p}$ if the disruption probability of the two suppliers is sufficiently low. This means that it is desirable to measure the probability of disruption and prepare for the threshold point due to the fact that the optimal order quantity changes according to the interruption probability of each supplier.

This study only considers a single period for deterministic demand for model simplification and is limited to a dual procurement policy. Thus, future research should consider expansion to a multi procurement model. In addition, we analyze the deterministic demand, but we expect that it will develop into a more realistic model considering the change of demand with time.

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