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SOLUTION METHOD FOR DELIVERY CENTER LOCATION PROBLEM CONSIDERING INVENTORY COST AND PROFIT RATIO

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Abstract. *Facility location and allocation decision problem LAPs (Location-Allocation Problems) aiming to minimize total expenses, such as transportation expenses, have been considered in conventional supply chain networks. Facility location and inventory decision problem PM-LIPs (Profit-Maximization Location-Inventory Problems), which aim to maximize profits, are also being studied. This study considers a case in which a customer's demand is sensitive to the price of the commodity, and presents the PM-LIPs of the distribution center, which considers inventory cost and profit ratio. It also verifies the efficiency of the solution through numerical experiments. In this paper the new efficient solution algorithm based on piecewise linear approximation is developed.*

Keywords: Profit-maximization location-inventory problems, Mixed integer programming, Piecewise linear approximation

1. **Introduction.** A supply chain [2, 6] is a series of systems that produce products at factories, store products as inventory, and deliver products, while meeting customer demand. In addition, total costs such as ordering, shipping, and shipping costs are minimized in the supply chain network. Therefore, it is necessary to plan the supply chain and design networks of suppliers, producers, wholesalers, retailers and customers.

The aim of conventional studies on supply chain networks is to minimize total expenses, such as transportation costs. However, other studies examine the facility location and allocation decision problem through PM-LAPs (Profit-Maximization Location-Allocation Problems), which aim at profit maximization. PM-LAPs are broadly classified into two categories: customer demand is sensitive to price or demand is flexible. The former indicates that only the price of the product affects a customer's demand. The latter indicates that a customer's demand does not depend on the price of the item.

In recent years, several studies have focused on facility location and inventory decision problem through PM-LIPs, which simultaneously determine facility location and volume of inventory to hold. Ahmadi-Javid and Hoseinpour [1] formulated PM-LIPs of a Delivery Center (DC) as a mixed integer nonlinear programming problem, considering inventory costs when customer demand is sensitive to the price of the commodity. Then, an approximate solution can be obtained using the Lagrangian relaxation method. However, the optimality of the solution is not considered. On the other hand, Shen [7] formulated the PM-LIPs of a DC as a set covering problem in the case of flexibility of demand, and solved it using the column generation method.

In this study, we consider that a customer's demand is sensitive to the price of the commodity. In Section 2, the problem is formulated. In Section 3, the solution technique is described. In Section 4, numerical experiments are presented. We summarize our results in Section 5.

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2. **Overview of PM-LAPs.** We formulate the problem of PM-LAPs in this section. The assumptions of the problem are as follows based on [1, 7].

2.1. **Problem setting.**

1) Profit ratio is the ratio of profit per unit to the wholesale price of goods and each DC has a finite number of profit ratio level scenarios.
2) Customer demand depends on fluctuations in the profit ratio and is sensitive to the price of products.
3) Each DC presents its own retail price to customers assigned to each item. The wholesale price is given; it varies between DCs.
4) According to the quantitative ordering method, the inventory of each DC is determined as the optimum lot size, by minimizing the inventory cost.
5) Each product is supplied from an arbitrary supplier.
6) The customer's demand for each product is supplied from only one DC, and it is not necessary to satisfy the demand of all customers.
7) Each DC is required to pay inventory cost.

2.2. **Definition of symbols.** The following symbols are used.

Sets

I	Set of DC
J	Set of customers
G	Set of profit ratio levels
K	Set of products

Parameters

f_i	Annual fixed cost on DC i
t_{ijk}	Delivery cost of product k between DC i and customer j
o_{ik}	Fixed order cost of product k on DC i
e_{ik}	Fixed delivery cost between supplier of product k and DC i
a_{ik}	Unit delivery cost between supplier of product k and DC i
h_{ik}	Annual unit inventory holding cost of product k at DC i
c_{ik}	Wholesale price of goods k at DC i
b_{ik}	Standard profit level of product k at DC i
p_{ik}	Standard unit retail price of product k at DC i
p'_{igk}	Unit retail price of product k at DC i at margin level g
d_{ijk}	The standard annual demand of customer j for goods k from DC i
d'_{igjk}	The annual demand of customer j for goods k from DC i at the gth margin level
σ_{igk}	Rate of change in profit level b_{ik}

Decision variables

X_i	1 if location DC i, 0 otherwise.
Y_{igjk}	1 if customer j is allocated to DC i of margin level g that satisfies the demand of product k, 0 otherwise.
Z_{igk}	1 if the item k is provided from DC i of the profit margin level g, 0 otherwise.
Q_{igk}	Order quantity of goods k at DC i of profit margin level g

2.3. **Objective function and constraints.** The formulation of the problem is as follows.

$$\text{(PM-LIPs): } \max \sum_{i\in I}\sum_{g\in G}\sum_{j\in J}\sum_{k\in K}(p'_{igk} - c_{ik} - t_{ijk})d'_{igjk}Y_{igjk} - \sum_{i\in I} f_i X_i$$

$$\sum_{i\in I}\sum_{g\in G}\sum_{k\in K}\left(o_{ik}\frac{\sum_{j\in J}d'_{igjk}Y_{igjk}}{Q_{igk}}+\frac{h_{ik}Q_{igk}}{2}\right.$$
$$\left.+\left(e_{ik}+a_{ik}Q_{igk}\right)\frac{\sum_{j\in J}d'_{igjk}Y_{igjk}}{Q_{igk}}\right) \tag{1}$$

$$\text{s.t.} \sum_{i\in I}\sum_{g\in G}Y_{igjk}\leq 1 \quad j\in J, \quad k\in K \tag{2}$$
$$\sum_{g\in G}Z_{igk}\leq 1 \quad i\in I, \quad k\in K \tag{3}$$
$$Y_{igjk}\leq Z_{igk} \quad i\in I, \quad g\in G, \quad j\in J, \quad k\in K \tag{4}$$
$$Z_{igk}\leq X_i \quad i\in I, \quad g\in G, \quad k\in K \tag{5}$$
$$X_i\in\{0,1\} \quad i\in I \tag{6}$$
$$Z_{igk}\in\{0,1\} \quad i\in I, \quad g\in G, \quad k\in K \tag{7}$$
$$Y_{igjk}\in\{0,1\} \quad i\in I, \quad g\in G, \quad j\in J, \quad k\in K \tag{8}$$
$$Q_{igk}\geq 0 \quad i\in I, \quad g\in G, \quad k\in K \tag{9}$$

The objective function (1) represents the maximum annual profit of DC. The inequality (2) is a constraint that the customer assigns to only one DC for each product, (3) states the restriction each placed DC has one profit ratio level to select. Two inequalities (4) (5) are constraints where a certain DC is placed and the goods are offered at a certain profit ratio level to satisfy the demand of each product with the specific profit ratio level. Constraints (6)-(8) are integer constraints of 0-1 variable, (9) is a nonnegative condition of the order quantity.

From Assumptions 1) and 2), we can show the retail price p'_{igk} and the demand d'_{igjk} as follows.

$$p'_{igk}=\{1+(1+\sigma_{igk})b_{ik}\}c_{ik}=(1+b_{ik})c_{ik}+\sigma_{igk}b_{ik}c_{ik}=p_{ik}+\sigma_{igk}b_{ik}c_{ik} \tag{10}$$
$$d'_{igjk}=(1-\sigma_{igk})d_{ijk} \tag{11}$$

The expression (10) indicates that the retail price varies according to the value of σ_{igk}. The expression (11) indicates that if the ratio increases by σ_{igk}, the customer's demand will decrease accordingly.

Based on the assumption and objective function (1), the optimum lot size Q^*_{igk}, which minimizes the annual inventory cost, is as follows.

$$Q^*_{igk}=\sqrt{\frac{2(o_{ik}+e_{ik})d'_{igjk}Y_{igjk}}{h_{ik}}} \tag{12}$$

Substituting the expression (12) into the objective function (1), the formulation of PM-LIPs is as follows.

$$\text{(PM-LIPs): max} \sum_{i\in I}\left\{\sum_{g\in G}\sum_{k\in K}\left(\sum_{j\in J}M_{igk}(j)Y_{igjk}-\sqrt{\sum_{j\in J}N_{igk}(j)Y_{igjk}}\right)-f_iX_i\right\} \tag{13}$$
$$\text{s.t. (2)-(8)}$$

The objective (13) is a function of X_i and Y_{igjk}. In addition, $M_{igk}(j)$, $N_{igk}(j)$ are substituted as follows.

$$M_{igk}(j)=(p'_{igk}-c_{ik}-a_{ik}-t_{ijk})d'_{igjk} \tag{14}$$
$$N_{igk}(j)=2h_{ik}(o_{ik}+e_{ik})d'_{igjk} \tag{15}$$

3. **Solution Algorithm for PM-LIPs.** As inventory costs are represented by non-convex functions of the 0-1 variable Y_{igjk}, the problem can be calculated in the following two ways.

3.1. **Approximate solution of inventory cost in PM-LAPs.** In the approximation method, we add the inventory cost after solving PM-LAPs without inventory cost.

Step 1: Determination of initial solution: We solve the PM-LAPs of a DC without considering inventory cost to obtain feasible solutions Y_{igjk}.

Step 2: Calculation of actual inventory cost: Exact inventory cost is computed as $\sum_{i\in I}\sum_{g\in G}\sum_{k\in K}\sqrt{\sum_{j\in J} N_{igk}(j)Y_{igjk}}$.

Step 3: Calculation of profit of DCs: From the objective function obtained in **Step 1**, subtract the inventory cost in **Step 2** and the DC's profit can be calculated.

$$\text{(PM-LAPs): } \max \sum_{i\in I}\left(\sum_{g\in G}\sum_{k\in K}\sum_{j\in J} M_{igk}(j)Y_{igjk} - f_i X_i\right) \tag{16}$$

$$\text{s.t. (2)-(8)}$$

3.2. **Solution by piecewise linear approximation.** We solve the nonlinear programming problem considering inventory cost. The content of the square root, which is a nonlinear term, is defined again.

$$u_{igk} = \sum_{j\in J} N_{igk}(j)Y_{igjk} \tag{17}$$

Inventory cost $F(u_{igk})$ is indicated as follows.

$$F(u_{igk}) = \sqrt{u_{igk}} = \sqrt{\sum_{j\in J} N_{igk}(j)Y_{igjk}} \tag{18}$$

The PM-LIPs of DC considering inventory cost $F(u_{igkl})$ is formulated as a mixed integer nonlinear programming problem. First, we transform the nonlinear programming problem of inventory cost into a mixed integer linear problem. By piecewise linear approximation [5], this problem is reduced to a mixed integer linear programming problem with 0-1 variables. Let $\bar{F}(u_{igkl})$ be the approximation of inventory cost $F(u_{igk})$ by piecewise linearization, l the index of split points, S_{igk} the number of split points, λ_{igkl} positive auxiliary variable, and the 0-1 variable μ_{igkl} is defined, representing the segment it belongs to. The approximation is shown in Figure 1.

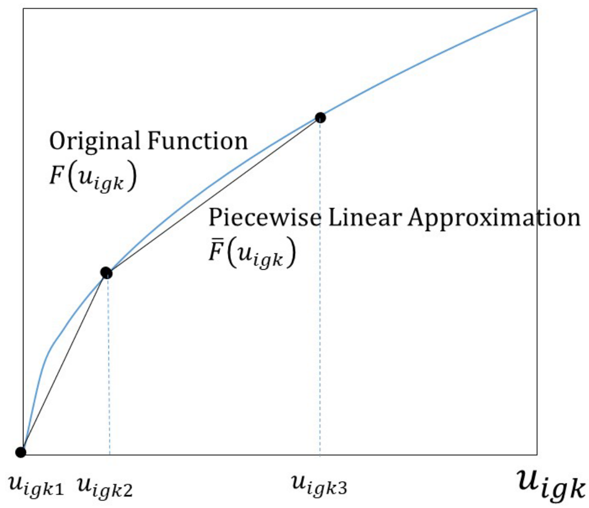

Figure 1. Piecewise linear approximation

The formulation of piecewise linear approximation of PM-LIPs is as follows.

$$\max \sum_{i\in I}\left\{\sum_{g\in G}\sum_{k\in K}\left(\sum_{j\in J} M_{igk}(j)Y_{igjk} - \bar{F}_{igk}(u_{igk})\right) - f_iX_i\right\} \tag{19}$$

$$\text{s.t.} \quad (2)\text{-}(8), (17)$$

$$u_{igk} = \sum_{l=1}^{S_{igk}} \lambda_{igkl}u_{igkl} \tag{20}$$

$$\bar{F}(u_{igk}) = \sum_{l=1}^{S_{igk}} \lambda_{igkl}F(u_{igkl}) \tag{21}$$

$$\sum_{l=1}^{S_{igk}} \lambda_{igkl} = 1, \quad \lambda_{igkl} \geq 0, \quad l = 1, \ldots, S_{igk} \tag{22}$$

$$\begin{aligned} &\lambda_{igk1} \leq \mu_{igk1}, \\ &\lambda_{igk2} \leq \mu_{igk1} + \mu_{igk2}, \\ &\cdots\cdots\cdots \\ &\lambda_{igkS_{igk}} \leq \mu_{igkS_{igk-1}} + \mu_{igkS_{igk}}, \end{aligned} \tag{23}$$

$$\sum_{l=1}^{S_{igk}} \mu_{igkl} = 1, \quad 0 \leq \mu_{igkl} \in Z, \quad l = 1, \ldots, S_{igk} \tag{24}$$

Equations (20), (21) are constraints for piecewise linear approximation. Expression (22) represents the determination of a point by convex combination of split points. The approximation point is determined by a point on a line segment connecting two adjacent points. Constraint (23) shows that the parameters λ for two adjacent points are positive, and their sum becomes 1 from (22). Expression (24) is a constraint for selecting only two neighboring points, indicating that λ for two adjacent points is positive.

In addition, as the number of split points increases, the accuracy of piecewise linear approximation improves. Using the solution of the piecewise linear problem, the exact value of the objective (13) can be calculated as follows. Using the solution $\bar{Y}_{igjk}$ and $\bar{X}_i$ of (19), the profit and the inventory cost can be recalculated.

$$\sum_{i\in I}\left\{\sum_{g\in G}\sum_{k\in K}\left(\sum_{j\in J} M_{igk}(j)\bar{Y}_{igjk} - \sqrt{\sum_{j\in J} N_{igk}(j)\bar{Y}_{igjk}}\right) - f_i\bar{X}_i\right\} \tag{25}$$

4. **Numerical Experiments.** The numbers of DC, profit ratio level, customer, product, split point used in piecewise linear method are shown in Table 1. Data on sales and expenses were uniformly distributed as in Ahmadi-Javid and Hoseinpour [1]. In addition, the rate of change in profit ratio was assumed to be a uniform distribution of $[-0.05, 0.05]$. The experimental environment is as follows, OS: Windows 10 64 bit, CPU: Core i 7-3770 S (3.10 GHz) memory: 8.00 GB. For optimization, we used AMPL/CPLEX.

Table 1. Problem setting

Instance	DC	Profit ratio	Customer	Products	Split points
1	10	5	400	1	3000
2	15	5	400	1	3000
3	10	10	400	1	3000
4	15	10	400	1	3000

The computational results of each solution are shown in Tables 2, 3. The profit is calculated as the value of the objective function (13) in the approximate solution, using PM-LAPs and the value of the objective function (25) in the piecewise linear solution.

Table 2. Results using PM-LAPs

Instance	Total profit (13)	Computing time (sec)
1	5,138,116	2
2	7,293,518	4
3	5,507,396	7
4	7,602,047	15

Table 3. Results using the piecewise linear method

Instance	Total profit (25)	Computing time (sec)
1	5,391,239	12
2	7,897,114	30
3	5,622,656	37
4	8,145,894	73

From Tables 2, 3, the greater the number of DC and margin level, the greater the profit. In addition, the larger the instance of each solution, the longer the computation time. Furthermore, from Tables 2, 3, the benefit of DC is higher using piecewise linear approximation. Both methods can solve problems in practical time, so it is possible to solve larger problems.

With regard to X_i and Y_{igjk} in the piecewise linear method, each DC has selected a high profit ratio level, and high-priced items were provided even when customer demand is low. Inventory cost is accurately calculated in the piecewise linear method. However, in the approximate solution method, without considering inventory cost, it can be said that the profit becomes lower than the linear approximation.

5. **Concluding Remarks.** In this study, we compared PM-LAPs of DC without considering inventory cost and PM-LIPs of DC, which is a mixed integer nonlinear programming problem. Nonlinear terms are omitted when solving the problem as PM-LAPs. Therefore, the solution can be obtained quickly, but the error is large. In the case of linear approximation of a problem, it takes a long time to calculate because extra 0-1 variables are added. However, on a practical scale problem with about 400 customers, this method was found to solve the problem quickly enough. The solution using the piecewise linear approximation is an efficient one, demonstrating that it is better to solve PM-LIPs considering inventory cost.

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