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RANKING MULTICHANNEL UNCERTAIN INTEGER QUANTITY DIGITAL INFORMATION BASED ON DISCRETE QUASI-FUZZY NUMBER VECTOR

Qingya Qi[1], Guixiang Wang[1], Chengjia Li[2] and Yifeng Xu[1]

[1]Institute of Operations Research and Cybernetics
[2]Information Engineering School
Hangzhou Dianzi University
No. 1158, 2nd Avenue, Xiasha Higher Education Zone, Hangzhou 310018, P. R. China
qi181070008@163.com; { g.x.wang; yfxu; chengjiali }@hdu.edu.cn

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Abstract. *In this paper, the concept of discrete quasi-fuzzy number is put forward, which is an extension of discrete fuzzy number, and a weak order on discrete quasi-fuzzy number vector space is defined based on centroid of special fuzzy set (its support is a finite set, and we call it a discrete fuzzy quantity) of real number field R. Then, a method of constructing discrete quasi-fuzzy number vector (and discrete fuzzy number vector) to express multichannel uncertain integer quantity digital information is set up. And then, a specific example is given to show how to use the defined order and the proposed method (for constructing discrete quasi-fuzzy number vector) to rank the multichannel uncertain integer quantity digital information, and ultimately provide decision-making reasons for decision-makers.*
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1. **Introduction.** It is known that fuzzy number can be used to express uncertain or imprecise digital information. Fuzzy numbers can be divided into two kinds. One is continuous type fuzzy number (called continuous fuzzy number, in short called fuzzy number) and the other is discrete type fuzzy number (called discrete fuzzy number). Continuous fuzzy number is used to express uncertain or imprecise real number digital information [1, 2], and discrete fuzzy number is used to express uncertain or imprecise discrete quantity (such as integer) digital information [3, 4].

The concept discrete fuzzy numbers was proposed by Voxman in [5] in 2001 as one kind of special fuzzy sets which have some application backgrounds. In 2007 and 2011, Casasnovas and Riera studied some characters of discrete fuzzy numbers in [6, 7]. Riera and Torrens studied the residual implications on the set of discrete fuzzy numbers in [8] in 2013, introduced aggregation functions on the set of discrete fuzzy numbers whose support is a set of consecutive natural numbers from a couple of discrete aggregation functions in [9] in 2014, used discrete fuzzy numbers to model complete and incomplete qualitative information, and proposed some methods to aggregate this kind of information in [10] in 2015. In 2014, Massanet et al. set up a new linguistic computational model based on discrete fuzzy numbers for computing with words in [11]. Recently, in [3], we defined two-dimensional discrete fuzzy numbers and gave their applications in two factor evaluation in the uncertain environment; in [4], we obtained some fixed point theorems for fuzzy integer value mapping, and applied these fixed point theorems to the optimal management in some balance problems; in [12], Alfonso et al. introduced a fuzzy regression procedure involving a class of fuzzy numbers defined by some level sets called finite fuzzy

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numbers, and used it to fuse some real-world financial data; in [13], Liao and Su studied parallel machine scheduling problems in consideration of real world uncertainty quantified based on fuzzy numbers; in [14], Saini et al. also studied the triangular intuitionistic fuzzy multiple criteria decision making problem for finding the best option where the phonetic factors for the criteria are pre-characterized; in [15], Ziemba dealt with the problem of constructing a fuzzy multiple criteria decision making method called neat fuzzy preference ranking organization method for enrichment evaluation characterized by conformity with the methodological assumptions of the classical preference ranking organization method and a relatively low degree of complexity; in [16], Prokopowicz presented an efficient use of the ordered fuzzy numbers model in the description of processes undergoing dynamic changes; in [17], Ma et al. studied the group consensus for group decision making with the linguistic model based on discrete fuzzy numbers.

In this paper, we are going to study the problem of ranking multichannel uncertain integer quantity digital information, and apply the method obtained in the study to evaluation or decision-making. The specific arrangements of this paper are as follows. In Section 2, we briefly review some basic notions, definitions about fuzzy set of real number field R. In Section 3, we propose the concepts of discrete quasi-fuzzy number and n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector, give the definition of the centroid vector of n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector, and set up a weak order on the n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector space. Then, in Section 4, we propose a method of constructing n-dimensional discrete quasi-fuzzy number vector and n-dimensional discrete fuzzy number vector to repress multichannel uncertain integer quantity digital information. And then, in Section 5, we give a specific example to show how to use the defined order and the proposed method (for constructing discrete quasi-fuzzy number vector) to rank the multichannel uncertain integer quantity digital information, and ultimately provide decision-making reasons for decision-makers. At last, we make a conclusion in Section 6.

2. **Basic Definitions and Notations.**

Let R be the real number set. A fuzzy set of R is a function $u : R \to [0, 1]$. For each fuzzy set u of R, let $[u]^r = \{x \in R : u(x) \geq r\}$ for any $r \in (0, 1]$ be its r-level set (or r-cut set). By supp$u$, we mean the support of u, i.e., the set $\{x \in R : u(x) > 0\}$. In addition, we denote the closure of supp$u$ by $[u]^0$, i.e., $[u]^0 = \overline{\{x \in R : u(x) > 0\}}$.

For a fuzzy set $u : R \to [0, 1]$, if supp$u$ is a finite set, then we say it to be a discrete fuzzy quantity.

Definition 2.1. [5] *A fuzzy set $u : R \to [0, 1]$ is called a discrete fuzzy number if its support is finite, i.e., there exist $x_1, x_2, \ldots, x_n \in R$ with $x_1 < x_2 < \cdots < x_n$ such that $[u]^0 = \{x_1, x_2, \ldots, x_n\}$, and there exist natural numbers s, t with $1 \leq s \leq t \leq n$ such that*

1) $u(x_i) = 1$ for any natural number i with $s \leq i \leq t$;

2) $u(x_i) \leq u(x_j)$ for any natural numbers i, j with $1 \leq i \leq j \leq s$;

3) $u(x_i) \geq u(x_j)$ for any natural numbers i, j with $t \leq i \leq j \leq n$.

We denote the collection for all discrete fuzzy numbers by D.

3. **Weak Order on $(D_q)^n$ and $(D)^n$ Based on Centroid Vector.**

Definition 3.1. *Let u be a fuzzy set of R. If supp$u$ is a finite set, and u is a normal, i.e., there exists $x_0 \in$ supp$u$ such that $u(x_0) = 1$, then we call u a discrete quasi-fuzzy number. And we use D_q to denote the collection of all discrete quasi-fuzzy numbers.*

Definition 3.2. *For $u_i \in D_q$ (resp. D), $i = 1, 2, \ldots, n$, we say the ordered array $(u_1, u_2, \ldots, u_n)$ to be an n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector. And we use $(D_q)^n$ (resp. $(D)^n$) to denote the collection of all n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vectors.*

It is well known that for a fuzzy set and an element, the greater the membership degree of the element to the fuzzy set, the greater the contribution of the element to the fuzzy set. So, for a discrete quasi-fuzzy (resp. fuzzy) number, we can regard the discrete fuzzy number as a system of particles whose mass is its membership degree. In this way, we can approximate the discrete fuzzy number by the centroid value of the particle system. So, we give the following definition.

Definition 3.3. *For any $u \in D_q$ (resp. D) with $suppu = \{x_1, x_2, \ldots, x_m\}$, we denote*

$$\mathrm{C}(u) = \frac{\sum_{i=1}^m u(x_i)x_i}{\sum_{i=1}^m u(x_i)} \tag{1}$$

and call it the centroid of the discrete quasi-fuzzy (resp. fuzzy) number u.

Definition 3.4. *For $u = (u_1, u_2, \ldots, u_n) \in (D_q)^n$ (resp. $(D)^n$), we denote*

$$\overrightarrow{\mathrm{C}}(u) = (\mathrm{C}(u_1), \mathrm{C}(u_2), \ldots, \mathrm{C}(u_n)) \tag{2}$$

and call it the centroid vector of the discrete quasi-fuzzy (resp. fuzzy) number vector u.

Let $p = (p_1, p_2, \ldots, p_n)$ with $p_i \geq 0$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^n p_i = 1$. For any $u = (u_1, u_2, \ldots, u_n) \in (D_q)^n$ (resp. $(D)^n$), we denote

$$\overline{\mathrm{C}}_p(u) = \sum_{i=1}^n p_i \mathrm{C}(u_i) \tag{3}$$

Definition 3.5. *Let $p = (p_1, p_2, \ldots, p_n)$ with $p_i \geq 0$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^n p_i = 1$. We define a binary relation on the n-dimensional discrete quasi-fuzzy number vector space $(D_q)^n$ (resp. the n-dimensional discrete fuzzy number vector space $(D)^n$), i.e., a subset $\prec_p$ of $(D_q)^n \times (D_q)^n$ (resp. $(D)^n \times (D)^n$) as follows:*

$$\prec_p = \left\{(u, v) \in (D_q)^n \times (D_q)^n \,(\textit{resp. } (D)^n \times (D)^n) : \overline{\mathrm{C}}_p(u) \leq \overline{\mathrm{C}}_p(v)\right\} \tag{4}$$

i.e.,

$$\prec_p = \left\{(u, v) \in (D_q)^n \times (D_q)^n \,(\textit{resp. } (D)^n \times (D)^n) : \sum_{i=1}^n p_i(\mathrm{C}(u_i) - \mathrm{C}(v_i)) \leq 0\right\} \tag{5}$$

If $(u, v) \in \prec_p$, then we say u to be smaller than v (about $\prec_p$), and denote it as $u \prec_p v$.

By Definition 3.4, we can directly obtain the following property.

Proposition 3.1. *Let $p = (p_1, p_2, \ldots, p_n)$ with $p_i \geq 0$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^n p_i = 1$. Then*

1) for any $u \in (D_q)^n$ (resp. $(D)^n$), $u \prec_p u$ (reflexivity);

2) for any $u, v, w \in (D_q)^n$ (resp. $(D)^n$), $u \prec_p v$ and $v \prec_p w \Longrightarrow u \prec_p w$ (transitivity);

3) for any $u, v \in (D_q)^n$ (resp. $(D)^n$), at least one of $u \prec_p v$ and $v \prec_p u$ is tenable (completeness).

The proposition tells us that $\prec_p$ is a weak order on the n-dimensional discrete quasi-fuzzy number vector space $(D_q)^n$ (resp. the n-dimensional discrete fuzzy number vector space $(D)^n$).

4. Constructing Discrete Quasi-Fuzzy (Fuzzy) Number Vector.

Problem description: Suppose that an object (denoted as O) to be processed is characterized by n feature attributes (O_i denotes the ith feature attribute $i = 1, 2, \ldots, n$), and each of these feature attributes corresponds to a discrete quantity with uncertain attributes (such as an integral quantity). For each feature attribute of the object O, we made multiple random observations (m_i denotes the observation times to the ith feature attribute O_i, $i = 1, 2, \ldots, n$), and obtain a set of data as follows:

$$O : \begin{array}{lllll} O_1 : & o_{11} & o_{12} & \cdots & o_{1m_1} \\ O_2 : & o_{21} & o_{22} & \cdots & o_{2m_2} \\ \vdots & \vdots & \vdots & & \vdots \\ O_n : & o_{n1} & o_{n2} & \cdots & o_{nm_n} \end{array} \tag{6}$$

where o_{ij} is the jth value for the ith feature attribute O_i of the object O ($i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m_i$). The problem to be solved is how to construct a suitable n-dimensional discrete quasi-fuzzy number vector or a suitable n-dimensional discrete fuzzy number vector to represent the object O which possesses some imprecise or uncertain attributes of discrete quantity.

In the following, we set up a concretely structuring method.

The constructing method:

The first step: For each fixed $i = 1, 2, \ldots, n$, we denote

$$X_i = \{x_{i1}, x_{i2}, \ldots, x_{ik_i}\} = \{x_{ij} : \text{ exists } o_{ij_0} \text{ such that } x_{ij} = o_{ij_0}\} \tag{7}$$

with $x_{i1} \le x_{i2} \le \cdots \le x_{ik_i}$ (Obviously $k_i \le m_i$).

The second step: For each fixed $i = 1, 2, \ldots, n$, we use n_{ij} to denote the number of the times x_{ij} ($j = 1, 2, \ldots, k_i$) occurs in the m_i observation values: $o_{i1}, o_{i2}, \ldots, o_{im_i}$ for the ith feature attribute o_i of the object O, and find out n_{ij_0} such that

$$N_i = \max\{n_{ij} : j = 1, 2, \ldots, k_i\} \tag{8}$$

The third step: For each fixed $i = 1, 2, \ldots, n$, we can define a discrete quasi-fuzzy number u_i with support supp$u \in X_i$ as the following:

$$u_i(x) = \begin{cases} \dfrac{n_{ij}}{N_i}, & x = x_{ij}, \quad j = 1, 2, \ldots, k_i \\ 0, & \text{others} \end{cases} \tag{9}$$

and then the n-dimensional discrete quasi-fuzzy number vector $u = (u_1, u_2, \ldots, u_n)$ can be used to represent the object O which possesses some imprecise or uncertain attributes of discrete quantity.

The third step:* For each fixed $i = 1, 2, \ldots, n$, finding $j_0 \in \{j : j = 1, 2, \ldots, k_i\}$ such that $n_{ij_0} = N_i$, and denoting

$$\begin{cases} r_1 = \dfrac{n_{i1}}{N_i} \\ r_2 = \max\left\{r_1, \dfrac{n_{i2}}{N_i}\right\} \\ \cdots \\ r_{j_0-1} = \max\left\{r_{j_0-2}, \dfrac{n_{i(j_0-1)}}{N_i}\right\} \\ r_{j_0} = 1 \\ r_{j_0+1} = \max\left\{r_{j_0+2}, \dfrac{n_{i(j_0+1)}}{N_i}\right\} \\ \cdots \\ r_{k_i-1} = \max\left\{r_{k_i}, \dfrac{n_{i(k_i-1)}}{N_i}\right\} \\ r_{k_i} = \dfrac{n_{ik_i}}{N_i} \end{cases} \tag{10}$$

we can also define a discrete fuzzy number u_i with support supp$u = X_i$ as the following:

$$u_i(x) = \begin{cases} r_j, & x = x_{ij}, \quad j = 1, 2, \ldots, k_i \\ 0, & \text{others} \end{cases} \tag{11}$$

and then the n-dimensional discrete fuzzy number vector $u = (u_1, u_2, \ldots, u_n)$ can be used to represent the object O which possesses some imprecise or uncertain attributes of discrete quantity.

5. **Ranking of Multichannel Uncertain Integer Information.** In this section, a specific example is going to be given to show us how to use the method of constructing discrete quasi-fuzzy (fuzzy) number vector and the weak order "$\prec_p$" which are respectively proposed by us in Section 4 and Section 3 to rank some objects which possess some imprecise or uncertain attributes of discrete quantity.

For convenience of writing, we use $\{(x_1 : r_1), (x_2 : r_2), \ldots, (x_m : r_m)\}$ to denote the discrete quasi-fuzzy (fuzzy) number u with support supp$u = \{x_1, x_2, \ldots, x_m\}$ defined by

$$u(x) = \begin{cases} r_i, & i = 1, 2, \ldots, m \\ 0, & \text{others} \end{cases}$$

Example 5.1. *A company intends to recruit two employees. For this purpose, the company set up a recruitment team composed of 13 experts (denoted by $\mathcal{E}_1, \mathcal{E}_2, \ldots, \mathcal{E}_{13}$) to interview the 7 people (denoted by $\mathcal{P}_1, \mathcal{P}_2, \ldots, \mathcal{P}_7$) who came to the interview. According to the company's regulations, each expert in the recruitment team should give the corresponding scores to each interviewee according to the performance of the 7 interviewees in the interview process. The scores are given in three aspects: 1) Personal conduct (denoted by $\mathcal{A}_1$) ; 2) Working ability (denoted by $\mathcal{A}_2$); 3) Development potential (denoted by $\mathcal{A}_3$). Scores of each aspect are all 10-point system, that is, according to the quality of the interviewee in a certain aspect, each expert selects a number from 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 that matches the interviewee as the interviewee's score in this aspect (the higher the score is, the better the interviewee is). We use matrix $\mathrm{P}_k = (x_{ij})_{3\times 13}$ to express the primitive scoring status of interviewee $\mathcal{P}_k$ ($k = 1, 2, \ldots, 7$), where x_{ij} is the score that is given by expert $\mathcal{E}_j$ to interviewee $\mathcal{P}_i$ ($i = 1, 2, 3$ and $j = 1, 2, \ldots, 13$).*

Suppose the data of the primitive scoring status of the 7 interviewees are as follows:

$$\mathrm{P}_1 = \begin{pmatrix} 9 & 9 & 7 & 9 & 8 & 8 & 10 & 8 & 8 & 9 & 9 & 8 & 9 \\ 6 & 8 & 8 & 5 & 7 & 7 & 8 & 7 & 7 & 7 & 8 & 8 & 5 \\ 8 & 5 & 5 & 5 & 4 & 4 & 6 & 7 & 7 & 6 & 5 & 5 & 6 \end{pmatrix}_{3\times 13} \tag{12}$$

$$\mathrm{P}_2 = \begin{pmatrix} 10 & 9 & 7 & 9 & 8 & 8 & 10 & 8 & 8 & 9 & 9 & 10 & 9 \\ 6 & 8 & 9 & 7 & 7 & 7 & 8 & 7 & 9 & 7 & 8 & 8 & 10 \\ 6 & 5 & 5 & 5 & 4 & 4 & 8 & 4 & 7 & 5 & 5 & 6 & 6 \end{pmatrix}_{3\times 13} \tag{13}$$

$$\mathrm{P}_3 = \begin{pmatrix} 10 & 9 & 7 & 9 & 8 & 8 & 10 & 8 & 8 & 9 & 9 & 10 & 9 \\ 6 & 7 & 8 & 7 & 6 & 7 & 6 & 7 & 8 & 7 & 6 & 7 & 8 \\ 9 & 7 & 7 & 7 & 6 & 6 & 9 & 6 & 9 & 7 & 7 & 8 & 7 \end{pmatrix}_{3\times 13} \tag{14}$$

$$\mathrm{P}_4 = \begin{pmatrix} 8 & 7 & 5 & 7 & 7 & 6 & 8 & 6 & 6 & 5 & 6 & 7 & 9 \\ 6 & 8 & 9 & 7 & 8 & 7 & 8 & 6 & 9 & 7 & 8 & 8 & 8 \\ 8 & 6 & 7 & 7 & 6 & 6 & 8 & 6 & 9 & 7 & 7 & 8 & 7 \end{pmatrix}_{3\times 13} \tag{15}$$

$$\mathrm{P}_5 = \begin{pmatrix} 6 & 8 & 9 & 8 & 8 & 7 & 8 & 6 & 9 & 7 & 7 & 8 & 8 \\ 8 & 7 & 5 & 7 & 7 & 6 & 8 & 6 & 6 & 5 & 6 & 7 & 9 \\ 9 & 7 & 7 & 7 & 6 & 6 & 9 & 6 & 9 & 7 & 7 & 8 & 7 \end{pmatrix}_{3\times 13} \tag{16}$$

$$\mathrm{P}_6 = \begin{pmatrix} 6 & 8 & 9 & 8 & 8 & 7 & 8 & 6 & 9 & 7 & 7 & 8 & 8 \\ 10 & 8 & 8 & 8 & 7 & 7 & 9 & 7 & 10 & 8 & 8 & 9 & 8 \\ 8 & 7 & 5 & 7 & 7 & 6 & 8 & 6 & 6 & 5 & 6 & 7 & 9 \end{pmatrix}_{3\times 13} \tag{17}$$

$$\mathrm{P}_7 = \begin{pmatrix} 8 & 9 & 9 & 10 & 9 & 8 & 8 & 7 & 9 & 8 & 8 & 9 & 10 \\ 10 & 8 & 9 & 8 & 8 & 7 & 10 & 7 & 10 & 9 & 9 & 9 & 8 \\ 9 & 8 & 8 & 8 & 7 & 8 & 9 & 7 & 6 & 6 & 7 & 8 & 10 \end{pmatrix}_{3\times 13} \tag{18}$$

In the following, we are first going to construct discrete quasi-fuzzy (fuzzy) number vector to represent the overall performance of the each person of the 7 interviewers based on the method proposed in Section 4. Then, using the weak order "$\prec_p$", we will give a ranking to the 7 interviewees. At last, based on the ranking, we will select two persons to be recruited.

First: For each $\mathcal{P}_i$ $(i = 1, 2, \ldots, 7)$ and $\mathcal{A}_j$ $(j = 1, 2, 3)$, by Formula (7), we can obtain the corresponding $X_{ij} = \{x_{ij1}, x_{ij2}, \ldots, x_{ijk_{ij}}\}$ as the following:

$$\begin{array}{lll} X_{11} = \{7,8,9,10\} & X_{12} = \{5,6,7,8\} & X_{13} = \{4,5,6,7,8\} \\ X_{21} = \{7,8,9,10\} & X_{22} = \{6,7,8,9,10\} & X_{23} = \{4,5,6,7,8\} \\ X_{31} = \{7,8,9,10\} & X_{32} = \{6,7,8\} & X_{33} = \{6,7,8,9\} \\ X_{41} = \{5,6,7,8,9\} & X_{42} = \{6,7,8,9\} & X_{43} = \{6,7,8,9\} \\ X_{51} = \{6,7,8,9\} & X_{52} = \{5,6,7,8,9\} & X_{53} = \{6,7,8,9\} \\ X_{61} = \{6,7,8,9\} & X_{62} = \{7,8,9,10\} & X_{63} = \{5,6,7,8,9\} \\ X_{71} = \{7,8,9,10\} & X_{72} = \{7,8,9,10\} & X_{73} = \{6,7,8,9,10\} \end{array}$$

Second: For each $\mathcal{P}_i$ $(i = 1, 2, \ldots, 7)$ and $\mathcal{A}_j$ $(j = 1, 2, 3)$, we denote $S_{ij} = \{n_{ij1}, n_{ij2}, \ldots, n_{ijk_{ij}}\}$, where n_{ijk} is the number of the times x_{ijk} occurs in the 13 scores which are given by the 13 experts to the jth aspect $\mathcal{A}_j$ of the ith interviewee $\mathcal{P}_i$. Then we can see that:

$$\begin{array}{lll} S_{11} = \{1,5,6,1\} & S_{12} = \{2,1,5,5\} & S_{13} = \{2,5,3,2,1\} \\ S_{21} = \{1,4,5,3\} & S_{22} = \{1,5,4,2,1\} & S_{23} = \{3,5,3,1,1\} \\ S_{31} = \{1,4,5,3\} & S_{32} = \{4,6,3\} & S_{33} = \{3,6,1,3\} \\ S_{41} = \{2,4,4,2,1\} & S_{42} = \{2,3,6,2\} & S_{43} = \{4,5,3,1\} \\ S_{51} = \{2,3,6,2\} & S_{52} = \{2,4,4,2,1\} & S_{53} = \{3,6,1,3\} \\ S_{61} = \{2,3,6,2\} & S_{62} = \{3,6,2,2\} & S_{63} = \{2,4,4,2,1\} \\ S_{71} = \{1,5,5,2\} & S_{72} = \{2,4,4,3\} & S_{73} = \{2,3,5,2,1\} \end{array}$$

Then, we have that

$$\begin{array}{lll} N_{11} = \max\{1,5,6,1\} = 6 & N_{12} = \max\{2,1,5,5\} = 5 & N_{13} = \max\{2,5,3,2,1\} = 5 \\ N_{21} = \max\{1,4,5,3\} = 5 & N_{22} = \max\{1,5,4,2,1\} = 5 & N_{23} = \max\{3,5,3,1,1\} = 5 \\ N_{31} = \max\{1,4,5,3\} = 5 & N_{32} = \max\{4,6,3\} = 6 & N_{33} = \max\{3,6,1,3\} = 6 \\ N_{41} = \max\{2,4,4,2,1\} = 4 & N_{42} = \max\{2,3,6,2\} = 6 & N_{43} = \max\{4,5,3,1\} = 5 \\ N_{51} = \max\{2,3,6,2\} = 6 & N_{52} = \max\{2,4,4,2,1\} = 4 & N_{53} = \max\{3,6,1,3\} = 6 \\ N_{61} = \max\{2,3,6,2\} = 6 & N_{62} = \max\{3,6,2,2\} = 6 & N_{63} = \max\{2,4,4,2,1\} = 4 \\ N_{71} = \max\{1,5,5,2\} = 5 & N_{72} = \max\{2,4,4,3\} = 4 & N_{73} = \max\{2,3,5,2,1\} = 5 \end{array}$$

Third: For each fixed $i = 1, 2, \ldots, 7$, by

$$u_{ij}(x) = \begin{cases} \dfrac{n_{ijk}}{N_{ij}}, & x = x_{ijk}, \quad k = 1, 2, \ldots, k_{ij} \\ 0, & \text{others} \end{cases}$$

for any $j = 1, 2, 3$, we can obtain the discrete quasi-fuzzy number u_{ij} as the following:

$$u_{11} = \left\{\left(7, \frac{1}{6}\right), \left(8, \frac{5}{6}\right), (9, 1), \left(10, \frac{1}{6}\right)\right\}$$

$$u_{12} = \left\{\left(5, \frac{2}{5}\right), \left(6, \frac{1}{5}\right), (7, 1), (8, 1)\right\}$$

$$u_{13} = \left\{\left(4, \frac{2}{5}\right), (5, 1), \left(6, \frac{3}{5}\right), \left(7, \frac{2}{5}\right), \left(8, \frac{1}{5}\right)\right\}$$

$$u_{21} = \left\{\left(7, \frac{1}{5}\right), \left(8, \frac{4}{5}\right), (9,1), \left(10, \frac{3}{5}\right)\right\}$$
$$u_{22} = \left\{\left(6, \frac{1}{5}\right), (7,1), \left(8, \frac{4}{5}\right), \left(9, \frac{2}{5}\right), \left(10, \frac{1}{5}\right)\right\}$$
$$u_{23} = \left\{\left(4, \frac{3}{5}\right), (5,1), \left(6, \frac{3}{5}\right), \left(7, \frac{1}{5}\right), \left(8, \frac{1}{5}\right)\right\}$$
$$u_{31} = \left\{\left(7, \frac{1}{5}\right), \left(8, \frac{4}{5}\right), (9,1), \left(10, \frac{3}{5}\right)\right\}$$
$$u_{32} = \left\{\left(6, \frac{2}{3}\right), (7,1), \left(8, \frac{1}{2}\right)\right\}$$
$$u_{33} = \left\{\left(6, \frac{1}{2}\right), (7,1), \left(8, \frac{1}{6}\right), \left(9, \frac{1}{2}\right)\right\}$$
$$u_{41} = \left\{\left(5, \frac{1}{2}\right), (6,1), (7,1), \left(8, \frac{1}{2}\right), \left(9, \frac{1}{4}\right)\right\}$$
$$u_{42} = \left\{\left(6, \frac{1}{3}\right), \left(7, \frac{1}{2}\right), (8,1), \left(9, \frac{1}{3}\right)\right\}$$
$$u_{43} = \left\{\left(6, \frac{4}{5}\right), (7,1), \left(8, \frac{3}{5}\right), \left(9, \frac{1}{5}\right)\right\}$$
$$u_{51} = \left\{\left(6, \frac{1}{3}\right), \left(7, \frac{1}{2}\right), (8,1), \left(9, \frac{1}{3}\right)\right\}$$
$$u_{52} = \left\{\left(5, \frac{1}{2}\right), (6,1), (7,1), \left(8, \frac{1}{2}\right), \left(9, \frac{1}{4}\right)\right\}$$
$$u_{53} = \left\{\left(6, \frac{1}{2}\right), (7,1), \left(8, \frac{1}{6}\right), \left(9, \frac{1}{2}\right)\right\}$$
$$u_{61} = \left\{\left(6, \frac{1}{3}\right), \left(7, \frac{1}{2}\right), (8,1), \left(9, \frac{1}{3}\right)\right\}$$
$$u_{62} = \left\{\left(7, \frac{1}{2}\right), (8,1), \left(9, \frac{1}{3}\right), \left(10, \frac{1}{3}\right)\right\}$$
$$u_{63} = \left\{\left(5, \frac{1}{2}\right), (6,1), (7,1), \left(8, \frac{1}{2}\right), \left(9, \frac{1}{4}\right)\right\}$$
$$u_{71} = \left\{\left(7, \frac{1}{5}\right), (8,1), (9,1), \left(10, \frac{2}{5}\right)\right\}$$
$$u_{72} = \left\{\left(7, \frac{1}{2}\right), (8,1), (9,1), \left(10, \frac{3}{4}\right)\right\}$$
$$u_{73} = \left\{\left(6, \frac{2}{5}\right), \left(7, \frac{3}{5}\right), (8,1), \left(9, \frac{2}{5}\right), \left(10, \frac{1}{5}\right)\right\}$$

and then the 3-dimensional discrete quasi-fuzzy number vector $u_i = (u_{i1}, u_{i2}, u_{i3})$ *can be used to represent the comprehensive evaluation score for the ith interviewee* $\mathcal{P}_i$, $i = 1, 2, \ldots, 7$.

Fourth: For each $i = 1, 2, \ldots, 7$ *and* $j = 1, 2, 3$*, by*

$$\mathrm{C}(u_{ij}) = \frac{\sum_{k=1}^{10} u_{ij}(k)k}{\sum_{k=1}^{10} u_{ij}(k)}$$

we can obtain that

$$\mathrm{C}(u_{11}) = \frac{\frac{1}{6}\cdot 7 + \frac{5}{6}\cdot 8 + 1\cdot 9 + \frac{1}{6}\cdot 10}{\frac{1}{6} + \frac{5}{6} + 1 + \frac{1}{6}} = 8.54$$

$$\mathrm{C}(u_{12}) = \frac{\frac{2}{5}\cdot 5 + \frac{1}{5}\cdot 6 + 1\cdot 7 + 1\cdot 8}{\frac{2}{5} + \frac{1}{5} + 1 + 1} = 7.00$$

$$\mathrm{C}(u_{13}) = \frac{\frac{2}{5}\cdot 4 + 1\cdot 5 + \frac{3}{5}\cdot 6 + \frac{2}{5}\cdot 7 + \frac{1}{5}\cdot 8}{\frac{2}{5} + 1 + \frac{3}{5} + \frac{2}{5} + \frac{1}{5}} = 5.62$$

$$\mathrm{C}(u_{21}) = \frac{\frac{1}{5}\cdot 7 + \frac{4}{5}\cdot 8 + 1\cdot 9 + \frac{3}{5}\cdot 10}{\frac{1}{5} + \frac{4}{5} + 1 + \frac{3}{5}} = 8.77$$

$$\mathrm{C}(u_{22}) = \frac{\frac{1}{5}\cdot 6 + 1\cdot 7 + \frac{4}{5}\cdot 8 + \frac{2}{5}\cdot 9 + \frac{1}{5}\cdot 10}{\frac{1}{5} + 1 + \frac{4}{5} + \frac{2}{5} + \frac{1}{5}} = 7.77$$

$$\mathrm{C}(u_{23}) = \frac{\frac{3}{5}\cdot 4 + 1\cdot 5 + \frac{3}{5}\cdot 6 + \frac{1}{5}\cdot 7 + \frac{1}{5}\cdot 8}{\frac{3}{5} + 1 + \frac{3}{5} + \frac{1}{5} + \frac{1}{5}} = 5.38$$

$$\mathrm{C}(u_{31}) = \frac{\frac{1}{5}\cdot 7 + \frac{4}{5}\cdot 8 + 1\cdot 9 + \frac{3}{5}\cdot 10}{\frac{1}{5} + \frac{4}{5} + 1 + \frac{3}{5}} = 8.77$$

$$\mathrm{C}(u_{32}) = \frac{\frac{2}{3}\cdot 6 + 1\cdot 7 + \frac{1}{2}\cdot 8}{\frac{2}{3} + 1 + \frac{1}{2}} = 6.92$$

$$\mathrm{C}(u_{33}) = \frac{\frac{1}{2}\cdot 6 + 1\cdot 7 + \frac{1}{6}\cdot 8 + \frac{1}{2}\cdot 9}{\frac{1}{2} + 1 + \frac{1}{6} + \frac{1}{2}} = 7.31$$

$$\mathrm{C}(u_{41}) = \frac{\frac{1}{2}\cdot 5 + 1\cdot 6 + 1\cdot 7 + \frac{1}{2}\cdot 8 + \frac{1}{4}\cdot 9}{\frac{1}{2} + 1 + 1 + \frac{1}{2} + \frac{1}{4}} = 6.69$$

$$\mathrm{C}(u_{42}) = \frac{\frac{1}{3}\cdot 6 + \frac{1}{2}\cdot 7 + 1\cdot 8 + \frac{1}{3}\cdot 9}{\frac{1}{3} + \frac{1}{2} + 1 + \frac{1}{3}} = 7.62$$

$$\mathrm{C}(u_{43}) = \frac{\frac{4}{5}\cdot 6 + 1\cdot 7 + \frac{3}{5}\cdot 8 + \frac{1}{5}\cdot 9}{\frac{4}{5} + 1 + \frac{3}{5} + \frac{1}{5}} = 7.08$$

$$\mathrm{C}(u_{51}) = \frac{\frac{1}{3}\cdot 6 + \frac{1}{2}\cdot 7 + 1\cdot 8 + \frac{1}{3}\cdot 9}{\frac{1}{3} + \frac{1}{2} + 1 + \frac{1}{3}} = 7.62$$

$$\mathrm{C}(u_{52}) = \frac{\frac{1}{2}\cdot 5 + 1\cdot 6 + 1\cdot 7 + \frac{1}{2}\cdot 8 + \frac{1}{4}\cdot 9}{\frac{1}{2} + 1 + 1 + \frac{1}{2} + \frac{1}{4}} = 6.69$$

$$\mathrm{C}(u_{53}) = \frac{\frac{1}{2}\cdot 6 + 1\cdot 7 + \frac{1}{6}\cdot 8 + \frac{1}{2}\cdot 9}{\frac{1}{2} + 1 + \frac{1}{6} + \frac{1}{2}} = 7.31$$

$$\mathrm{C}(u_{61}) = \frac{\frac{1}{3}\cdot 6 + \frac{1}{2}\cdot 7 + 1\cdot 8 + \frac{1}{3}\cdot 9}{\frac{1}{3} + \frac{1}{2} + 1 + \frac{1}{3}} = 7.62$$

$$\mathrm{C}(u_{62}) = \frac{\frac{1}{2}\cdot 7 + 1\cdot 8 + \frac{1}{3}\cdot 9 + \frac{1}{3}\cdot 10}{\frac{1}{2} + 1 + \frac{1}{3} + \frac{1}{3}} = 8.23$$

$$\mathrm{C}(u_{63}) = \frac{\frac{1}{2}\cdot 5 + 1\cdot 6 + 1\cdot 7 + \frac{1}{2}\cdot 8 + \frac{1}{4}\cdot 9}{\frac{1}{2} + 1 + 1 + \frac{1}{2} + \frac{1}{4}} = 6.69$$

$$\mathrm{C}(u_{71}) = \frac{\frac{1}{5}\cdot 7 + 1\cdot 8 + 1\cdot 9 + \frac{2}{5}\cdot 10}{\frac{1}{5} + 1 + 1 + \frac{2}{5}} = 8.62$$

$$\mathrm{C}(u_{72}) = \frac{\frac{1}{2}\cdot 7 + 1\cdot 8 + 1\cdot 9 + \frac{3}{4}\cdot 10}{\frac{1}{2} + 1 + 1 + \frac{3}{4}} = 8.62$$

$$\mathrm{C}(u_{73}) = \frac{\frac{2}{5}\cdot 6 + \frac{3}{5}\cdot 7 + 1\cdot 8 + \frac{2}{5}\cdot 9 + \frac{1}{5}\cdot 10}{\frac{2}{5}+\frac{3}{5}+1+\frac{2}{5}+\frac{1}{5}} = 7.77$$

Taking $p = \left(\frac{2}{5}, \frac{2}{5}, \frac{1}{5}\right)$*, by*

$$\overline{\mathrm{C}}_p(u_i) = \sum_{j=1}^{3} p_j \mathrm{C}(u_{ij})$$

for $i = 1, 2, \ldots, 7$*, we have that*

$$\overline{\mathrm{C}}_p(u_1) = \frac{2}{5}\cdot 8.54 + \frac{2}{5}\cdot 7.00 + \frac{1}{5}\cdot 5.62 = 7.34$$
$$\overline{\mathrm{C}}_p(u_2) = \frac{2}{5}\cdot 8.77 + \frac{2}{5}\cdot 7.77 + \frac{1}{5}\cdot 5.38 = 7.69$$
$$\overline{\mathrm{C}}_p(u_3) = \frac{2}{5}\cdot 8.77 + \frac{2}{5}\cdot 6.92 + \frac{1}{5}\cdot 7.31 = 7.74$$
$$\overline{\mathrm{C}}_p(u_4) = \frac{2}{5}\cdot 6.69 + \frac{2}{5}\cdot 7.62 + \frac{1}{5}\cdot 7.08 = 7.14$$
$$\overline{\mathrm{C}}_p(u_5) = \frac{2}{5}\cdot 7.62 + \frac{2}{5}\cdot 6.69 + \frac{1}{5}\cdot 7.31 = 7.17$$
$$\overline{\mathrm{C}}_p(u_6) = \frac{2}{5}\cdot 7.62 + \frac{2}{5}\cdot 8.23 + \frac{1}{5}\cdot 6.69 = 7.68$$
$$\overline{\mathrm{C}}_p(u_7) = \frac{2}{5}\cdot 8.62 + \frac{2}{5}\cdot 8.62 + \frac{1}{5}\cdot 7.77 = 8.45$$

Fifth: It is obvious that $\overline{\mathrm{C}}_p(u_4) \le \overline{\mathrm{C}}_p(u_5) \le \overline{\mathrm{C}}_p(u_1) \le \overline{\mathrm{C}}_p(u_6) \le \overline{\mathrm{C}}_p(u_2) \le \overline{\mathrm{C}}_p(u_3) \le \overline{\mathrm{C}}_p(u_7)$*. So by the definition of weak order "*$\prec_p$*" (i.e., Formula (4)), we can obtain the following ranking of the 7 interviewees as follows:*

$$u_4 \prec_p u_5 \prec_p u_1 \prec_p u_6 \prec_p u_2 \prec_p u_3 \prec_p u_7$$

Thus, according to the ranking, we can select interviewee $\mathcal{P}_7$ *and interviewee* $\mathcal{P}_3$ *as the two persons to be recruited.*

6. **Conclusions.** In this paper, we proposed the concepts of discrete quasi-fuzzy number (Definition 3.1) and n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector (Definition 3.2), gave the definition of the centroid vector (Definitions 3.3 and 3.4) of n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector, set up a weak order (Definition 3.5) on the n-dimensional discrete quasi-fuzzy (resp. fuzzy) number vector space, and gave some properties of the weak order (Proposition 3.1). Then, we established a method (Equations (7)-(11)) of constructing n-dimensional discrete quasi-fuzzy number vector and a method of n-dimensional discrete fuzzy number vector to repress multichannel uncertain integer quantity digital information. And then, we used a specific example (Example 5.1) to show how to rank the multichannel uncertain integer quantity digital information with the weak order and the proposed constructing method, and ultimately provide decision-making reasons for decision-makers. In the future research work, we can discuss the problem of establishing fuzzy order in high-dimensional discrete fuzzy number space, so that we can give a dynamic ranking (the ranking results cab vary with the given level values) method for multi-channel uncertain integer quantity information.

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