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EVENT-TRIGGERED H_∞ CONTROL OF SEMI-MARKOVIAN JUMP SYSTEMS WITH NONLINEARITIES

Chaoxu Guan, Liu Yang and Zhongyang Fei

Research Institute of Intelligent Control and Systems
Harbin Institute of Technology
No. 92, West Dazhi Street, Nangang District, Harbin 150001, P. R. China
guanchaoxu@sina.com

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Abstract. *This paper investigates the H_∞ control for a class of semi-Markovian jump nonlinear systems with an event-triggered control mechanism. In order to reduce the data packets sent to the communication network, a mode-dependent event-triggered condition is adopted to determine which packet can be transmitted. According to the triggering mechanism, a switched closed-loop system model is established for stability analysis and control synthesis. Then by employing a new Lyapunov functional, sufficient criteria are obtained to ensure that the resulting closed-loop system is stochastically stable with a prescribed H_∞ performance. Finally, a numerical example is utilized to illustrate the usefulness and advantages of the proposed theorems.*

Keywords: Semi-Markovian jump systems, Event-triggered control, Nonlinearity

1. **Introduction.** Markovian jump systems, as an important component of stochastic switched systems, have been excessively studied during the past decades. Since they can successfully model the random perturbations and abrupt changes in practice, numerous methods have been proposed for the stability analysis and system synthesis for different kinds of Markovian jump systems. However, due to the exponential distribution of Markovian jump process, it is inevitable that some limitations and conservatism may exist in the results based on Markovian jump systems. To handle this problem, increasing attention has been paid to the research on semi-Markovian jump systems since its relaxed limitations on the probability distributions. Quite a few results have been reported on the related topics of semi-Markovian jump systems in recent years [1-4]. To mention a few, the stochastic stability is analyzed for semi-Markovian jump systems with mode-dependent delays by exploiting a piecewise analysis approach [5]. Via a mode-transition-dependent sojourn-time distribution, a state feedback controller is designed to achieve the stabilization of continuous-time semi-Markovian jump systems [6].

It is well known that most control processes are implemented based on the digital signals, which thus prompts the study on sampled-data control systems. Traditionally, the system state or output is sampled with a certain sampling period, which is easy to be implemented while may bring some redundant data packets to communication network. Recently, event-triggered control systems have drawn considerable attention owing to its capacity in dealing with the resource waste of traditional sampled-data systems [7-9]. Under an event-triggered control strategy, only the data packets that satisfy a certain triggering condition will be transmitted to the network and hence, the communication resource is efficiently saved. In [10], the event-triggered H_∞ filtering is presented for distributed parameter systems with Markovian switching topology, which reduces the communication burden and also achieves the H_∞ disturbance attenuation performance. By employing a relaxed Lyapunov functional and applying a mode-dependent event-triggered

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strategy, a desired filter together with triggering parameters is co-designed for Markovian jump systems with general transition probabilities [11].

Based on the discussion above, this paper aims to design an event-triggered controller to guarantee the stochastic stability and H_∞ performance for a class of semi-Markovian jump systems. Since the nonlinear characteristics are inherent in practical applications, a nonlinear representation is taken into account in the system model. By establishing a new Lyapunov functional, which is a switched form subject to the resulting closed-loop system and also involves the nonlinearity, sufficient conditions are obtained to ensure that the system is stochastically stable with a desired H_∞ performance. In the end, numerical simulations are provided to show the effectiveness and advantages of the proposed results.

The rest of this paper is organized as follows. The problem description and some definitions are presented in Section 2. Section 3 establishes the criteria on the stability analysis and control synthesis. In Section 4, a numerical example is used to show the effectiveness of the obtained theorems. Finally, Section 5 concludes this work.

2. **Problem Statement and Preliminaries.** Consider the following semi-Markovian jump nonlinear system described by

$$\dot{x}(t) = A(r(t))x(t) + B(r(t))u(t) + E(r(t))f(x(t)) + F(r(t))w(t), \tag{1}$$

$$z(t) = G(r(t))x(t) + H(r(t))w(t), \tag{2}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $z(t) \in R^p$ is the output signal and $w(t) \in R^q$ is the external disturbance belonging to square-integrable function space $L_2[0,\infty)$. $\{r(t), t \geq 0\}$ represents a continuous-time semi-Markov process and takes values in a finite set $S = \{1, 2, \ldots, N\}$, in which N denotes the number of subsystems. The probability transition of the semi-Markov process $r(t)$ is administrated by

$$\Pr\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}(\Delta)\Delta + o(\Delta), & i \neq j \\ 1 + \pi_{ii}(\Delta)\Delta + o(\Delta), & i = j \end{cases},$$

where $\pi_{ij}(\Delta) \geq 0$ is the transition rate from mode i at instant t to mode j at instant $t + \Delta$ for $i \neq j$ and $\pi_{ii}(\Delta) = -\sum_{i\in S, j\neq i} \pi_{ij}(\Delta)$. $\Delta > 0$ refers to the sojourn time and $\lim\limits_{\Delta\to 0} o(\Delta)/\Delta = 0$. $A(r(t))$, $B(r(t))$, $E(r(t))$, $F(r(t))$, $G(r(t))$, $H(r(t))$ are constant real matrices and simplified as A_i, B_i, E_i, F_i, G_i, H_i for each $r(t) = i$, $i \in S$. $f(\cdot)$ indicates the nonlinearity satisfying $f(0) = 0$ and the following Lipschitz condition $\|f(x_1) - f(x_2)\| \leq C\|x_1 - x_2\|$, where C is known positive diagonal matrix.

In this paper, an event-triggered control mechanism is adopted to determine the instants $\{s_k, k = 0, 1, 2, \ldots\}$ of sending system measurements. Then the controller updates the control input according to the received measurements and sends it back to the system plant for the stabilization goal. The event-triggered control mechanism is described as follows

$$s_{k+1} = \min\left\{s \geq s_k + h | (x(s) - x(s_k))^T \Omega_i (x(s) - x(s_k)) \geq \varepsilon x^T(s)\Omega_i x(s)\right\}, \tag{3}$$

where $h > 0$, $0 \leq \varepsilon < 1$ are given constants and $\Omega_i > 0$, $\forall i \in S$ is the mode-dependent weighting matrix. Based on this mechanism, the controller for system (1) is given as

$$u(t) = -K_i x(s_k), \quad t \in [s_k, s_{k+1}), \tag{4}$$

where K_i, $i \in S$ is the state feedback controller gain to be designed.

Some definitions are restated here for latter use.

Definition 2.1. [5] *The semi-Markovian jump system (1) with $w(t) = 0$ is said to be stochastically stable if for any initial values $x(0)$, $r(0)$, $\lim\limits_{t\to\infty} E\left\{\int_0^t \|x(s)\|^2 ds | x(0), r(0)\right\} < \infty$ holds, where $E\{.\}$ is the mathematics expectation operation.*

Definition 2.2. [12] *For a given scalar $\gamma > 0$, the semi-Markovian jump system (1) is said to be stochastically stable with an H_∞ performance index γ, if system (1) is stochastically stable and under the zero initial values, $E\left\{\int_0^\infty z^T(t)z(t)dt\right\} \le \gamma^2 \int_0^\infty w^T(t)w(t)dt$ holds for any nonzero $w(t) \in L_2[0,\infty)$.*

3. **Main Results.** According to the section above, the semi-Markovian jump nonlinear system (1) under event-triggered mechanism (3) and controller (4) can be rewritten as [4]

$$\dot{x}(t) = (A_i - B_iK_i)x(t) + \varphi(t)\tau(t)B_iK_iv_1 + E_if(x(t)) - (1-\varphi(t))B_iK_ie(t) + F_iw(t), \quad (5)$$

where

$$\varphi(t) = \begin{cases} 1, & t \in [s_k, s_k + h) \\ 0, & t \in [s_k + h, s_{k+1}) \end{cases}, \quad v_1 = \frac{1}{\tau(t)}\int_{t-\tau(t)}^{t} \dot{x}(s)ds,$$

$$\tau(t) = t - s_k \le h, \quad t \in [s_k, s_k + h), \quad e(t) = x(s_k) - x(t), \quad t \in [s_k + h, s_{k+1}).$$

The Lyapunov candidate for system (5) is chosen as

$$V(t) = x^T(t)P_ix(t) + \varphi(t)(V_u(t) + V_x(t)) + \frac{1}{\lambda^2}\int_0^t \left(\|Cx(s)\|^2 - \|f(x(s))\|^2\right) ds,$$

$$V_u(t) = (h-\tau(t))\int_{t-\tau(t)}^{t} \dot{x}^T(s)U\dot{x}(s)ds, \xi(t) = \left[\begin{array}{cc} x^T(t) & x^T(t-\tau(t)) \end{array}\right]^T,$$

$$V_x(t) = (h-\tau(t))\xi^T(t)\left[\begin{array}{cc} \frac{X+X^T}{2} & -X+X_1 \\ * & -X_1 - X_1^T + \frac{X+X^T}{2} \end{array}\right]\xi(t).$$

Due to the limitation of pages, the following theorems are proposed for stability analysis and control synthesis with the proof omitted.

Theorem 3.1. *For $h > 0$, $0 \le \varepsilon < 1$, $\gamma > 0$, the semi-Markovian jump nonlinear system (1) under the event trigger (3) is stochastically stable with an H_∞ performance index if there exist matrices $P_i > 0$, $\Omega_i > 0$, $\forall i \in S$, $U > 0$, $Q > 0$ and X, X_1, Y_1, Y_2, Y_3 such that for any $i \in S$,*

$$\Theta_i \ge 0, \quad \Psi_{1i} < 0, \quad \Psi_{2i} < 0, \quad \Phi_i < 0, \quad (6)$$

where

$$\Theta_i = \left[\begin{array}{cc} P_i + h\frac{X+X^T}{2} & hX_1 - hX \\ * & -hX_1 - hX_1^T + h\frac{X+X^T}{2} \end{array}\right],$$

$$\Psi_{1i} = \left[\begin{array}{ccc} \Psi_{1i}(1,1) & \cdots & \Psi_{1i}(1,5) \\ & \ddots & \vdots \\ * & & \Psi_{1i}(5,5) \end{array}\right],$$

$$\Psi_{2i} = \left[\begin{array}{ccc} \Psi_{2i}(1,1) & \cdots & \Psi_{2i}(1,6) \\ & \ddots & \vdots \\ * & & \Psi_{2i}(6,6) \end{array}\right], \quad \Phi_i = \left[\begin{array}{ccc} \Phi_i(1,1) & \cdots & \Phi_i(1,5) \\ & \ddots & \vdots \\ * & & \Phi_i(5,5) \end{array}\right],$$

$$\Psi_{1i}(1,1) = \sum_{j\in S}\pi_{ij}(\Delta)P_j + A_i^TQ + QA_i - Y_1 - Y_1^T - \frac{X+X^T}{2} + G_i^TG_i + \frac{1}{\lambda^2}C^TC,$$

$$\Psi_{1i}(1,2) = P_i + h\frac{X+X^T}{2} - Q + A_i^TQ - Y_2,$$

$$\Psi_{1i}(1,3) = Y_1^T - QB_iK_i - Y_3 + (X - X_1),$$

$$\Psi_{1i}(1,4) = QE_i, \quad \Psi_{1i}(1,5) = G_i^TH_i + QF_i, \quad \Psi_{1i}(2,2) = -2Q + hU,$$

$\Psi_{1i}(2,3) = Y_2^T - QB_iK_i - (X - X_1), \quad \Psi_{1i}(2,4) = QE_i, \quad \Psi_{1i}(2,5) = QF_i,$
$\Psi_{1i}(3,3) = Y_3 + Y_3^T - \frac{1}{2}\left(X + X^T - 2X_1 - 2X_1^T\right), \quad \Psi_{1i}(3,4) = \Psi_{1i}(3,5) = 0,$
$\Psi_{1i}(4,4) = -\frac{1}{\lambda^2}I, \quad \Psi_{1i}(4,5) = 0, \quad \Psi_{1i}(5,5) = H_i^TH_i - \gamma^2 I,$
$\Psi_{2i}(1,1) = \Psi_{1i}(1,1), \quad \Psi_{2i}(1,2) = P_i - Q + A_i^TQ - Y_2, \quad \Psi_{2i}(1,3) = \Psi_{1i}(1,3),$
$\Psi_{2i}(1,4) = hY_1^T, \quad \Psi_{2i}(1,5) = QE_i, \quad \Psi_{2i}(1,6) = G_i^TH_i + QF_i, \quad \Psi_{2i}(2,2) = -2Q,$
$\Psi_{2i}(2,3) = Y_2^T - QB_iK_i, \quad \Psi_{2i}(2,4) = hY_2^T, \quad \Psi_{2i}(2,5) = QE_i, \quad \Psi_{2i}(2,6) = QF_i,$
$\Psi_{2i}(3,3) = \Psi_{1i}(3,3), \quad \Psi_{2i}(3,4) = hY_3^T, \quad \Psi_{2i}(3,5) = \Psi_{2i}(3,6) = 0,$
$\Psi_{2i}(4,4) = -hU, \quad \Psi_{2i}(4,5) = \Psi_{2i}(4,6) = 0,$
$\Psi_{2i}(5,5) = -\frac{1}{\lambda^2}I, \quad \Psi_{2i}(5,6) = 0, \quad \Psi_{2i}(6,6) = \Psi_{1i}(5,5),$
$\Phi_i(1,1) = \sum_{j\in S}\pi_{ij}(\Delta)P_j + G_i^TG_i + \frac{1}{\lambda^2}C^TC + \varepsilon\Omega_i + Q(A_i - B_iK_i) + (A_i - B_iK_i)^TQ,$
$\Phi_i(1,2) = P_i - Q + (A_i - B_iK_i)^TQ, \quad \Phi_i(1,3) = -QB_iK_i, \quad \Phi_i(1,4) = QE_i,$
$\Phi_i(1,5) = G_i^TH_i + QF_i, \quad \Phi_i(2,2) = -2Q, \quad \Phi_i(2,3) = -QB_iK_i, \quad \Phi_i(2,4) = QE_i,$
$\Phi_i(2,5) = QF_i, \quad \Phi_i(3,3) = -\Omega_i, \quad \Phi_i(3,4) = \Phi_i(3,5) = 0, \quad \Phi_i(4,4) = -\frac{1}{\lambda^2}I,$
$\Phi_i(4,5) = 0, \quad \Phi_i(5,5) = H_i^TH_i - \gamma^2 I.$

Notice that the inequalities in Theorem 3.1 are difficult to be solved since the time-varying transition rate $\pi_{ij}(\Delta)$ and the coupling of variables. By referring to [13], we assume that

$$\pi_{ij}(\Delta) = \sum_{k=1}^{T}\alpha_k\pi_{ij,k}, \quad \sum_{k=1}^{T}\alpha_k = 1, \quad \alpha_k \geq 0$$

$$\text{and } \pi_{ij,k} = \begin{cases} \hat{\pi}_{ij} + (k-1)\dfrac{\breve{\pi}_{ij} - \hat{\pi}_{ij}}{T-1}, & i \neq j,\ j \in S \\ \breve{\pi}_{ij} - (k-1)\dfrac{\breve{\pi}_{ij} - \hat{\pi}_{ij}}{T-1}, & i = j,\ j \in S \end{cases},$$

where $\hat{\pi}_{ij}$ and $\breve{\pi}_{ij}$ are the lower and upper bounds of $\pi_{ij}(\Delta)$, respectively. Based on this, the following theorem proposes the co-design on controller gains and event-triggered weighting matrix for the semi-Markovian jump nonlinear system.

Theorem 3.2. *For $h > 0$, $0 \leq \varepsilon < 1$, $\gamma > 0$, the semi-Markovian jump nonlinear system (1) under the event trigger (3) is stochastically stable with an H_∞ performance index if there exist matrices $\tilde{P}_i > 0$, $\tilde{\Omega}_i > 0$, $\tilde{K}_i$, $\forall i \in S$, $\tilde{U} > 0$, $\tilde{Q} > 0$ and $\tilde{X}$, $\tilde{X}_1$, $\tilde{Y}_1$, $\tilde{Y}_2$, $\tilde{Y}_3$ such that for any $i \in S$, $k = 1, 2, \ldots, T$,*

$$\tilde{\Theta}_i \geq 0, \quad \tilde{\Psi}_{1i,k} < 0, \quad \tilde{\Psi}_{2i,k} < 0, \quad \tilde{\Phi}_{i,k} < 0, \tag{7}$$

where

$$\tilde{\Theta}_i = \begin{bmatrix} \tilde{P}_i + h\dfrac{\tilde{X} + \tilde{X}^T}{2} & h\tilde{X}_1 - h\tilde{X} \\ * & -h\tilde{X}_1 - h\tilde{X}_1^T + h\dfrac{\tilde{X} + \tilde{X}^T}{2} \end{bmatrix},$$

$$\tilde{\Psi}_{1i,k} = \begin{bmatrix} \tilde{\Psi}_{1i,k}(1,1) & \cdots & \tilde{\Psi}_{1i}(1,7) \\ & \ddots & \vdots \\ * & & \tilde{\Psi}_{1i}(7,7) \end{bmatrix},$$

$$\tilde{\Psi}_{2i,k} = \begin{bmatrix} \tilde{\Psi}_{2i,k}(1,1) & \cdots & \tilde{\Psi}_{2i,k}(1,8) \\ & \ddots & \vdots \\ * & & \tilde{\Psi}_{2i,k}(8,8) \end{bmatrix}, \quad \tilde{\Phi}_{i,k} = \begin{bmatrix} \tilde{\Phi}_{i,k}(1,1) & \cdots & \tilde{\Phi}_{i,k}(1,7) \\ & \ddots & \vdots \\ * & & \tilde{\Phi}_{i,k}(7,7) \end{bmatrix},$$

$$\tilde{\Psi}_{1i,k}(1,1) = \sum_{j\in S} \pi_{ij,k}\tilde{P}_j + \tilde{Q}A_i^T + A_i\tilde{Q} - \tilde{Y}_1 - \tilde{Y}_1^T - \frac{\tilde{X}+\tilde{X}^T}{2},$$

$$\tilde{\Psi}_{1i,k}(1,2) = \tilde{P}_i + h\frac{\tilde{X}+\tilde{X}^T}{2} - \tilde{Q} + \tilde{Q}A_i^T - \tilde{Y}_2,$$

$$\tilde{\Psi}_{1i,k}(1,3) = \tilde{Y}_1^T - B_i\tilde{K}_i - \tilde{Y}_3 + \left(\tilde{X} - \tilde{X}_1\right), \quad \tilde{\Psi}_{1i,k}(1,4) = E_i,$$

$$\tilde{\Psi}_{1i,k}(1,5) = \tilde{Q}G_i^T H_i + F_i, \quad \tilde{\Psi}_{1i,k}(1,6) = \tilde{Q}G_i^T, \quad \tilde{\Psi}_{1i,k}(1,7) = \tilde{Q}C^T,$$

$$\tilde{\Psi}_{1i,k}(2,2) = -2\tilde{Q} + h\tilde{U}, \quad \tilde{\Psi}_{1i,k}(2,3) = \tilde{Y}_2^T - B_i\tilde{K}_i - \left(\tilde{X} - \tilde{X}_1\right),$$

$$\tilde{\Psi}_{1i,k}(2,4) = E_i, \quad \tilde{\Psi}_{1i,k}(2,5) = F_i, \quad \tilde{\Psi}_{1i,k}(2,6) = \tilde{\Psi}_{1i,k}(2,7) = 0,$$

$$\tilde{\Psi}_{1i,k}(3,3) = \tilde{Y}_3 + \tilde{Y}_3^T - \frac{1}{2}\left(\tilde{X} + \tilde{X}^T - 2\tilde{X}_1 - 2\tilde{X}_1^T\right),$$

$$\tilde{\Psi}_{1i,k}(3,4) = \tilde{\Psi}_{1i,k}(3,5) = \tilde{\Psi}_{1i,k}(3,6) = \tilde{\Psi}_{1i,k}(3,7) = 0, \quad \tilde{\Psi}_{1i,k}(4,4) = -\frac{1}{\lambda^2}I,$$

$$\tilde{\Psi}_{1i,k}(4,5) = \tilde{\Psi}_{1i,k}(4,6) = \tilde{\Psi}_{1i,k}(4,7) = 0, \quad \tilde{\Psi}_{1i,k}(5,5) = H_i^T H_i - \gamma^2 I,$$

$$\tilde{\Psi}_{1i,k}(5,6) = \tilde{\Psi}_{1i,k}(5,7) = 0, \quad \tilde{\Psi}_{1i,k}(6,6) = -I, \quad \tilde{\Psi}_{1i,k}(6,7) = 0,$$

$$\tilde{\Psi}_{1i,k}(7,7) = -\lambda^2 I, \quad \tilde{\Psi}_{2i,k}(1,1) = \tilde{\Psi}_{1i,k}(1,1), \quad \tilde{\Psi}_{2i,k}(1,2) = \tilde{P}_i - \tilde{Q} + \tilde{Q}A_i^T - \tilde{Y}_2,$$

$$\tilde{\Psi}_{2i,k}(1,3) = \tilde{\Psi}_{1i,k}(1,3), \quad \tilde{\Psi}_{2i,k}(1,4) = h\tilde{Y}_1^T, \quad \tilde{\Psi}_{2i,k}(1,5) = E_i,$$

$$\tilde{\Psi}_{2i,k}(1,6) = \tilde{Q}G_i^T H_i + F_i, \quad \tilde{\Psi}_{2i,k}(1,7) = \tilde{Q}G_i^T, \quad \tilde{\Psi}_{2i,k}(1,8) = \tilde{Q}C^T,$$

$$\tilde{\Psi}_{2i,k}(2,2) = -2\tilde{Q}, \quad \tilde{\Psi}_{2i,k}(2,3) = \tilde{Y}_2^T - B_i\tilde{K}_i, \quad \tilde{\Psi}_{2i,k}(2,4) = h\tilde{Y}_2^T,$$

$$\tilde{\Psi}_{2i,k}(2,5) = E_i, \quad \tilde{\Psi}_{2i,k}(2,6) = F_i, \quad \tilde{\Psi}_{2i,k}(2,7) = \tilde{\Psi}_{2i,k}(2,8) = 0,$$

$$\tilde{\Psi}_{2i,k}(3,3) = \tilde{\Psi}_{1i,k}(3,3), \quad \tilde{\Psi}_{2i,k}(3,4) = h\tilde{Y}_3^T,$$

$$\tilde{\Psi}_{2i,k}(3,5) = \tilde{\Psi}_{2i,k}(3,6) = \tilde{\Psi}_{2i,k}(3,7) = \tilde{\Psi}_{2i,k}(3,8) = 0, \quad \tilde{\Psi}_{2i,k}(4,4) = -h\tilde{U},$$

$$\tilde{\Psi}_{2i,k}(4,5) = \tilde{\Psi}_{2i,k}(4,6) = \tilde{\Psi}_{2i,k}(4,7) = \tilde{\Psi}_{2i,k}(4,8) = 0, \quad \tilde{\Psi}_{2i,k}(5,5) = -\frac{1}{\lambda^2}I,$$

$$\tilde{\Psi}_{2i,k}(5,6) = \tilde{\Psi}_{2i,k}(5,7) = \tilde{\Psi}_{2i,k}(5,8) = 0, \quad \tilde{\Psi}_{2i,k}(6,6) = \tilde{\Psi}_{1i,k}(5,5),$$

$$\tilde{\Psi}_{2i,k}(6,7) = \tilde{\Psi}_{2i,k}(6,8) = 0, \quad \tilde{\Psi}_{2i,k}(7,7) = -I, \quad \tilde{\Psi}_{2i,k}(7,8) = 0,$$

$$\tilde{\Psi}_{2i,k}(8,8) = -\lambda^2 I,$$

$$\tilde{\Phi}_{i,k}(1,1) = \sum_{j\in S} \pi_{ij,k}\tilde{P}_j + \varepsilon\tilde{\Omega}_i + \left(A_i\tilde{Q} - B_i\tilde{K}_i\right) + \left(A_i\tilde{Q} - B_i\tilde{K}_i\right)^T,$$

$$\tilde{\Phi}_{i,k}(1,2) = \tilde{P}_i - \tilde{Q} + \left(A_i\tilde{Q} - B_i\tilde{K}_i\right)^T, \quad \tilde{\Phi}_{i,k}(1,3) = -B_i\tilde{K}_i, \quad \tilde{\Phi}_i(1,4) = E_i,$$

$$\tilde{\Phi}_{i,k}(1,5) = \tilde{Q}G_i^T H_i + F_i, \quad \tilde{\Phi}_{i,k}(1,6) = \tilde{Q}G_i^T, \quad \tilde{\Phi}_{i,k}(1,7) = \tilde{Q}C^T, \quad \tilde{\Phi}_{i,k}(2,2) = -2\tilde{Q},$$

$$\tilde{\Phi}_{i,k}(2,3) = -B_i\tilde{K}_i, \quad \tilde{\Phi}_{i,k}(2,4) = E_i, \quad \tilde{\Phi}_{i,k}(2,5) = F_i, \quad \tilde{\Phi}_{i,k}(2,6) = \tilde{\Phi}_{i,k}(2,7) = 0,$$

$$\tilde{\Phi}_{i,k}(3,3) = -\tilde{\Omega}_i, \quad \tilde{\Phi}_{i,k}(3,4) = \tilde{\Phi}_{i,k}(3,5) = \tilde{\Phi}_{i,k}(3,6) = \tilde{\Phi}_{i,k}(3,7) = 0,$$

$$\tilde{\Phi}_{i,k}(4,4) = -\frac{1}{\lambda^2}I, \quad \tilde{\Phi}_{i,k}(4,5) = \tilde{\Phi}_{i,k}(4,6) = \tilde{\Phi}_{i,k}(4,7) = 0,$$

$$\tilde{\Phi}_{i,k}(5,5) = H_i^T H_i - \gamma^2 I, \quad \tilde{\Phi}_{i,k}(5,6) = \tilde{\Phi}_{i,k}(5,7) = 0, \quad \tilde{\Phi}_{i,k}(6,6) = -I,$$

$$\tilde{\Phi}_{i,k}(6,7) = 0, \quad \tilde{\Phi}_{i,k}(7,7) = -\lambda^2 I.$$

Then the controller gains can be given by $K_i = \tilde{K}_i\tilde{Q}^{-1}$, $\forall i \in S$.

4. **A Numerical Example.** Consider a semi-Markovian jump nonlinear system with the following parameters:

$$A_1 = \begin{bmatrix} 1.1555 & -0.4006 \\ -0.4470 & -0.7930 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.0274 & 0.0249 \\ -0.3835 & 0.3026 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.01 \\ 0.02 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix},$$

$$G_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.02 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.01 & 0 \\ 0.01 & 0.01 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} 0.01 \\ 0.01 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.02 \\ -0.01 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}.$$

The time-varying transition rates are set as $\pi_{12}(\Delta) \in [0.76, 0.8]$, $\pi_{21}(\Delta) \in [0.45, 0.5]$. The nonlinear function is chosen to be $f(x(t)) = x_1(t)\cos(t)$ and the external disturbance is $w(t) = 0.1e^{-3t}$. With $h = 0.1$, $\varepsilon = 0.1$, $\gamma = 1$, we solve the inequalities in Theorem 3.2 and obtain the following controller gains $K_1 = [\ 4.1436 \quad -0.8932\ ]$, $K_2 = [\ 1.6738 \quad -0.8494\ ]$. The corresponding state trajectory of the closed-loop system and the evolution of event triggering are drawn in Figure 1. Clearly, the semi-Markovian jump nonlinear system is stabilized with a prescribed H_∞ performance under the designed controllers and event-triggered mechanism. In this case, the amount of data transmission is computed to be 83 over the running time, while by using the traditional periodic sampling method, the amount will be 150 with the same period $h = 0.1$. Hence, the superiority and advantages of the proposed results are demonstrated by the numerical simulations.

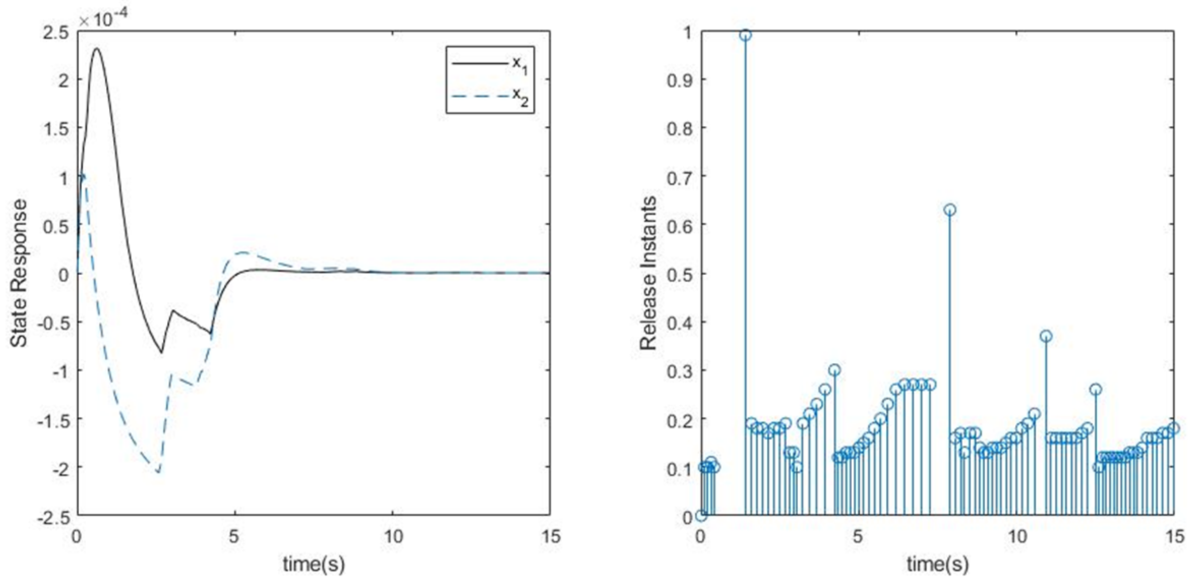

FIGURE 1. State trajectory of closed-loop system and the evolution of event trigger

5. **Conclusions.** This paper is concerned with the event-triggered H_∞ control for a class of continuous-time semi-Markovian jump systems with nonlinearities. A mode-dependent event-triggered mechanism is introduced to reduce the amounts of data sending, which thus saves the limited communication resource. By establishing a switched closed-loop system model and employing a new Lyapunov functional, sufficient criteria are developed to guarantee the stochastic stability and H_∞ performance of the event-triggered control

system. In the end, the usefulness and superiority of the obtained results are verified by a numerical example. In the further study, we would extend the current research to more complex cases, such as singular semi-Markovian jump systems, fuzzy semi-Markovian jump systems and semi-Markovian jump systems with unknown probability transitions.

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