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COOPERATIVE CONTROL FOR UNKNOWN NONLINEAR LEADER-FOLLOWING SYSTEMS WITH UNKNOWN PARAMETERIZABLE FRICTION

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Abstract. *In this paper, the problem of the cooperative adaptive fault fuzzy tracking control for networked unknown nonlinear leader-following systems with unknown nonlinear friction is discussed. Based on the principle of sliding mode control, an adaptive fault tolerant control scheme is proposed, which guarantees that all followers asymptotically synchronize a leader node with tracking errors converging to a small adjustable neighborhood of the origin and a distributed sliding mode adaptive controller is designed for each follower node to make tracking errors uniformly terminated and bounded. Based on algebraic graph theory and Lyapunov theory, the stability and parameter convergence of the algorithm are analyzed. Finally, the simulation results show the effectiveness of the scheme.*

Keywords: Fuzzy tracking control, Leader-following, Sliding mode control, Friction

1. **Introduction.** In recent years, the cooperative control of leader-following systems has achieved fruitful research results [1-3] under the extensive attention of experts and scholars. Generally speaking, the control problem of such systems can be categorized into two classes, namely, the cooperative regulator problem and the cooperative tracking problem. As stated in [4], for the first problem, distributed controller is respectively designed for each follower, such that all followers are eventually driven to a leader. This problem is known as (leaderless) consensus, synchronization, or rendezvous in literature. For the later problem, a leader agent is considered, and it acts as a command generator, which generates the desired reference trajectory and ignores information from the follower agents. All other agents attempt to follow the trajectory of the leader agent. This problem is known as leader-following consensus, synchronization to a leader, model reference consensus, leader-following control, or pinning control. [5] investigates the output consensus control problem of uncertain second-order nonlinear multi-agent systems with unknown nonlinear dead zone, [6] considers the consensus tracking control problem for general linear multi-agent systems with unknown dynamics in both the leader and all followers, [7] investigates the cooperative control problem of uncertain high-order nonlinear multi-agent system on directed graph with a fixed topology. By using distributed observer approach, the cooperative output regulation problem of linear multi-agent systems has been solved under the assumption that each follower knows the system matrix of the leader system [8]. However, the aforementioned works do not take friction occurring in the systems into account, which motivates this work. Friction as a complex uncertain nonlinear phenomenon, encounters in various practical systems, such as servo-mechanisms. In the paper, a cooperative adaptive fuzzy tracking control scheme for unknown nonlinear leader-following systems is proposed, which guarantees that all followers can asymptotically synchronize

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the leader with tracking errors being cooperative uniform ultimate bounded (CUUB). Compared with existing works, the following main contributions are worth being emphasized. 1) Differing from some of the literature, the dynamic leader and all followers considered in this paper are high-order and have unknown nonlinear dynamics; 2) the approach does not require the assumption that the parameters and frictions must be known [9]; 3) furthermore, the condition that the friction parameters must be known, is removed by approximating online.

The rest of this paper is organized as follows. In Section 2, basic graph theory and notations, the problem formulation are introduced. A cooperative adaptive tracking controller is proposed for each follower in Section 3. Finally, Section 4 draws the conclusion.

2. Problem Statement and Preliminaries.

Consider a multi-agent system consisting of a leader and the followers. To solve the coordination problems and model the information exchange between agents, graph theory is introduced here. Let $O = (v, E)$ be a weighted graph, $v = (v_1, \dots, v_N)$ is the nonempty set of nodes/agents, $E \subseteq v \times v$ is the set of edges/arcs, $(v_j, v_i) \in E$ means there is an edge from node i to node j. The topology of a weighted graph G is often represented by the adjacency matrix $A = [a_{ij}] \in R^{N\times N}$, and $a_{ij} > 0$ if $(v_j, v_i) \in E$; otherwise $a_{ij} = 0$. Throughout this paper, it is assumed that $a_{ii} = 0$ and the topology is fixed, i.e., A is time-invariant. O is directed graph. Define $d_i = \sum_{j=1}^{N} a_{ij}$ as the weighted in-degree of node i and $D = diag(d_1, \dots, d_N)$ as in-degree matrix. The graph Laplacian matrix is $L = [l_{ij}] = D - A$. Let $\underline{1} = [1, \dots, 1]$ with appropriate dimension; then $L\underline{1} = 0$. The set of neighbors of node i is denoted as $N_j = \{j|(v_j, v_i) \in E\}$. If node j is a neighbor of node i, then node i can get information from node j, not necessarily vice versa for directed graph. For undirected graph, neighbor is mutual relation. A direct path from node i to node j is a sequence of successive edges in the form $\{(v_i, v_l), (v_l, v_k), \dots, (v_m, v_j)\}$.

Notations: In this paper, R, R_n and $R^{n\times m}$ denote, respectively, the real numbers, the real n-vectors, and the real $n\times m$ matrices; $|\Delta|$ is the absolute value of a real number; $\|\Delta\|$ is the Euclidean norm of a vector; $\|\Delta\|_F$ is the Frobenius norm of a matrix; $tr\{\cdot\}$ is the trace of a matrix; $s(\cdot)$ is the set of singular values of a matrix, with the maximum singular value $\bar{s}(\cdot)$, matrix $P > 0$ ($P \geq 0$) means P is positive definite (positive semidefinite); I denotes the identity matrix with appropriate dimensions.

Consider N ($N \geq 2$) agents with distinct dynamics. Dynamics of the kth agent is described in Brunovsky form

$$\begin{cases} \dot{x}_{k,i}(t) = x_{k,i+1}(t) \\ \dot{x}_{k,n_k}(t) = f_k(\bar{x}_k) + g_{k,1}u_k + g_{k,2}F_{r,k} + d_k(\bar{x}_k, t) \end{cases}, \quad i = 1, \dots, n_k - 1,\ k = 1, \dots, N \tag{1}$$

where n_k, $x_{k,i} \in R$ and $\bar{x}_k = [x_{k,1}, \dots, x_{k,n_k}]^T \in R^{n_k}$ denote the ith state, the order number and the state vector of node k; $f_k(\bar{x}_k)$: $R^{n_k} \to R$ is locally Lipschitz in R^{n_k} with $f_k(0) = 0$ and it is assumed to be unknown; $u_k \in R$ is the control input/protocol; and $d_k(\bar{x}_k, t) \in R$ is an external disturbance, which is unknown but bounded. $g_{k,1}, g_{k,2} \in R^+$ denote unknown constants; $F_{r,k}$ denotes the nonlinear friction term. The friction term F_r is assumed to have the following form as in $F_{r,k} = \gamma_1(\tanh_k(\gamma_2\dot{x}_{k,1}(t)) - \tanh_k(\gamma_3\dot{x}_{k,1}(t)) + \gamma_4 \tanh_k(\gamma_5\dot{x}_{k,1}(t)) + \gamma_6\dot{x}_{k,1}(t))$, here, $\gamma_i \in R^+$, $i = 1, 2, 3, 4, 5, 6$ are unknown constants.

In this paper, it is assumed that $n_0 = n_1 = n_2 = \cdots = n_k = n$, where n_0 is the order number of the following leader node, labeled 0. Define $x_i = [x_{1,i}, \dots, x_{N,i}]^T \in R^N$, $i = 1, \dots, n$, then the above agents' dynamics can be re-written in the following compact form:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t) \\ \dot{x}_n(t) = f(\bar{x}) + g_{k,1}u(t) + g_{k,2}F_{r,k} + d(\bar{x}, t) \end{cases}, \quad i = 1, \dots, n - 1 \tag{2}$$

where $\bar{x} = \left[\bar{x}_1^T, \ldots, \bar{x}_N^T\right]^T$, $f(\bar{x}) = [f_1(\bar{x}_1), \ldots, f_N(\bar{x}_N)]^T$, $u(t) = [u_1(t), \ldots, u_N(t)]^T$, and $d(\bar{x}, t) = [d_1(\bar{x}_1, t), \ldots, d_N(\bar{x}_N, t)]^T$.

The dynamics of the leader node, labeled 0, is described as follows:

$$\begin{cases} \dot{x}_{0,i}(t) = x_{0,i+1}(t) \\ \dot{x}_{0,n}(t) = f_0(\bar{x}_0, t) \end{cases}, \quad i = 1, \ldots, n-1 \tag{3}$$

where $x_{0,i} \in R$ and $\bar{x}_0 = [x_{0,1}, \ldots, x_{0,n}]^T \in R^n$ denote the ith state and the state vector of the leader node k; $f_0(\bar{x}, t)$: $[0, \infty) \times R^n \to R$ is piecewise continuous in time t and locally Lipschitz in $\bar{x}_0$ with $f_0(0, t) = 0$ for all $\forall t \geq 0$ and $\bar{x}_0 \in R^n$, and it is unknown for all follower nodes.

System (3) is assumed to be forward complete, i.e., for every initial condition, the solution $\bar{x}_0(t)$ exists for all $\forall t \geq 0$. In other words, there is no finite escape time. The leader node dynamics (3) can be considered as an exosystem that generates a desired command trajectory. Define the ith order tracking error (disagreement variable) for node k $(k = 1, \ldots, N)$ as follows: $\delta_{k,i}(t) = x_{k,i}(t) - x_{0,i}(t)$, $i = 1, \ldots, n$, $k = 1, \ldots, N$. Let $\delta_i = [\delta_{1,i}, \ldots, \delta_{N,i}]^T \in R^N$, $i = 1, \ldots, n$, then $\delta_i = x_i - \underline{x}_{0,i}$ where $\underline{x}_{0,i} = [x_{0,1}, \ldots, x_{0,i}]^T \in R^N$.

The control objective: The distributed controllers are designed for all follower nodes such that the tracking error δ_i converges to small neighborhoods of the origin, for all i $(i = 1, \ldots, n)$.

The following definition is introduced to illustrate the control problem, which extends the standard concept of uniform ultimate boundedness to cooperative control systems.

Note that, it is assumed that only relative state information can be used for the follower's controller design in this paper. More precisely, for the kth node, the only obtainable information is the neighborhood synchronization error

$$e_{k,i}(t) = \sum_{j \in N_k} a_{kj}(x_{j,i} - x_{k,i}) + b_k(x_{0,i} - x_{k,i})$$

where $i = 1, \ldots, n$, $k = 1, \ldots, N$ and $b_k \geq 0$ is the weight of edge from the leader node to node k $(k = 1, \ldots, N)$, $b_k > 0$ if there is an edge from the leader node to node k.

Define the following notations:

$$\begin{aligned} &e_i = [e_{1,i}, \ldots, e_{N,i}]^T \in R^N, \quad \underline{f}_0 = [f_0(\bar{x}_0, t), \ldots, f_0(\bar{x}_0, t)]^T \in R^N, \\ &B = diag\{b_1, \ldots, b_N\} \in R^{N \times N} \\ &g_1 = diag\{g_{1,1}, \ldots, g_{N,1}\} \in R^{N \times N}, \quad g_2 = diag\{g_{1,2}, \ldots, g_{N,2}\} \in R^{N \times N} \end{aligned}$$

Similar to (2), the above tracking error can be re-written in the following compact form:

$$\begin{cases} \dot{e}_i(t) = e_{i+1}(t) \\ \dot{e}_n(t) = -(L + B)\left(f + g_1 u(t) + g_2 F_{r,k} + d - \underline{f}_0\right) \end{cases}, \quad i = 1, \ldots, n-1 \tag{4}$$

where the property $L\underline{1} = 0$ is used.

Define the augmented graph as $\bar{O} = \left\{\bar{v}, \bar{E}\right\}$, $\bar{v} = \{v_0, v_1, \ldots, v_N\}$ and $\bar{E} \subseteq \bar{v} \times \bar{v}$. In practical applications, the actuators may become faulty. The following assumptions on the graph topology and the dynamics of leader node are made for co-operative tracking problem.

Assumption 2.1. *The augmented graph $\bar{O}$ contains a spanning tree with the root node being the leader node 0.*

Assumption 2.2. *There exists a positive constant $M_0 > 0 \in R$, $M_{f_0} \in R$ such that $||x_0(t)|| \leq M_0$ and $|f_0(\bar{x}_0, t)| \leq M_{f_0}$, $\forall t \geq t_0$.*

Assumption 2.3. *For each node k, an unknown constant $M_{d,k} > 0 \in R$, so $|d(\bar{x}, t)| \leq M_{d,k}$.*

Lemma 2.1. *$||\delta_i|| \leq \frac{||e_i||}{\underline{s}(L+B)}$, $i = 1, \ldots, n$, where $\underline{s}(L+B)$ is a minimum singular value of matrix $L+B$ [4].*

Lemma 2.2. *Define $q = [q_1, \ldots, q_N] = (L+B)^{-1}\underline{1}$, $P = diag\{p_i\} = diag\left\{\frac{1}{q_i}\right\}$, $Q = P(L+B) + (L+B)^T P$, then $P > 0$ and $Q > 0$.*

In this paper, the fuzzy logic system (FLS) is used to approximate $f(x)$. Let $f(x)$ be a continuous function that is defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists $\sup\left|f(x) - \theta^T\xi(x)\right| \leq \varepsilon$. FLSs are universal approximations, i.e., they can approximate any smooth function on a compact space. Because of this approximation capability, we can assume that the nonlinear term $f(x)$ can be approximated as

$$f(x, \theta) = \theta^T\xi(x) \tag{5}$$

Define the optimal parameter θ^* as $\theta^* = \arg\min_{\theta\in\Omega}\left[\sup\left|f(x) - f\left(x, \theta^*\right)\right|\right]$ where Ω and U are compact regions for θ and x, respectively. Also the FLS minimum approximation error is defined as

$$\varepsilon = f(x) - \theta^{*T}\xi(x) \tag{6}$$

In this paper, we use the aforementioned FLS to approximate the unknown functions $f_k(\bar{x}_k)$, $k = 1, \ldots, N$, namely, there exist θ_k^*, ε_k such that $f_k(\bar{x}_k) = \theta_k^{*T}\xi_k(\bar{x}_k) + \varepsilon_k$.

Assumption 2.4. *There exist unknown constants $M_{\varepsilon,k} > 0 \in R$, $k = 1, \ldots, N$ such that $|\varepsilon_k| \leq M_{\varepsilon,k}$.*

3. **Main Results.** Define the filtered error σ for the kth node as follows:

$$\sigma_k = \left(\frac{d}{dt} + \lambda\right)^{n-1} e_{k,1}(t) = \sum_{i=1}^{n-1} c_{k,i}e_{k,i}(t) + e_{k,n}(t) \tag{7}$$

where $c_{k,i} = C_{n-1}^{i-1}\alpha^{n-i}$, $i = 1, \ldots, n-1$, $\alpha_k > 0$ denotes a designed parameter. Let $\underline{e}_k(t) = [e_{k,1}(t), \ldots, e_{k,n}(t)]^T$.

Lemma 3.1. *Let σ_k be defined by (7), and then,*
1) if $\sigma_k = 0$, then $\lim_{t\to\infty}\underline{e}_k(t) = 0$;
2) if $|\sigma_k| \leq a_k$, $\underline{e}_k(0) \in \Omega_{a_k}$, then $e_k(t) \in \Omega_{a_k}$, $\forall t \geq 0$;
3) if $|\sigma_k| \leq a_k$, $\underline{e}_k(0) \notin \Omega_{a_k}$, then $\exists T_k = (m_k - 1)/\lambda_k$, $\exists\forall t \geq T_k$, $\underline{e}_k(t) \in \Omega_{a_k}$,
where $\Omega_{a_k} = \left\{\underline{e}_k(t)\middle||e_{k,i}| \leq 2^{(j-1)}\lambda_k^{j-m_k}a_k\right\}$, $i = 1, \ldots, n$, $j = 1, 2, \ldots, m_k$.

For simplification, let $c_{1,i} = \cdots = c_{N,i} = \lambda_i$, $\lambda_n = 1$, then $\sigma_k = \lambda_1 e_{k,1} + \cdots + \lambda_n e_{k,n}$.

Define the global sliding mode error $\sigma = [\sigma_1, \ldots, \sigma_N]^T$, then $\sigma = \lambda_1 e_1 + \cdots + \lambda_n e_n$.

Recalling the overall tracking error dynamics

$$\begin{cases} \dot{e}_i(t) = e_{i+1}(t) \\ \dot{e}_n(t) = -(L+B)\left(f\left(\bar{x}\right) + u(t) + d\left(\bar{x}, t\right) - \underline{f}_0\right) \end{cases}, \quad i = 1, \ldots, n \tag{8}$$

one has

$$\dot{\sigma} = \lambda_1\dot{e}_1 + \cdots + \lambda_n\dot{e}_n = \sum_{i=1}^{n-1}\lambda_i e_i + \dot{e}_n = \gamma - (L+B)\left(f\left(\bar{x}\right) + g_1 u + g_2 F_r + d(\bar{x}, t) - \underline{f}_0\right)$$

where $\gamma = \sum_{i=1}^{n-1}\lambda_i e_i$, $\dot{e}_i = [\dot{e}_{1,i}, \ldots, \dot{e}_{N,i}]^T$.

Define the following Lyapunov function

$$V_s = 1/2\left(\sigma^T P\sigma\right) \tag{9}$$

where $P = P^T > 0$.

Differentiating V_s with respect to time t, one has

$$\dot{V}_s = \sigma^T P\gamma - \sigma^T P(D+B)\theta^{*T}\xi - \sigma^T P(D+B)\left(\varepsilon + d - \underline{f}_0\right) - \sigma^T P(L+B)(g_1 u)$$

$$+\sigma^T PA\theta^{*T}\xi + \sigma^T PA\left(\varepsilon + d - \underline{f}_0\right) - \sigma^T P(L+B)(g_2 F_r) \tag{10}$$

First, using the mean values theorem we have $g(x) = \tanh(x) = \dot{g}(\lambda x)(x-0)$, where $\lambda \in [0,1]$.

And using $|\tanh(\bullet)| < 1$, we have $\dot{g}(\lambda x) = \lambda\dot{x}\left[1 - (\tanh(\lambda x))^2\right] < \lambda|\dot{x}| \le |\dot{x}|$.

Then, (10) can be transformed to the following form:

$$\begin{aligned}\dot{V}_s &< \sigma^T P\gamma - \sigma^T P(D+B)\theta^{*T}\xi - \sigma^T P(D+B)(\varepsilon + d - \underline{f}_0)\\ &\quad - \sigma^T P(L+B)(g_1 u) + \sigma^T PA\theta^{*T}\xi + \sigma^T PA(\varepsilon + d - \underline{f}_0)\\ &\quad + \sigma^T P(L+B)|\dot{x}_2||\dot{x}_1|[\gamma_{g_1} + \gamma_{g_2} + \gamma_{g_3}] + \sigma^T P(L+B)|\dot{x}_1|\gamma_{g_4}\end{aligned} \tag{11}$$

where $|\dot{x}_1| = diag\{|\dot{x}_{1,1}|, \ldots, |\dot{x}_{N,1}|\} \in R^{N\times N}$, $|\dot{x}_2| = diag\{|\dot{x}_{1,2}|, \ldots, |\dot{x}_{N,2}|\} \in R^{N\times N}$, $\gamma_{k,g_1} = g_{k,2}\gamma_{k,1}\gamma_{k,2}$, $\gamma_{k,g_2} = g_{k,2}\gamma_{k,1}\gamma_{k,3}$, $\gamma_{k,g3} = g_{k,2}\gamma_{k,4}\gamma_{k,5}$, $\gamma_{k,g_4} = g_{k,2}\gamma_{k,6}$, $\gamma_{g_1} = [\gamma_{1,g1}, \ldots, \gamma_{N,g1}]^T$, $\gamma_{g_2} = [\gamma_{1,g2}, \ldots, \gamma_{N,g2}]^T$, $\gamma_{g_3} = [\gamma_{1,g3}, \ldots, \gamma_{N,g3}]^T$, $\gamma_{g_4} = [\gamma_{1,g4}, \ldots, \gamma_{N,g4}]^T$.

Define control law as follows:

$$\begin{aligned}u = g_{\min}^{-1}\Big\{&(D+B)^{-1}\gamma - \hat{f} - \operatorname{sgn}\left(s^T P(D+B)\right)\hat{\bar{M}}_{d\varepsilon f} + cs\\ &+ |\dot{x}_2||\dot{x}_1|[\hat{\gamma}_{g1} + \hat{\gamma}_{g2} + \hat{\gamma}_{g3}] + |\dot{x}_1|\hat{\gamma}_{g4}\Big\}\end{aligned} \tag{12}$$

where $g_{\min}^{-1} = diag\left\{g_{\min 1,1}^{-1}, \ldots, g_{\min N,1}^{-1}\right\} \in R^{N\times N}$, $\hat{f} = \left[\hat{f}_1, \ldots, \hat{f}_n\right]^T$, $\hat{f}_k = \hat{\theta}_k^T \xi_k(\bar{x}_k)$, $k = 1, \ldots, N$, are the estimate of $f_k(\bar{x}_k)$, $P_{s,k}$ is the ith element of $s^T P(D+B)$, $\operatorname{sgn}\left(s^T P(D+B)\right) = diag(\operatorname{sgn}(P_{s,1}), \ldots, \operatorname{sgn}(P_{s,N}))$, $\hat{\bar{M}}_{d\varepsilon f} = \left[\hat{M}_{d\varepsilon f,1}, \ldots, \hat{M}_{d\varepsilon f,N}\right]^T$, $\hat{M}_{d\varepsilon f,k}$ is the estimate of $M_{d\varepsilon f,k} = M_{d,k} + M_{\varepsilon,k} + M_{f_0}$, $k = 1, \ldots, N$, and γ_{g_1}, γ_{g_2}, γ_{g_3}, γ_{g_4} are the estimate of $\hat{\gamma}_{g_1}$, $\hat{\gamma}_{g_2}$, $\hat{\gamma}_{g_3}$, $\hat{\gamma}_{g_4}$, $c > 0 \in R$ is a design parameter, which satisfies $c\underline{s}(Q)/2 - \left(4r + \bar{\lambda}^2/\left(4r\underline{\lambda}^2\right)\right)\bar{s}(P)\bar{s}(Q) > 0$. Let $\hat{\theta}^T = diag\left(\hat{\theta}_1^T, \ldots, \hat{\theta}_N^T\right)$, and $\hat{\theta}_k^T$, $k = 1, \ldots, N$ are the estimates of $\hat{\theta}_k^{*T}$, in the following, define the notation: $\tilde{\bullet} = \bullet - \hat{\bullet}$. Substituting the control law (12) into (11), one has

$$\begin{aligned}\dot{V}_s = &-c\sigma^T P(L+B)\sigma + \sigma^T P(D+B)\tilde{\theta}^T\xi + \sigma^T P(D+B)\operatorname{sgn}\left(\sigma^T P(D+B)\right)\tilde{\bar{M}}_{d\varepsilon f}\\ &+ \sigma^T P(D+B)\left(|\dot{x}_2||\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right) + \sigma^T PA\tilde{\theta}^T\xi + \sigma^T PA\left(\varepsilon + d - \underline{f}_0\right)\\ &- \sigma^T PA\operatorname{sgn}\left(\sigma^T P(D+B)\right)\hat{\bar{M}}_{d\varepsilon f} - \sigma^T PA\left(|\dot{x}_2||\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)\\ &+ \sigma^T PA(D+B)^{-1}\gamma\end{aligned}$$

Since using Young inequality, we have

$$\sigma^T PA\tilde{\theta}^T\xi \le \bar{s}(P)\bar{s}(A)r\sigma^T\sigma + \frac{\bar{s}(P)\bar{s}(A)}{4r}\xi^T\tilde{\theta}\tilde{\theta}^T\xi$$

$$-\sigma^T PA\operatorname{sgn}\left(\sigma^T P(D+B)\right)\hat{\bar{M}}_{d\varepsilon f} \le \bar{s}(P)\bar{s}(A)r\sigma^T\sigma + \bar{s}(P)\bar{s}(A)\Big/(4r)\hat{\bar{M}}_{d\varepsilon f}^T\hat{\bar{M}}_{d\varepsilon f}$$

$$\sigma^T PA\left(\varepsilon + d - \underline{f}_0\right) \le \bar{s}(P)\bar{s}(A)r\sigma^T\sigma + \frac{\bar{s}(P)\bar{s}(A)}{4r}\bar{M}_{d\varepsilon f}^T\bar{M}_{d\varepsilon f}$$

$$\sigma^T PA(D+B)^{-1}\gamma \le \bar{s}(P)\bar{s}(A)r\sigma^T\sigma + \frac{\bar{s}(P)\bar{s}(A)}{4r}\gamma^T(D+B)^{-2}\gamma$$

$$\sigma^T PA\left(|\dot{x}_2||\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)$$

$$\leq \bar{s}(P)\bar{s}(A)r\sigma^T\sigma + \frac{\bar{s}(P)\bar{s}(A)}{4r}\left(\sum_{i=1}^{3}|\dot{x}_2||\dot{x}_1|\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)^T\left(\sum_{i=1}^{3}|\dot{x}_2||\dot{x}_1|\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)$$

where $r > 0 \in R$ is a design parameter and $\bar{M}_{d\varepsilon f} = [M_{d\varepsilon f,1}, \ldots, M_{d\varepsilon f,N}]^T$ one has

$$\begin{aligned}\dot{V}_s = & -c\sigma^T P(L+B)\sigma + \sigma^T P(D+B)\tilde{\theta}^T\xi + \sigma^T P(D+B)\mathrm{sgn}\left(\sigma^T P(D+B)\right)\tilde{\bar{M}}_{d\varepsilon f} \\ & + \sigma^T P(D+B)\left(|\dot{x}_2||\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right) + 5r\bar{s}(P)\bar{s}(A)r\sigma^T\sigma \\ & + \bar{s}(P)\bar{s}(A)\Big/(4r)\left(\xi^T\tilde{\theta}\tilde{\theta}^T\xi + \hat{\bar{M}}_{d\varepsilon f}^T\hat{\bar{M}}_{d\varepsilon f} + \bar{M}_{d\varepsilon f}^T\bar{M}_{d\varepsilon f} + \gamma^T(D+B)^{-2}\gamma\right. \\ & \left. + \left(\sum_{i=1}^{3}|\dot{x}_2||\dot{x}_1|\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)^T\left(\sum_{i=1}^{3}|\dot{x}_2||\dot{x}_1|\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)\right)\end{aligned}$$

Notice that, since θ_K^* and $\hat{\theta}_K$ are bounded, which are guaranteed by Assumption 2.4 and the adaptive law (14), and $\xi_k(\bar{x}_k) \leq 1$, $\tilde{\theta}^T\xi$ is bounded. From Assumptions 2.2 and 2.3, i.e., ε_k, d_k and f_0 are bounded, $\bar{M}_{d\varepsilon f}$ is bounded as well. The adaptive law (16) ensures that $\hat{\bar{M}}_{d\varepsilon f}$ is bounded. Because $\tilde{\theta}^T\xi$, $\bar{M}_{d\varepsilon f}$, $\tilde{\gamma}_{g_i}$, $|\dot{x}_2|$, $|\dot{x}_1|$ and $\hat{\bar{M}}_{d\varepsilon f}^T$ are bounded, if $r > 0 \in R$ is chosen to be large enough, then

$$\begin{aligned}& \bar{s}(P)\bar{s}(A)\Big/(4r)\left(\xi^T\tilde{\theta}\tilde{\theta}^T\xi + \hat{\bar{M}}_{d\varepsilon f}^T\hat{\bar{M}}_{d\varepsilon f} + \bar{M}_{d\varepsilon f}^T\bar{M}_{d\varepsilon f}\right. \\ & \left. + \left(\sum_{i=1}^{3}|\dot{x}_2||\dot{x}_1|\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)^T\left(\sum_{i=1}^{3}|\dot{x}_2||\dot{x}_1|\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right)\right) \leq \mu_0\end{aligned}$$

where $\mu_0 > 0 \in R$ is a design parameter, and since

$$\gamma_k^2 = \sum_{i=1}^{n-1}\lambda_i^2 e_{k,i+1}^2 \leq \bar{\lambda}^2\sigma_k^2\Big/\underline{\lambda}^2$$

where $\bar{\lambda} = \max\{\lambda_1, \ldots, \lambda_n\}$, $\underline{\lambda} = \min\{\lambda_1, \ldots, \lambda_n\}$, one has

$$\gamma^T\gamma = \sum_{k=1}^{N}\gamma_k^2 \leq \sum_{k=1}^{N}\frac{\bar{\lambda}^2}{\underline{\lambda}^2}\sigma_k^2 = \frac{\bar{\lambda}^2}{\underline{\lambda}^2}\sum_{k=1}^{N}\sigma_k^2 = \frac{\bar{\lambda}^2}{\underline{\lambda}^2}\sigma^T\sigma$$

So one has

$$\begin{aligned}\dot{V}_s \leq & -c\sigma^T P(L+B)\sigma + \sigma^T P(D+B)\tilde{\theta}^T\xi + \sigma^T P(D+B)\mathrm{sgn}\left(\sigma^T P(D+B)\right)\tilde{\bar{M}}_{d\varepsilon f} \\ & + \sigma^T P(D+B)\left(|\dot{x}_2||\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i} + |\dot{x}_1|\tilde{\gamma}_{g_4}\right) + \left(4r + \bar{\lambda}^2\big/\left(4r\bar{\lambda}\right)\right)\bar{s}(P)\bar{s}(A)\sigma^T\sigma + \mu_0\end{aligned}$$

Define

$$V_0 = tr\left\{\tilde{\theta}^T\tilde{\theta}\right\}\Big/(2\eta_1) + \tilde{\bar{M}}_{d\varepsilon f}^T\tilde{\bar{M}}_{d\varepsilon f}\Big/(2\eta_2) + \frac{\sum_{i=1}^{3}\tilde{\gamma}_{g_i}^T\tilde{\gamma}_{g_i}}{2\eta_3} + \frac{\tilde{\gamma}_{g_4}^T\tilde{\gamma}_{g_4}}{2\eta_4}$$

where $\eta_i > 0 \in R$, $i = 1, 2, 3, 4$ are design parameters.

Define $V = V_s + V_0$, differentiating V with respect to time t, one has

$$\dot{V} \leq -c\sigma^T P(L+B)\sigma + \sigma^T P(D+B)\tilde{\theta}^T\xi + \sigma^T P(D+B)\mathrm{sgn}\left(\sigma^T P(D+B)\right)\tilde{\bar{M}}_{d\varepsilon f}$$

$$+\sigma^T P(D+B)\left(|\dot{x}_2|\,|\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i}+|\dot{x}_1|\,\tilde{\gamma}_{g_4}\right)+\left(4r+\bar{\lambda}^2\big/\left(4r\bar{\lambda}\right)\right)\bar{s}(P)\bar{s}(A)\sigma^T\sigma+\mu_0$$

$$-tr\left\{\tilde{\theta}^T\dot{\hat{\theta}}\right\}\Big/\eta_1-\tilde{\bar{M}}_{d\varepsilon f}^T\dot{\hat{\bar{M}}}_{d\varepsilon f}\Big/\eta_2-\frac{\sum_{i=1}^{3}\tilde{\gamma}_{g_i}^T\dot{\hat{\gamma}}_{g_i}}{\eta_3}-\frac{\tilde{\gamma}_{g_4}^T\dot{\hat{\gamma}}_{g_4}}{\eta_4} \tag{13}$$

Define the adaptive laws as follows:

$$\dot{\hat{\theta}}=\begin{cases}\eta_1\xi P(D+B)\sigma+\eta_0\hat{\theta}, & \left\|\hat{\theta}\right\|<M_\theta \text{ or } \left\|\hat{\theta}\right\|=M_\theta \text{ and } \sigma\hat{\theta}^T\xi(x)\le 0\\ \eta_1\xi P(D+B)\sigma+\eta_0\hat{\theta}-\left[\eta_1\xi P(D+B)\sigma+\eta_0\hat{\theta}\right]\dfrac{\hat{\theta}\hat{\theta}^T}{\left\|\hat{\theta}\right\|^2}, & \left\|\hat{\theta}\right\|=M_\theta \text{ and } \sigma\hat{\theta}^T\xi(x)>0\end{cases} \tag{14}$$

$$\dot{\hat{\bar{M}}}_{d\varepsilon f}=\begin{cases}\eta_2 P(D+B)\mathrm{sgn}\left(\sigma^T P(D+B)\right)\sigma+\eta_M\hat{\bar{M}}_{d\varepsilon f}, & \left\|\hat{\theta}\right\|<M_\theta \text{ or } \left\|\hat{\theta}\right\|=M_\theta \text{ and } \sigma\hat{\theta}^T\xi(x)\le 0\\ \eta_2 P(D+B)\mathrm{sgn}\left(\sigma^T P(D+B)\right)\sigma+\eta_M\hat{\bar{M}}_{d\varepsilon f} & \\ -\left[\eta_2 P(D+B)\mathrm{sgn}\left(\sigma^T P(D+B)\right)\sigma+\eta_M\hat{\bar{M}}_{d\varepsilon f}\right]\dfrac{\hat{\theta}\hat{\theta}^T}{\left\|\hat{\theta}\right\|^2}, & \left\|\hat{\theta}\right\|=M_\theta \text{ and } \sigma\hat{\theta}^T\xi(x)>0\end{cases} \tag{15}$$

$$\dot{\hat{\gamma}}_{g_i}=\eta_3\sigma^T P(D+B)\,|\dot{x}_2|^T\,|\dot{x}_1|^T,\quad i=1,2,3,\quad \dot{\hat{\gamma}}_{g_4}=\eta_4\sigma^T P(D+B)\,|\dot{x}_1|^T \tag{16}$$

where $\eta_0>0\in R$, $\eta_M>0\in R$ are design parameters, and $\hat{\theta}$, $\hat{\bar{M}}_{d\varepsilon}$ are bounded.

Proof: Substituting the adaptive laws into (13), as a result

$$\begin{aligned}\dot{V}\le&-c\sigma^T P(L+B)\sigma+\sigma^T P(D+B)\left(|\dot{x}_2|\,|\dot{x}_1|\sum_{i=1}^{3}\tilde{\gamma}_{g_i}+|\dot{x}_1|\,\tilde{\gamma}_{g_4}\right)\\&+\left(4r+\bar{\lambda}^2\big/\left(4r\bar{\lambda}\right)\right)\bar{s}(P)\bar{s}(A)\sigma^T\sigma+\mu_0-tr\left\{\tilde{\theta}^T\dot{\hat{\theta}}\right\}\Big/\eta_1-\tilde{\bar{M}}_{d\varepsilon f}^T\dot{\hat{\bar{M}}}_{d\varepsilon f}\Big/\eta_2\\&-\sum_{i=1}^{3}\tilde{\gamma}_{g_i}\frac{\eta_3\sigma^T P(D+B)\,|\dot{x}_2|\,|\dot{x}_1|}{\eta_3}-\tilde{\gamma}_{g_4}\frac{\eta_4\sigma^T P(D+B)\,|\dot{x}_1|}{\eta_4}\\\le&-c\sigma^T P(L+B)\sigma+\left(4r+\bar{\lambda}^2\big/\left(4r\bar{\lambda}\right)\right)\bar{s}(P)\bar{s}(A)\sigma^T\sigma+\mu_0-tr\left\{\tilde{\theta}^T\hat{\theta}\right\}\eta_0\Big/\eta_1\\&-\tilde{\bar{M}}_{d\varepsilon f}^T\hat{\bar{M}}_{d\varepsilon f}\eta_M/\eta_2\end{aligned}$$

Because

$$-tr\left\{\tilde{\theta}^T\hat{\theta}\right\}\eta_0\Big/\eta_1\le\frac{tr\left\{\theta^{*T}\theta^*\right\}\eta_0-tr\left\{\tilde{\theta}^T\tilde{\theta}\right\}\eta_0}{2\eta_1},$$

$$-\tilde{\bar{M}}_{d\varepsilon f}^T\hat{\bar{M}}_{d\varepsilon f}\eta_M\Big/\eta_2\le\frac{\eta_M\left(\bar{M}_{d\varepsilon f}^T\bar{M}_{d\varepsilon f}-\tilde{\bar{M}}_{d\varepsilon f}^T\tilde{\bar{M}}_{d\varepsilon f}\right)}{\eta_2}$$

one has

$$\begin{aligned}\dot{V}\le&\frac{-c\sigma^T P(L+B)\sigma}{2}+\left(4r+\bar{\lambda}^2\big/\left(4r\bar{\lambda}\right)\right)\bar{s}(P)\bar{s}(A)\sigma^T\sigma+\mu_0\\&+\frac{tr\left\{\theta^{*T}\theta^*\right\}\eta_0-tr\left\{\tilde{\theta}^T\tilde{\theta}\right\}\eta_0}{2\eta_1}+\frac{\eta_M\left(\bar{M}_{d\varepsilon f}^T\bar{M}_{d\varepsilon f}-\tilde{\bar{M}}_{d\varepsilon f}^T\tilde{\bar{M}}_{d\varepsilon f}\right)}{\eta_2}\end{aligned}$$

We know θ^*, $\bar{M}_{d\varepsilon f}$ are bounded. So, if η_0, η_1, η_M, η_2, η_3 and η_4 are chosen appropriately, then $\frac{tr\left\{\theta^{*T}\theta^*\right\}\eta_0}{2\eta_1}+\frac{\eta_M\left(\bar{M}_{d\varepsilon f}^T\bar{M}_{d\varepsilon f}\right)}{2\eta_2}\le\mu_1$, where $\mu_1>0\in R$ is a design parameter.

4. **Conclusions.** In the paper, for networked unknown nonlinear multi-agent systems with unknown parameterizable friction, a cooperative adaptive fuzzy tracking controller is proposed. By sliding mode control technique and the function approximation capability of fuzzy logic system, using the relative state information between each follower node and its neighbors, a cooperative adaptive fault fuzzy tracking control scheme is proposed such that the tracking errors are semi-globally uniform ultimate bounded.

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