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RELIABLE STABILIZATION OF A TWO-STAGE COMPENSATOR SYSTEM

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Abstract. *In this paper, we consider a SISO two-stage compensator system which satisfies reliable stabilization by a pair of stabilizing controllers $C_1(s)$ and $C_2(s)$ of the given plant $G(s)$. And the controllers must be designed to guarantee stability of the system, even if one of the controllers fails. Finding controllers which together stabilize the system can be regarded as the reliable stabilization problem. This problem is often considered when designing such as power plants and chemical plants which need redundant stability from the idea of fail-safe designing. And due to the remarkable progress of motorization and integration technologies, control systems become more complicated, and the demand of reliability is increasing. The purpose of this paper is to give a solution to design stabilizing controllers of the two-stage compensator system which has two controllers that one of them has the factor of the other one. And we give a part of solution to the reliable stabilization problem.*

Keywords: Reliable stabilization, Two-stage compensator system

1. Introduction.

In recent years, more remarkable progress of integration than the past few decades is bringing cheaper, lighter and smarter, devices and vehicles to our daily life. For a while, mechanical connections of those systems are getting less, and the demands of reliably controlled system are more increasing. Reliable control has some different approaches to make systems keep working or shut down safely against system failures. Passive redundancy is one of the approaches, and we consider a passive redundant system with a two-stage compensator system. Main idea of reliable stabilization is to design control systems under every possible system breakdown, in advance.

There are two methods to solve reliable control problems proposed. One of the methods is to give redundancy to systems by designing multiple stabilizing controllers for the systems [1, 2, 3]. The other one is based on the problem of integrity. In this method, the problem to be considered is about designing the pole of the closed-loop system so that it is designed in any range of stability. These methods are useful to stabilize the system if failures occur to the systems. In the former method, Šiljak proposed a mathematical framework for constructing reliable schemes by the redundant use of unreliable controllers. However, the general procedures for design of reliable control were not given. Vidyasagar and Viswanadham considered reliable stabilization using a multi-controller configuration. They considered multi-controller configurations with passive redundancy and active redundancy. And they gave a solution of reliable stabilization problem with factorization approach. As an extension of those studies Niemann and Stoustrup examined various types of faults models on reliable control problem [4]. And they introduced that dual Youla parameterization is useful to analyze how large faults can be tolerated

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without losing stability of the system. And various types of two-stage compensator systems are studied and proposed by Mori [5, 6, 7]. And parameterizations of all stabilizing controllers or stabilizable plants in feedback systems are proposed [8, 9, 10]. Their parameterization makes designing stable feedback systems easier. Yet, they have not given the reliable stability condition to SISO two-stage compensator systems that have one of the two controllers with the same factor.

In this paper, we examine the reliable stabilization problem of one of the two-stage compensator systems with passive redundancy and give the relationship of a pair of controllers together to stabilize the given plant of the system.

This paper is organized as follows. In Section 2, we show the problem statement. In Section 3, we describe the relationship between redundant stabilizing controllers. Section 4 gives conclusions.

Notations

$R(s)$ The set of real rational functions with s.
RH_∞ The set of stable proper real rational functions.

2. **Problem Statement.** The single-input, single-output (SISO) two-stage compensator system, which is shown in Figure 1, is used for the reliable stabilization problem considered here. Assuming this system needs to be reliably stabilized by two stabilizing controllers, in case that one of the controllers fails.

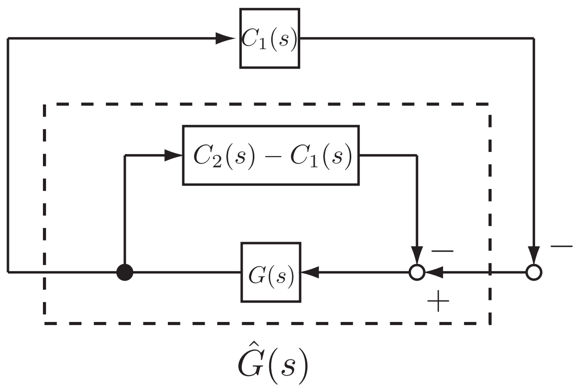

Figure 1. Block diagram of the two-stage compensator system

We define reliable stabilization of the system we considered as follows.

(A) The system is stable when the two stabilizing controllers are working at the same time.

(B) The system is stable when one of the stabilizing controllers is working.

(B-1) The system is stabilized by only a controller installed in the inner-loop.

(B-2) The system is stabilized by only a controller installed in the outer-loop.

In this system, $G(s) \in RH_\infty(s)$ represents the transfer function of the given stable plant, and $C_1(s) \in R(s)$ and $C_2(s) \in RH_\infty(s)$ represent the transfer functions of stabilizing controllers of the plant.

The problem considered in this paper is to clarify the relationship of the stabilizing controllers of the given plant in the system.

3. **Main Results.** In this section, as we discuss in Section 2, we consider necessary and sufficient conditions of which the modified two-stage compensator system achieves reliable stability.

Here, we assume that $\hat{C}_1(s) = C_2(s) - C_1(s)$, $\hat{C}_2(s) = C_2(s)$. Then, the two-stage compensator system in Figure 1 can be redrawn as Figure 2.

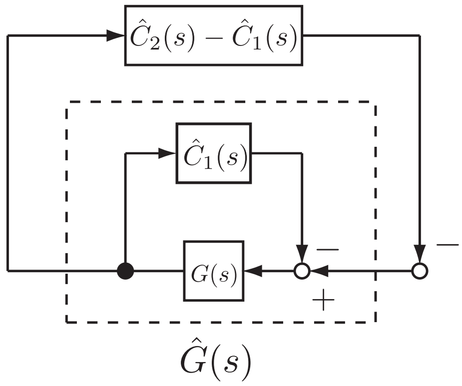

FIGURE 2. Redrawn block diagram of the two-stage compensator system

We will consider the modified two-stage compensator system in Figure 2. We firstly consider the necessary and sufficient condition which solves the problem (A) and (B-1). And we have the theorem as follows.

Theorem 3.1. *Stabilizing controllers, $\hat{C}_1(s)$ and $\hat{C}_2(s)$ together stabilize a plant $G(s)$ if and only if $\hat{C}_2(s) - \hat{C}_1(s)$ stabilizes*

$$\hat{G}(s) = \frac{G(s)}{1 + \hat{C}_1(s)G(s)} \in RH_\infty, \tag{1}$$

where $\hat{C}_1(s) = C_2(s) - C_1(s)$, $\hat{C}_2(s) = C_2(s)$.

Proof: First, the necessity is shown. That is, we show that if $\hat{C}_1(s)$ and $\hat{C}_2(s)$ together stabilize a plant $G(s)$, $\hat{C}_2(s) - \hat{C}_1(s)$ stabilizes $\hat{G}(s)$ in (1). From the assumption that $\hat{C}_1(s)$ and $\hat{C}_2(s)$ together stabilize $G(s)$, we can obtain both stabilizing controllers by Youla parameterization. And they can be written as $\hat{C}_1(s) = Q_1(s)/(1-Q_1(s)G(s))$ and $\hat{C}_2(s) = Q_2(s)/(1 - Q_2(s)G(s))$, where $Q_1(s), Q_2(s) \in RH_\infty$. Thus, all the transfer functions of $\left(G, \hat{C}_1\right)$ and $\left(G, \hat{C}_2\right)$ can be written as, $1\Big/\left(1 + \hat{C}_1(s)G(s)\right)$, $\hat{C}_1(s)\Big/\left(1 + \hat{C}_1(s)G(s)\right)$, $G(s)\Big/\left(1 + \hat{C}_1(s)G(s)\right)$, $\hat{C}_1(s)G(s)\Big/\left(1 + \hat{C}_1(s)G(s)\right)$, $1\Big/\left(1 + \hat{C}_2(s)G(s)\right)$, $\hat{C}_2(s)\Big/\left(1 + \hat{C}_2(s)G(s)\right)$, $G(s)\Big/\left(1 + \hat{C}_2(s)G(s)\right)$ and $\hat{C}_2(s)G(s)\Big/\left(1 + G(s)\hat{C}_2(s)\right)$, and they are stable.

All the transfer functions of $\left(G, \hat{C}_2 - \hat{C}_1\right)$ can be written as follows.

$$\frac{1}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)} = \left(1 + \hat{C}_1(s)G(s)\right)(1 - Q_2(s)G(s)), \tag{2}$$

$$\frac{\hat{G}(s)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)} = G(s)(1 - Q_2(s)G(s)), \tag{3}$$

$$\frac{\hat{C}_2(s) - \hat{C}_1(s)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)} = Q_2(s)(1 + C_1(s)G(s)) - C_1(s)(1 + C_1(s)G(s))(1 - Q_2(s)G(s)) \tag{4}$$

and

$$\frac{\left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)} = Q_2(s)G(s) - C_1(s)G(s)(1 - Q_2(s)G(s)). \tag{5}$$

From the assumption that $\hat{C}_1(s)$ and $\hat{C}_2(s)$ together stabilize a plant $G(s)$ and manipulation results of (2), (3), (4) and (5), we prove all the transfer functions of $\left(G, \hat{C}_2 - \hat{C}_1\right)$ are stable. Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, we show that if $\hat{C}_2(s) - \hat{C}_1(s)$ stabilizes $\hat{G}(s)$, then $\hat{C}_1(s)$ and $\hat{C}_2(s)$ stabilize $G(s)$. From the assumption that $\hat{C}_2(s) - \hat{C}_1(s)$ stabilizes $\hat{G}(s)$, all the transfer functions of $\left(\hat{G}(s), \hat{C}_2(s) - \hat{C}_1(s)\right)$ is stable. So, $1\Big/\left(1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)\right) \in RH_\infty$, $\left(\hat{C}_2(s) - \hat{C}_1(s)\right)\Big/\left(1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)\right) \in RH_\infty$, $\hat{G}(s)\Big/\left(1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)\right) \in RH_\infty$ and $\left(\left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)\right)\Big/\left(1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)\right) \in RH_\infty$ hold. And from Equation (1), we obtain a transfer function of the plant $G(s)$ as follows,

$$G(s) = \frac{\hat{G}(s)}{1 - \hat{C}_1(s)\hat{G}(s)}. \tag{6}$$

Thus from (6) and with simple manipulation, we have all transfer functions of $\left(G, \hat{C}_1\right)$ and $\left(G, \hat{C}_2\right)$ as follows,

$$\frac{1}{1 + \hat{C}_1(s)G(s)} = 1 - \hat{C}_1(s)\hat{G}(s), \tag{7}$$

$$\frac{G(s)}{1 + \hat{C}_1(s)G(s)} = \hat{G}(s), \tag{8}$$

$$\frac{\hat{C}_1(s)}{1 + \hat{C}_1(s)G(s)} = \hat{C}_1(s)\left(1 - \hat{C}_1(s)\hat{G}(s)\right), \tag{9}$$

$$\frac{\hat{C}_1(s)G(s)}{1 + \hat{C}_1(s)G(s)} = \hat{C}_1(s)\hat{G}, \tag{10}$$

$$\frac{1}{1 + \hat{C}_2(s)G(s)} = \frac{1 - \hat{C}_1(s)\hat{G}(s)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)}, \tag{11}$$

$$\frac{G(s)}{1 + \hat{C}_2(s)G(s)} = \frac{\hat{G}(s)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)}, \tag{12}$$

$$\frac{\hat{C}_2(s)}{1 + \hat{C}_2(s)G(s)} = \frac{\hat{C}_2(s)\left(1 - \hat{C}_1(s)\hat{G}(s)\right)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)}, \tag{13}$$

and

$$\frac{\hat{C}_2(s)G(s)}{1 + \hat{C}_2(s)G(s)} = \frac{\hat{C}_2(s)\hat{G}(s)}{1 + \left(\hat{C}_2(s) - \hat{C}_1(s)\right)\hat{G}(s)}. \tag{14}$$

Since $\hat{C}_1(s) \in RH_\infty$, $\hat{G}(s) \in RH_\infty$ and the assumption that $\hat{C}_2(s) - \hat{C}_1(s)$ stabilizes $\hat{G}(s)$, all transfer functions in (7)-(14) are stable. From the above, the sufficiency has been shown.

We have thus proved Theorem 3.1. □

Next, we will present the main results.

Theorem 3.2. *$\hat{C}_1(s)$ and $\hat{C}_2(s)$ together stabilize a plant $G(s)$ if and only if $\hat{C}_2(s)-\hat{C}_1(s)$ is written by the form in*

$$\hat{C}_2(s)-\hat{C}_1(s)=C_1(s)=\frac{Q(s)}{1-Q(s)\hat{Q}(s)}, \tag{15}$$

where $Q(s)\in RH_\infty$ and $\hat{Q}(s)\in RH_\infty$ are any functions.

Proof: First, the necessity is shown. That is, we show that if $\hat{C}_1(s)$ and $\hat{C}_2(s)$ together stabilize $G(s)$, $\hat{C}_2(s)-\hat{C}_1(s)$ takes the form in (15). From Theorem 3.1, this is equivalent to that if $\hat{C}_2(s)-\hat{C}_1(s)$ stabilizes $\hat{G}(s)$ in (1), $\hat{C}_2(s)-\hat{C}_1(s)$ takes the form in (15). From the assumption that $\hat{C}_2(s)-\hat{C}_1(s)$ stabilizes $\hat{G}(s)$ in (1), all of the transfer functions can be written as, $1\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$, $\hat{G}(s)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}\right)$, $\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$ and $\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$, and these belong to RH_∞.

Therefore, using $Q(s)\in RH_\infty$, $\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$ can be rewritten as

$$\frac{\hat{C}_2(s)-\hat{C}_1(s)}{1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)}=Q(s). \tag{16}$$

From simple manipulation, we have

$$\hat{C}_2(s)-\hat{C}_1(s)=\frac{Q(s)}{1-Q(s)\hat{G}(s)}. \tag{17}$$

Since $\hat{G}(s)$ is stable, using $\hat{Q}(s)\in RH_\infty$ and let $\hat{G}(s)=\hat{Q}(s)$, (16) is rewritten as

$$\hat{C}_2(s)-\hat{C}_1(s)=\frac{Q(s)}{1-Q(s)\hat{Q}(s)}. \tag{18}$$

Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, if $\hat{C}_2(s)-\hat{C}_1(s)$ takes the form in (15), $\hat{C}_2(s)-\hat{C}_1(s)$ makes $\hat{G}(s)$ stable. We set $\hat{G}(s)$ as

$$\hat{G}(s)=\hat{Q}(s), \tag{19}$$

where $\hat{Q}(s)\in RH_\infty$. Then transfer functions, $1\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$, $\hat{G}(s)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}\right)$, $\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$ and $\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\Big/\left(1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)\right)$ are rewritten as

$$\frac{1}{1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)}=1-Q(s)\hat{Q}(s), \tag{20}$$

$$\frac{\hat{C}_2(s)-\hat{C}_1(s)}{1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)}=Q(s), \tag{21}$$

$$\frac{\hat{G}(s)}{1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)}=\left(1-Q(s)\hat{Q}(s)\right)\hat{Q}(s), \tag{22}$$

$$\frac{\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)}{1+\left(\hat{C}_2(s)-\hat{C}_1(s)\right)\hat{G}(s)}=Q(s)\hat{Q}(s). \tag{23}$$

Since $Q(s) \in RH_\infty$ and $\hat{Q}(s) \in RH_\infty$, (20), (21), (22) and (23) are stable. Thus, the sufficiency has been shown.

Therefore, we have proved Theorem 3.2. □

4. **Conclusions.** In this paper, we describe the necessary and sufficient condition of the problem (A) and (B-1) and the relationship between stabilizing controllers of the two-stage compensator system we considered. Given a pair of stabilizing controllers $\hat{C}_1(s)$ and $\hat{C}_2(s)$, thus $C_1(s)$ and $C_2(s) - C_1(s)$ have the relationship satisfying (15). Yet, we have not given the necessary and sufficient condition to achieve (B-2). We will clarify the condition and controller parameterization in another article.

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