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ADAPTIVE FAULT TOLERANT CONTROL FOR A CLASS OF HIGH-ORDER NONLINEAR SYSTEMS

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Abstract. *In this paper, the fault tolerant control problem is investigated for a class of high-order nonlinear systems. By using the approximation capability of the fuzzy logic system, an adaptive fault tolerant stability control scheme is proposed. Based on Lyapunov stability theory, all of the signals in the closed-loop system are proved to be semi-globally uniformly ultimately bounded. The simulation results demonstrate the effectiveness of the approach.*

Keywords: Fault tolerant control, High-order nonlinear systems, Adaptive control

1. **Introduction.** Adaptive stabilizing control design of nonlinear systems is always one of hot issues in current control theory research. Compared with linear systems, the nonlinear systems are more accurate in describing the practical systems. However, the research in nonlinear systems is more complex. The stabilization problem of high-order nonlinear systems has received a lot of attention. The stability control or output tracking problem of a class of high-order nonlinear systems has been studied [1-10]. In [4], a continuous feedback method was proposed to solve the global strong stability problem of a nonlinear system which may be unstable even locally. In [5], the output-feedback stabilization problem for a class of stochastic high-order nonlinear systems was investigated. In [6], the generalization of the homogeneous idea in the global adaptive stabilization of high-order uncertain nonlinear systems was improved. In [7], the finite-time stabilization problem for a class of high-order uncertain nonlinear systems was researched. In [10], the problem of global asymptotic stabilization control design was solved for a class of high-order uncertain nonlinear systems. It is well-known that the control problem of high-order systems is more complex. Recently, high-order systems have been investigated, and many results have been obtained [1-10]. However, the faults occurring in the controlled systems are not considered in the above-mentioned results [4-7,10]. In fact, actuator faults often occur in the practical applications, which will cause system performance deterioration and lead to instability that can further produce catastrophic accidents.

In the past decades, fault tolerant control (FTC) has attracted wide attention, and abundant results are achieved in [11-17]. In [15], a new sensor fault model was designed to deal with practical problem in sensor fault diagnosis and estimation of near-space hypersonic vehicle. In [16], the problem of fault-tolerant dynamic surface control was discussed for a class of uncertain nonlinear systems with actuator faults, and an active fault-tolerant control scheme was proposed. In [17], the problem of the time delay due to fault diagnosis on system performance was investigated, and a new fault diagnosis algorithm was proposed to avoid the negative effects. However, the above studies only consider low-order systems and do not consider high-order systems.

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In this paper, an adaptive fault-tolerant stability control scheme is proposed for a class of high-order nonlinear systems with actuator faults. The approximation capability of fuzzy logic system is utilized to approximate the unknown function. The following contributions in this paper should be emphasized: 1) the existing fault-tolerant control results are extended to uncertain high-order nonlinear systems, and a fault-tolerant control scheme based on fuzzy logic systems is proposed; 2) actuator bias faults are considered and compensated.

2. Problem Statement and Preliminaries.

Consider the following high-order nonlinear systems

$$\begin{cases} \dot{x} = f(x) + g(x)u^p \\ y = x \end{cases} \tag{1}$$

where $x \in R$, $u \in R$, $y \in R$ denote the state, control input and output, respectively; $f(x) \in R$ is assumed to be an unknown continuous function; $g(x) \in R$ is the control gain function, which is unknown but bounded; $p > 1 \in N$ is an odd positive integer.

In practical applications, actuator may become faulty. The actuator fault considered in this paper is bias fault and its model can be described as

$$u_f = u + u_b, \quad t > t_f \tag{2}$$

where $u_b \in R$ denotes an unknown bounded signal, and t_f is unknown fault occurrence time.

The control objective is to design an adaptive fault tolerant control scheme such that the system (1) is asymptotically stable.

In order to design a stable adaptive fuzzy controller, the following assumptions are made for the system.

Assumption 2.1. *$g_{\max} \geq g(x) \geq g_{\min}$, $\forall x \in R$, where $g_{\max} \in R$ and $g_{\min} \in R$ are known positive constants.*

Assumption 2.2. *There exists a known positive constant b_0, such that $|u_b| \leq b_0$.*

Lemma 2.1. *Let c, d be positive real numbers and $r(x, y) > 0$ a real-valued function. Then,*

$$|x|^c|y|^d \leq \frac{c}{c+d} r(x, y)|x|^{c+d} + \frac{d}{c+d} r^{\frac{-c}{d}}(x, y)|y|^{c+d}$$

3. Fuzzy Logic Systems.

According to [18], a fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy If-then rules of the following form,

R^i: If x_1 is ${F^1}_1$ and x_2 is ${F^1}_2$ and … and x_n is ${F^1}_n$, then y is G^i, $i = 1, 2, \ldots, N$, where $x = [x_1, x_2, \ldots, x_n]^T$ and y are the input and output, respectively. ${F^i}_j$ and G^i denote fuzzy sets. N is the rule number.

The FLS can be described as

$$y(x) = \sum_{i=1}^{N} \theta_i \prod_{j=1}^{n} \mu {F^i}_j(x_j) \Bigg/ \sum_{i=1}^{N} \left[\prod_{j=1}^{n} \mu {F^i}_j(x_j) \right]$$

where $\theta_i = \max_{y \in R} \mu G^i(y)$. $\mu {F^i}_j(x_j)$ and $\mu G^i(y)$ are the membership functions.

Define the fuzzy basis functions as

$$\xi_i(x) = \prod_{j=1}^{n} \mu {F^i}_j(x_j) \Bigg/ \sum_{i=1}^{N} \left[\prod_{j=1}^{n} \mu {F^i}_j(x_j) \right], \quad i = 1, 2, \ldots, N$$

Let $\theta^T = [\theta_1, \theta_2, \ldots, \theta_N]^T$ and $\xi(x) = (\xi_1(x), \ldots, \xi_N(x))^T$, and then the FLS can be rewritten as

$$f(x,\theta) = \theta^T \xi(x)$$

Lemma 3.1. *Let $f(x)$ be a continuous function defined on a compact set U. Then for any positive constant ε_w, there exists an FLS*

$$\sup_{x \in U} \left| f(x) - \theta^{*T}\xi(x) \right| \le \varepsilon_w$$

4. **Control Design.** Consider the following Lyapunov function:

$$V_1 = \frac{1}{2}x^2 \tag{3}$$

By using Lemma 2.1, the time derivative of V_1 is

$$\begin{aligned} \dot{V}_1 &= x\dot{x} \\ &= x\left(f(x) + g(x)(u+u_b)^p\right) \\ &= xf(x) + xg(x)(u+u_b)^p \\ &= xf(x) + xg(x)\left(u^p + {u_b}^p + \sum_{i=1}^{p-1} C_p^i u^i {u_b}^{p-i}\right) \\ &\le xf(x) + xg(x)\left(u^p + {u_b}^p\right) + |x|g(x)\sum_{i=1}^{p-1} C_p^i \left|u^i\right| \left|{u_b}^{p-i}\right| \\ &\le xf(x) + xg(x)\left(u^p + {u_b}^p\right) + |x|g(x)\sum_{i=1}^{p-1} C_p^i \left(\frac{i}{p}|u|^p\gamma + \frac{p-i}{p}|u_b|^p\gamma^{\frac{-i}{p-i}}\right) \end{aligned} \tag{4}$$

where $\gamma > 0$ is a real-valued function, and $C_p^i = \frac{p!}{i!(p-i)}$.

Define the control law as

$$u = -\text{sgn}(x)u'$$

where $u' \ge 0$, and the specific definition will be given in (8).

From (4), we have

1) if $x \ge 0$, $u = -u' \le 0$, $|u|^p = u'^p$

$$\begin{aligned} \dot{V}_1 &\le xf(x) - xg(x)u'^p + xg(x){u_b}^p + xg(x)\sum_{i=1}^{p-1} C_p^i \frac{i}{p}u'^p\gamma + |x|g(x)\sum_{i=1}^{p-1} C_p^i \frac{p-i}{p}|u_b|^p\gamma^{\frac{-i}{p-i}} \\ &\le xf(x) - xg(x)\left(1 - \sum_{i=1}^{p-1} C_p^i \frac{i}{p}\gamma\right)u'^p + xg(x){u_b}^p + |x|g(x)\sum_{i=1}^{p-1} C_p^i \frac{p-i}{p}|u_b|^p\gamma^{\frac{-i}{p-i}} \end{aligned} \tag{5}$$

2) if $x < 0$, $u = u' \ge 0$, $|u|^p = u'^p$

$$\begin{aligned} \dot{V}_1 &\le xf(x) + xg(x)u'^p + xg(x){u_b}^p - xg(x)\sum_{i=1}^{p-1} C_p^i \frac{i}{p}u'^p\gamma + |x|g(x)\sum_{i=1}^{p-1} C_p^i \frac{p-i}{p}|u_b|^p\gamma^{\frac{-i}{p-i}} \\ &\le xf(x) + xg(x)\left(1 - \sum_{i=1}^{p-1} C_p^i \frac{i}{p}\gamma\right)u'^p + xg(x){u_b}^p + |x|g(x)\sum_{i=1}^{p-1} C_p^i \frac{p-i}{p}|u_b|^p\gamma^{\frac{-i}{p-i}} \end{aligned} \tag{6}$$

From the above discussion, one has

$$\dot{V}_1 \le xf(x) - |x|g(x)\left(1 - \sum_{i=1}^{p-1} C_p^i \frac{i}{p}\gamma\right)u'^p + xg(x){u_b}^p + |x|g(x)\sum_{i=1}^{p-1} C_p^i \frac{p-i}{p}|u_b|^p\gamma^{\frac{-i}{p-i}} \tag{7}$$

Define the control law and adaptive law as

$$u' = \left(\frac{1}{\rho_1 g_{\min}}\left(\left|f\left(x,\hat{\theta}\right)\right| + k|x| + \hat{\varepsilon}_w + \frac{\rho_1}{\rho_2}{b_0}^p\right)\right)^{\frac{1}{p}} \tag{8}$$

$$\dot{\hat{\theta}} = \begin{cases} \eta_1 x\xi(x), & \left\|\hat{\theta}\right\| < M_\theta \text{ or } \left\|\hat{\theta}\right\| = M_\theta \text{ and } x\hat{\theta}^T\xi(x) \le 0 \\ \eta_1 x\xi(x) - \eta_1 x\frac{\hat{\theta}\hat{\theta}^T}{\|\hat{\theta}\|^2}\xi(x), & \left\|\hat{\theta}\right\| = M_\theta \text{ and } x\hat{\theta}^T\xi(x) > 0 \end{cases} \tag{9}$$

$$\dot{\hat{\varepsilon}}_w = \eta_2|x| \tag{10}$$

where $\rho_1 = 1-\sum_{i=1}^{p-1} C_p^i \frac{i}{p}\gamma$, $\rho_2 = 1+\sum_{i=1}^{p-1} C_p^i \frac{p-i}{p}\gamma^{\frac{-i}{p-i}}$; $k > 0$ is a design parameter; $\hat{\theta}$ is the estimate of θ^*; $\hat{\varepsilon}_w$ is the estimate of ε_w; $\eta_1 > 0$ and $\eta_2 > 0$ are adaptive rates; $\tilde{\theta} = \theta^* - \hat{\theta}$, $\tilde{\varepsilon}_w = \varepsilon_w - \hat{\varepsilon}_w$.

5. **Stability Analysis.** For the system (1), using the above control law and adaptive law, the following theorem is proposed.

Theorem 5.1. *Considering the system (1) under Assumption 2.1 and Assumption 2.2, the control law (8) and adaptive laws (9) and (10), then the closed-loop system is asymptotically stable, with all signals being semi-globally uniformly ultimately bounded.*

Proof: Define the following Lyapunov function

$$V = V_1 + \frac{1}{2\eta_1}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2\eta_2}\tilde{\varepsilon}_w^2 \tag{11}$$

Differentiating V with time t, one has

$$\begin{aligned} \dot{V} &\le xf(x) - |x|g(x)\rho_1 u'^p + |x|g(x){b_0}^p\rho_2 - \frac{1}{\eta_1}\tilde{\theta}^T\dot{\hat{\theta}} - \frac{1}{\eta_2}\tilde{\varepsilon}_w\dot{\hat{\varepsilon}}_w \\ &\le x\tilde{\theta}^T\xi(x) + |x|\tilde{\varepsilon}_w + |x|\left|f\left(x,\hat{\theta}\right)\right| + |x|\hat{\varepsilon}_w - |x|g(x)\rho_1 u'^p + |x|g(x){b_0}^p\rho_2 \\ &\quad - \frac{1}{\eta_1}\tilde{\theta}^T\dot{\hat{\theta}} - \frac{1}{\eta_2}\tilde{\varepsilon}_w\dot{\hat{\varepsilon}}_w \\ &\le \tilde{\theta}^T\left(x\xi(x) - \frac{1}{\eta_1}\dot{\hat{\theta}}\right) + \tilde{\varepsilon}_w\left(|x| - \frac{1}{\eta_2}\dot{\hat{\varepsilon}}_w\right) + |x|\left|f\left(x,\hat{\theta}\right)\right| + |x|\hat{\varepsilon}_w \\ &\quad - |x|g(x)\rho_1 u'^p + |x|g(x){b_0}^p\rho_2 \end{aligned}$$

Substituting the control law (8) and adaptive laws (9) and (10) into the above equation, we have

$$\dot{V} \le -kx^2 + \tilde{\theta}^T\left(x\xi(x) - \frac{1}{\eta_1}\dot{\hat{\theta}}\right) + \tilde{\varepsilon}_w\left(x - \frac{1}{\eta_2}\dot{\hat{\varepsilon}}_w\right) \le -kx^2 \le 0 \tag{12}$$

Integrating (12) over $[0,t]$, we obtain $\int_0^t \dot{V}(\tau)d\tau \le -\int_0^t kx^2(\tau)d\tau$, i.e., $\int_0^t kx^2(\tau)d\tau \le V(0) - V(t)$. Then, $\int_0^t kx^2(\tau)d\tau \le V(0) - V(+\infty)$. Therefore, $x \in L_2$, $|x| \le \sqrt{2V(0)}$. Since $x \in L_2$, $x \in L_\infty$ and $\dot{x} \in L_\infty$, then, $\lim_{t\to\infty} x = 0$. Theorem 5.1 is proved.

6. **Simulation.** In order to verify the effectiveness of the proposed solution, consider the following nonlinear system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.2x_1 - 0.5x_2 - 5\sin x_2 - 2\sin x_1 + (-x_1 - 2x_2)u^3 \end{cases} \tag{13}$$

The initial states of the system are taken as $x_1(0) = 0.5$, $x_2(0) = 0$, $\hat{\varepsilon}(0) = 0$, $\hat{\theta}^T = (1, 0)$. The parameters are designed as $\eta_1 = 0.1$, $\eta_2 = 0.1$, $\rho_1 = 0.3$, $\rho_2 = 0.1$, $g_{\min} = 0.1$, $k = 0.1$. The simulation results are presented in Figures 1 and 2. From Figure 1, it can be seen to eventually reach zero and reach stability.

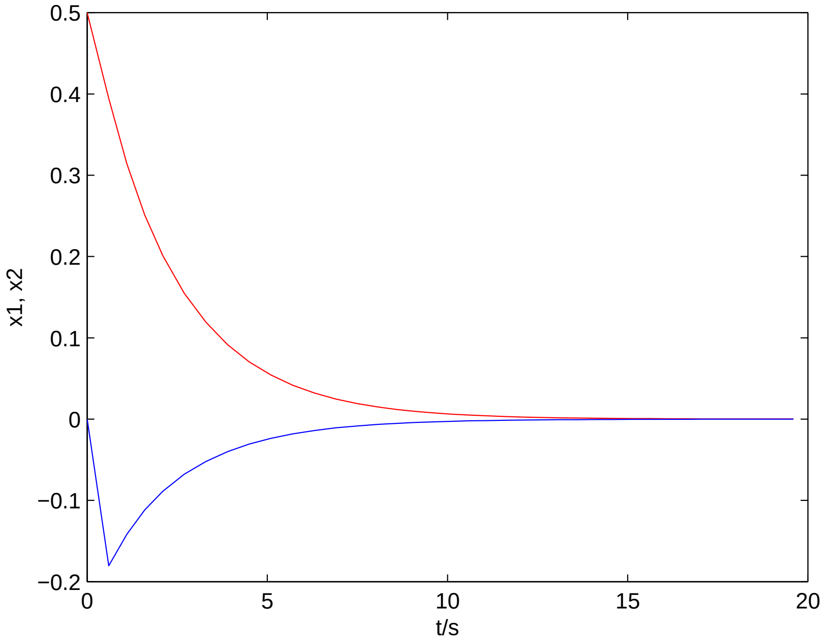

Figure 1. State of x_1, x_2

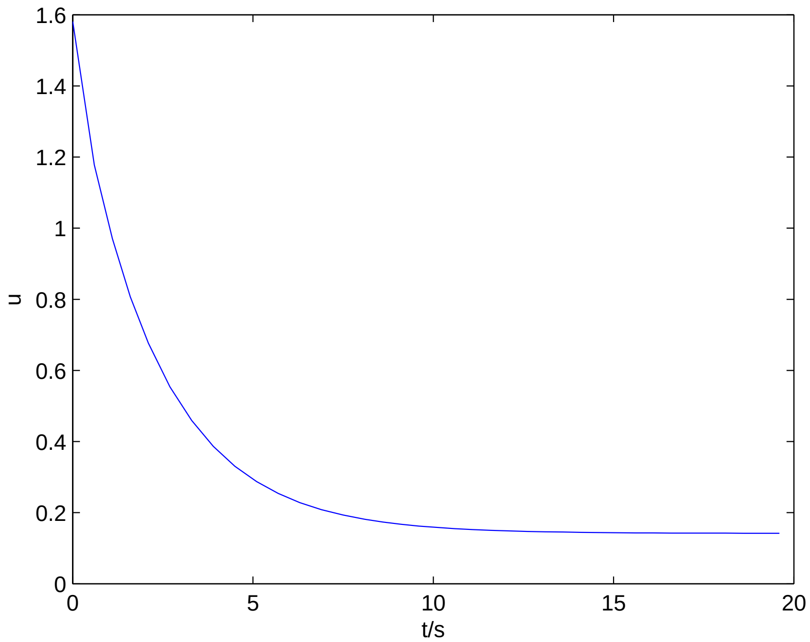

Figure 2. Control law u

The simulation shown in Figure 1 demonstrates the stability of the closed-loop system.

7. **Conclusions.** In this paper, for a class of high-order nonlinear systems with actuator faults, the fuzzy logic system approximation capability is used to approximate the unknown nonlinear term, and an adaptive fault tolerant control scheme is designed. This scheme can guarantee all signals in the closed-loop system semi-globally uniformly ultimately bounded. However, the system considered in this paper includes the first order integrators. Furthermore, it is well-known that many practical systems are modeled by such high-order systems, and how to deal with the control problem is very challenging and will be studied in our further research.

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