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RELATIVE POSITION COORDINATED CONTROL OF SPACECRAFT FORMATION BASED ON EVENT TRIGGERED TRANSMISSION

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Abstract. *This paper considers the position coordination control of distributed spacecraft formation system with event triggered mechanism. Firstly, a modified event triggered mechanism for multiple spacecraft system is introduced to alleviate communication pressure. Then, based on this event triggered mechanism, an adaptive liner sliding model control algorithm is employed to achieve spacecraft formation position coordination with bounded external disturbance and uncertain mass. Finally, the numerical simulation is presented to demonstrate the performance of the proposed event triggered mechanism and control algorithm.*

Keywords: Spacecraft formation, Relative position, Coordination control, Event triggered

1. **Introduction.** Recently, spacecraft formation flying (SFF) technology has attracted much attention due to its technical advantages over single spacecraft. During most SFF missions, not only the whole formation needs to track the expected translation and rotation, but also each spacecraft needs to translate and rotate in coordination with other spacecraft. To meet this demand, many effective approaches to achieving formation coordination have been proposed [1,2]. And coordination control algorithms considering external disturbances, uncertainties, velocity information lacking have also been proposed [3-7].

The above-mentioned control algorithms ignore communication limitations, which is impractical. Restricted by hardware, there are communication limitations of formation spacecraft. To solve this problem, a non-time-periodic sampling mechanism is proposed, called event triggered mechanism, which has been proved to be an effective way to reduce bandwidth occupancy [8,9]. In event triggered mechanism, only when the system states satisfy the preset conditions can communication be carried out, thus reducing the bandwidth occupation. Up to now, there are not many research results of event triggered control scheme of spacecraft. In [10,11], event triggered mechanism is introduced to spacecraft attitude control system. In [12], an event-triggered based adaptive decentralized attitude coordination control law is developed. It is worth noting that all these previous works concentrate on attitude control problem.

This paper focuses on event based relative position coordination control problem of distributed SFF system under undirected topology, and is organized as follows. In Section 1, the research background of this paper is briefly introduced. In Section 2, the relative position coordination problem of distributed SFF system with undirected communication topology is described. In Section 3, a modified event triggered mechanism is proposed, and an adaptive position coordination control algorithm based on event triggered mechanism is designed, considering the external disturbances and uncertainty of mass. In Section 4,

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the numerical simulation is presented to demonstrate the performance of the proposed algorithm. Finally, the conclusion of this paper is given in Section 5.

2. Problem Statement and Preliminaries.

In this section, some useful basic theories are introduced and the control problem is described.

2.1. Graph theory.

Generally, the communication topology of formation spacecraft can be expressed by graph theory. In graph $\boldsymbol{\Lambda} = (\boldsymbol{\upsilon}, \boldsymbol{\varsigma}, \boldsymbol{G})$, $\boldsymbol{\upsilon} = \{\upsilon_1, \ldots, \upsilon_n\}$ is a limited non-empty node set, $\boldsymbol{\varsigma} \subseteq \boldsymbol{\upsilon} \times \boldsymbol{\upsilon}$ is an edge set composed of ordered pairs of nodes and $\boldsymbol{G} = [g_{ij}] \in \boldsymbol{R}^{n\times n}$ is a weight adjacency matrix. If node υ_i can get the information of node υ_j, it can be denoted as $(\upsilon_i, \upsilon_j) \in \boldsymbol{\varsigma}$, and the corresponding element in the weighted adjacency matrix satisfies $g_{ij} > 0$. In undirected graph, there are $g_{ij} = g_{ji} > 0$ while node υ_i and node υ_j are connected, and $g_{ii} = 0$.

Assumption 2.1. *The communication topology of the distributed SFF system in this paper is undirected and connected.*

2.2. Zeno behavior.

In an event triggered system, if there are countless triggering events in a limited time interval, then there is Zeno behavior in the system. Apparently, it is hard to save communication resources if Zeno behavior exists. However, Zeno behavior can be avoided by offering reasonable trigger criteria. And it can be proved that there is no Zeno behavior, if the lower bound of interval between triggered events exists and is greater than zero.

2.3. Relative position dynamics of SFF.

The nonlinear relative position dynamics of distributed SFF system is introduced in this subsection. Figure 1 shows the relative motion schematic of SFF system, in which S_0 is the leader spacecraft and S_i $(i = 1, 2, \ldots, n)$ is the ith following spacecraft, *C-OXYZ* is the geocentric inertial coordinate frame, and *C-oxyz* is the orbit coordinate frame of the leader spacecraft. It is worth noting that S_0 is a virtual leader spacecraft for distributed SFF system. In the orbital coordinate frame *C-oxyz*, the nonlinear relative dynamics equation of the ith spacecraft relative to the virtual leader spacecraft can be expressed as

$$\dot{\boldsymbol{\rho}}_i = \boldsymbol{v}_i \tag{1}$$

$$m_i\dot{\boldsymbol{v}}_i = \boldsymbol{C}_i\left(\dot{\theta}\right)\boldsymbol{v}_i + \boldsymbol{D}_i\left(\dot{\theta}, \ddot{\theta}, \boldsymbol{r}_i\right)\boldsymbol{\rho}_i + \boldsymbol{N}_i\left(\boldsymbol{r}_i, \boldsymbol{r}_0\right) + \boldsymbol{f}_{di} + \boldsymbol{f}_{ai} \tag{2}$$

$$\boldsymbol{C}_i\left(\dot{\theta}\right) = 2m_i\dot{\theta}\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

$$\boldsymbol{D}_i\left(\dot{\theta}, \ddot{\theta}, \boldsymbol{r}_i\right) = -m_i\frac{\mu}{r_i^3}\boldsymbol{I}_3 + m_i\begin{bmatrix} \dot{\theta}^2 & \ddot{\theta} & 0 \\ -\ddot{\theta} & \dot{\theta}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{N}_i(\boldsymbol{r}_i, \boldsymbol{r}_0) = \mu m_i\begin{bmatrix} -\dfrac{\boldsymbol{r}_0}{\boldsymbol{r}_i} + \dfrac{1}{\boldsymbol{r}_0^2} & 0 & 0 \end{bmatrix}^T \tag{5}$$

where $\boldsymbol{r}_0$ is the position vector from the center of earth to the center of the virtual leader spacecraft, $\boldsymbol{r}_i$ is the position vector from earth to the ith spacecraft, $\boldsymbol{\rho}_i = \boldsymbol{r}_i - \boldsymbol{r}_0$ is the relative position vector between the ith spacecraft and the virtual leader spacecraft, μ is the gravitational constant of earth, m_i is the mass of the ith spacecraft, θ is the angular velocity of the virtual leader spacecraft, and $\boldsymbol{f}_{di}$, $\boldsymbol{f}_{ai}$ are the external disturbance and control force of the ith spacecraft.

Let $\boldsymbol{\rho}_i^d$ be the expecting position vector of the ith spacecraft, and $\boldsymbol{v}_i^d = \dot{\boldsymbol{\rho}}_i^d$ be the expecting velocity vector of the ith spacecraft. The position and velocity tracking errors of

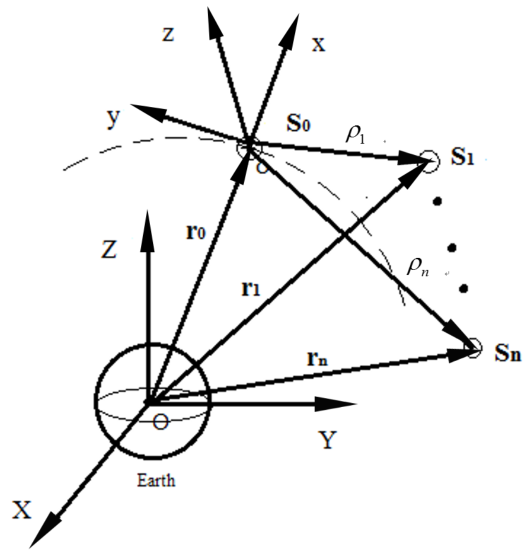

Figure 1. Schematic representation of the SFF system

the ith spacecraft to the expecting trajectory are defined as $\boldsymbol{e}_{\rho i} = \boldsymbol{\rho}_i - \boldsymbol{\rho}_i^d$ and $\boldsymbol{e}_{vi} = \boldsymbol{v}_i - \boldsymbol{v}_i^d$, and then we can obtain the relative error dynamics equation of the ith spacecraft

$$\dot{\boldsymbol{e}}_{\rho i} = \boldsymbol{e}_{vi} \tag{6}$$

$$m_i \dot{\boldsymbol{e}}_{vi} = \boldsymbol{C}_i\left(\dot{\theta}\right)\boldsymbol{v}_i + \boldsymbol{D}_i\left(\dot{\theta}, \ddot{\theta}, \boldsymbol{r}_i\right)\boldsymbol{\rho}_i + \boldsymbol{N}_i(\boldsymbol{r}_i, \boldsymbol{r}_0) - m_i \dot{\boldsymbol{v}}_i^d + \boldsymbol{f}_{di} + \boldsymbol{f}_{ai} \tag{7}$$

2.4. **Problem formulation.** The intention of this paper is to design a relative position coordination controller of distributed SFF system based on event triggered mechanism, which means we need to ensure that $\boldsymbol{e}_{\rho i} \to \boldsymbol{0^p}$, $\boldsymbol{e}_{vi} \to \boldsymbol{0^p}$ and $\boldsymbol{e}_{\rho i} \to \boldsymbol{e}_{\rho j}$, $\boldsymbol{e}_{vi} \to \boldsymbol{e}_{vj}$, as $t \to \infty$.

Assumption 2.2. *Assume that the external disturbance is upper bounded, which satisfies $k \le d_i$, where $d_i \ge 0$ is a constant. And there are*

$$\frac{d}{dt} p_i^2 = -3\alpha k p_i \tag{8}$$

Lemma 2.1. *Supposing x_i $(i = 1, 2, \ldots, n)$, $q \in (0, 1)$ are all positive constants, then the following inequalities hold*

$$\sum_{i=1}^{n} |x_i|^{1+q} \ge \left(\sum_{i=1}^{n} |x_i|^2\right)^{\frac{1+q}{2}} \tag{9}$$

$$(|x_1| + \cdots + |x_n|)^q \le |x_1|^q + \cdots + |x_n|^q \tag{10}$$

Lemma 2.2. *If x is an arbitrary real number and y is a non-zero real number. For x and y, the following inequalities hold*

$$0 \le |x|\left(1 - \tanh(|x/y|)\right) \le \alpha |y| \tag{11}$$

where $\alpha > 0$. Let $\alpha^* = x^*\left(1 - \tanh x^*\right)$ be the minimum value of α and x^* satisfies $e^{-2x^*} + 1 - 2x^* = 0$.

3. **Main Results.** In this section, an event trigger based adaptive control algorithm is proposed to achieve relative position coordination with external disturbances and mass uncertainties.

First, a sliding surface is defined as

$$\boldsymbol{s}_i = \gamma_i \boldsymbol{e}_{\rho i} + \boldsymbol{e}_{vi} \tag{12}$$

Suppose the mass of the ith spacecraft is unknown, and the estimated value is represented by $\hat{m}_i$. $\tilde{m}_i = m_i - \hat{m}_i$ is the mass estimation error, where

$$\dot{\hat{m}}_i = \lambda_i \boldsymbol{s}_i^T \boldsymbol{W}_i \tag{13}$$

$$\boldsymbol{W}_i = \boldsymbol{C}_i\left(\dot{\theta}\right)\boldsymbol{v}_i + \boldsymbol{D}_i\left(\dot{\theta}, \ddot{\theta}, \boldsymbol{r}_i\right)\boldsymbol{\rho}_i + \boldsymbol{N}_i(\boldsymbol{r}_i, \boldsymbol{r}_0) - m_i \dot{\boldsymbol{v}}_i^d + m_i \gamma_i \boldsymbol{e}_{vi} \tag{14}$$

Then a static threshold was introduced into traditional event update rule

$$\boldsymbol{e}_i(t) = \boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_i(t) \tag{15}$$

$$t_{k_i+1}^i = \min\left\{t : t > t_{k_i}^i, q_i(t, \boldsymbol{\zeta}_i(t), \boldsymbol{h}_{ij}(t)) \geq 0\right\} \tag{16}$$

$$\begin{aligned} &q_i(t, \boldsymbol{e}_i(t)) \\ &= 2\left\|\sum_{j \in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right]\right\|\left(\left\|\boldsymbol{e}_i^T(t)\right\| - \Delta_0\right) - \sigma \sum_{j \in N_i} g_{ij}\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\|^2 \end{aligned} \tag{17}$$

where $\boldsymbol{e}_i(t)$ represents the triggered error of the ith spacecraft, $q_i(t, \boldsymbol{\zeta}_i(t), \boldsymbol{h}_{ij}(t))$ is the trigger criteria, σ is a constant which satisfies $\sigma \in (0, 1)$ and static threshold $\Delta_0 > 0$ is a very small positive constant, g_{ij} is the weight coefficient of the weighted adjacency matrix G, $\boldsymbol{s}_i\left(t_{ki}^i\right)$ is the triggered state of the ith spacecraft by the k_ith updating, and t_{ki}^i satisfies $0 \leq t_0^i \leq t_1^i \leq t_2^i \leq \cdots$.

Then an event trigger based adaptive coordination control algorithm is given by

$$\boldsymbol{f}_{ai} = -\hat{m}_i \boldsymbol{W}_i - \beta \boldsymbol{s}_i - k \tanh\left(\boldsymbol{s}_i / p_i^2\right) - \sum_{j \in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] \tag{18}$$

Theorem 3.1. *Consider the SFF system (1), (2) with external disturbance and mass uncertainty. If Assumption 2.1 and Assumption 2.2 hold and the event triggered control law is provided by (18), the relative position error can converge to the neighborhood near the origin.*

Proof: By substituting the controller (18) into the error dynamic model (6), (7), we have

$$\begin{aligned} m_i \dot{\boldsymbol{s}}_i &= m_i\left(\gamma_i \dot{\boldsymbol{e}}_{\rho i} + \dot{\boldsymbol{e}}_{vi}\right) = m_i \gamma_i \dot{\boldsymbol{e}}_{\rho i} + m_i \dot{\boldsymbol{e}}_{vi} \\ &= m_i \gamma_i \dot{\boldsymbol{e}}_{\rho i} + \boldsymbol{C}_i\left(\dot{\theta}\right)\boldsymbol{v}_i + \boldsymbol{D}_i\left(\dot{\theta}, \ddot{\theta}, \boldsymbol{r}_i\right)\boldsymbol{\rho}_i + \boldsymbol{N}_i\left(\boldsymbol{r}_i, \boldsymbol{r}_0\right) - m_i \dot{\boldsymbol{v}}_i^d + \boldsymbol{f}_{di} + \boldsymbol{f}_{ai} \\ &= m_i \gamma_i \dot{\boldsymbol{e}}_{\rho i} + m_i \boldsymbol{W}_i - m_i \gamma_i \boldsymbol{e}_{vi} + m_i \boldsymbol{d}_i \\ &\quad + \left(-\hat{m}_i \boldsymbol{W}_i - \beta \boldsymbol{s}_i - \sum_{j \in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right]\right) \\ &= \tilde{m}_i \boldsymbol{W}_i + m_i \boldsymbol{d}_i - \beta \boldsymbol{s}_i - \sum_{j \in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] \end{aligned} \tag{19}$$

From Lemma 2.2 we have

$$\begin{aligned} -k \boldsymbol{s}_i^T \tanh\left(\boldsymbol{s}_i / p_i^2\right) + s_i^T \boldsymbol{d}_i &\leq \sum_{l=1}^{3} s_{i,l}\left(d_{i,l} - k \tanh\left(\boldsymbol{s}_{i,l} / p_i^2\right)\right) \\ &\leq k \sum_{l=1}^{3}\left\|\boldsymbol{s}_{i,l}\right\|\left(1 - \tanh\left(\boldsymbol{s}_{i,l} / p_i^2\right)\right) = 3\alpha k p_i^2 \end{aligned} \tag{20}$$

Then we have $\sum_{i=1}^n \boldsymbol{s}_i^T\left(m_i \boldsymbol{d}_i - k \tanh\left(\boldsymbol{s}_i / p_i^2\right)\right) = \sum_{i=1}^n 3\alpha k p_i$.

Consider the following Lyapunov function

$$\boldsymbol{V} = \frac{1}{2}\sum_{i=1}^n \boldsymbol{s}_i^T m_i \boldsymbol{s}_i + \frac{1}{2\lambda_i}\sum_{i=1}^n \tilde{m}_i^2 + \sum_{i=1}^n p_i^2 \tag{21}$$

Take the time derivative of $\boldsymbol{V}$

$$\begin{aligned}
\dot{\boldsymbol{V}} &= \sum_{i=1}^{n} \boldsymbol{s}_i^T m_i \dot{\boldsymbol{s}}_i - \frac{1}{\lambda_i}\sum_{i=1}^{n} \tilde{m}_i \hat{m}_i + \sum_{i=1}^{n} \frac{d}{dt} p_i^2 \\
&= \sum_{i=1}^{n} \boldsymbol{s}_i^T \left(\tilde{m}_i \boldsymbol{W}_i + m_i \boldsymbol{d}_i - \beta \boldsymbol{s}_i - k \tanh\left(\boldsymbol{s}_i/p_i^2\right)\right) - \sum_{i=1}^{n} \boldsymbol{s}_i^T \sum_{j\in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] \\
&\quad + \sum_{i=1}^{n} \frac{d}{dt} p_i^2 - \frac{1}{\lambda_i}\sum_{i=1}^{n} \tilde{m}_i \hat{m}_i \\
&= -\sum_{i=1}^{n} \boldsymbol{s}_i^T \beta \boldsymbol{s}_i - \sum_{i=1}^{n} \boldsymbol{s}_i^T \sum_{j\in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] + \sum_{i=1}^{n} \boldsymbol{s}_i^T \left(m_i \boldsymbol{d}_i - k \tanh\left(\boldsymbol{s}_i/p_i^2\right)\right) \\
&\quad - \sum_{i=1}^{n} 3\alpha k p_i + \sum_{i=1}^{n} \boldsymbol{s}_i^T \tilde{m}_i \boldsymbol{W}_i - \frac{1}{\lambda_i}\sum_{i=1}^{n} \tilde{m}_i \hat{m}_i
\end{aligned} \tag{22}$$

From (13) we can obtain

$$\sum_{i=1}^{n} \boldsymbol{s}_i^T \tilde{m}_i \boldsymbol{W}_i = \sum_{i=1}^{n} \boldsymbol{s}_i^T \tilde{m}_i \left(\frac{\dot{\hat{m}}_i}{\lambda_i \boldsymbol{s}_i^T}\right) = \sum_{i=1}^{n} \frac{1}{\lambda_i} \tilde{m}_i \dot{\hat{m}}_i \tag{23}$$

Substituting the triggered error and (20), (23) into (22), and considering Lemma 2.1

$$\begin{aligned}
\dot{\boldsymbol{V}} &= -\sum_{i=1}^{n} \boldsymbol{s}_i^T \beta \boldsymbol{s}_i - \sum_{i=1}^{n} \boldsymbol{s}_i^T \sum_{j\in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] \\
&= -\sum_{i=1}^{n} \boldsymbol{s}_i^T \beta \boldsymbol{s}_i - \sum_{i=1}^{n} \boldsymbol{s}_i^T\left(t_k^i\right) \sum_{j\in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] \\
&\quad + \sum_{i=1}^{n} e_i^T(t) \sum_{j\in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right] \\
&\le -\sum_{i=1}^{n} \boldsymbol{s}_i^T \beta \boldsymbol{s}_i - \frac{1}{2}\sum_{i=1}^{n} \sum_{j\in N_i} g_{ij}\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\|^2 \\
&\quad + \sum_{i=1}^{n} \left\|\boldsymbol{e}_i^T(t)\right\| \left\|\sum_{j\in N_i}^{g_{ij}} \left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right]\right\|
\end{aligned} \tag{24}$$

Under the proposed event triggered mechanism, we have

$$2\left\|\sum_{j\in N_i} g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right]\right\| \left\|\boldsymbol{e}_i^T(t) - \Delta_0\right\| \le \sigma \sum_{j\in N_i} g_{ij}\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\|^2 \tag{25}$$

Then we have

$$\begin{aligned}
\dot{\boldsymbol{V}} &\le -\sum_{i=1}^{n} \boldsymbol{s}_i^T \beta \boldsymbol{s}_i - \frac{(\sigma-1)}{2}\sum_{i=1}^{n} \sum_{j\in N_i} g_{ij}\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\|^2 \\
&\quad + \sum_{i=1}^{n} \sum_{j\in N_i} \Delta_0 \left\|g_{ij}\left[\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right]\right\|
\end{aligned} \tag{26}$$

Suppose $\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\| < l_m$ and Δ_0 is sufficiently small. Then

$$\dot{\boldsymbol{V}} \le -\sum_{i=1}^{n} \boldsymbol{s}_i^T \beta \boldsymbol{s}_i + \frac{(\sigma-1)}{2}\sum_{i=1}^{n} \sum_{j\in N_i} g_{ij}\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\|^2 + \sum_{i=1}^{n} \sum_{j\in N_i} \Delta_0 g_{ij} l_m \tag{27}$$

Then by using bounding theory, it can be proved that $\|\boldsymbol{s}_i\| \to \sqrt{\frac{\Delta_0 l_m \sum_{j\in N_i} g_{ij}}{\beta}} = l_s$ and $\left\|\boldsymbol{s}_i\left(t_{ki}^i\right) - \boldsymbol{s}_j\left(t_{kj}^j\right)\right\| \to \sqrt{\frac{2\Delta_0 l_m}{(1-\sigma)}}$ as $t \to \infty$. Back to the (15), (25), we can obtain that $\boldsymbol{e}_i$ and $\boldsymbol{s}_i\left(t_k^i\right)$ can also converge to the neighbor of the origin as $t \to \infty$.

Consider a new Lyapunov function

$$\boldsymbol{V}_1 = \frac{1}{2}\sum_{i=1}^{n} \boldsymbol{e}_{\rho i}^T \boldsymbol{e}_{\rho i} \tag{28}$$

The derivation of Lyapunov functions $\boldsymbol{V}_1$ is

$$\dot{\boldsymbol{V}}_1 = \sum_{i=1}^{n} \boldsymbol{e}_{\rho i}^T \boldsymbol{e}_{vi} \tag{29}$$

While $t \to \infty$, $\|\boldsymbol{s}_i\| = \|\gamma \boldsymbol{e}_{pi} + \boldsymbol{e}_{vi}\| \to l_s$ equivalent to $\|\boldsymbol{e}_{vi}\| \to -\|\gamma \boldsymbol{e}_{pi}\| + l_s$.

$$\dot{\boldsymbol{V}}_1 = \sum_{i=1}^{n} \boldsymbol{e}_{\rho i}^T \left(-\gamma \boldsymbol{e}_{\rho i}\right) \le -\sum_{i=1}^{n} \gamma \boldsymbol{e}_{\rho i}^T \boldsymbol{e}_{\rho i} + l_s \sum_{i=1}^{n} \boldsymbol{e}_{\rho i}^T \tag{30}$$

Similarly, we have $\|\boldsymbol{e}_{\rho i}\| \to \frac{l_s}{\gamma}$ and $\|\boldsymbol{e}_{vi}\| \to 0$ as $t \to \infty$.

4. **Simulation.** In this section, the distributed formation system with three spacecraft is simulated. Assuming that three spacecraft have equal mass $m_i = 100$ kg, and this mass only be used in dynamic model. The perturbation term introduced in the dynamic model is taken as $\boldsymbol{f}_{di} = 0.0001 * [\sin(3t); \cos(4t); \sin(5t)]$ N.

The initial relative position and velocity errors are respectively set as $\boldsymbol{e}_{\rho 1} = [10\ \ 11\ \ 12]$ m, $\boldsymbol{e}_{\rho 2} = [13\ \ 14\ \ 15]$ m, $\boldsymbol{e}_{\rho 3} = [16\ \ 17\ \ 18]$ m, $\boldsymbol{e}_{vi} = [0\ \ 0\ \ 0]$ m/s, $(i = 1, 2, 3)$. And the controller parameters are chosen as follows, $\gamma_i = 2$, $\beta_i = 3$, $\lambda = [0.065\ \ 0.04\ \ 0.04]$, $k = 5$, $\alpha = 1$. The weighted adjacency matrix in communication topology is

$$G = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

As for the parameters of event update condition, $\sigma = 0.5$, $\Delta_0 = 0.001$ m. In order to be more practical, we limit the control force to 0.25 N in the simulation. The simulation results are shown in Figures 2-6.

At the same time, we simulate the system under traditional event triggered mechanism $(\Delta_0 = 0)$ based control algorithm at the same conditions, shown in Figure 7, to verify the advantages of the modified event triggered mechanism.

As can be seen from Figure 2, the adaptive mass estimates of three spacecraft converge near the true values. From Figures 3-5, it can be seen that the formation can converge to the desired configuration coordinately within 500 s under the proposed adaptive relative position controller based on modified event triggered strategy. The relative position control accuracy of modified event triggered control is better than 2×10^{-5} m, and the relative velocity control accuracy of modified event triggered control is better than 2×10^{-6} m, almost the same as the control accuracy of traditional event triggered control system.

From Figure 6, it can be noticed that before the system reaches stability, the modified event triggered mechanism can significantly reduce the communication frequency, and after the system reaches stability, long-time non-communication can be achieved within the allowable range of control accuracy.

However, from Figure 7, we notice that although traditional event triggered mechanism can reduce the communication before the system reaches stability, Zeno behavior will occur when the system is about to stabilize or stabilized. Obviously, the superiority of the modified event triggered mechanism is verified in the simulation.

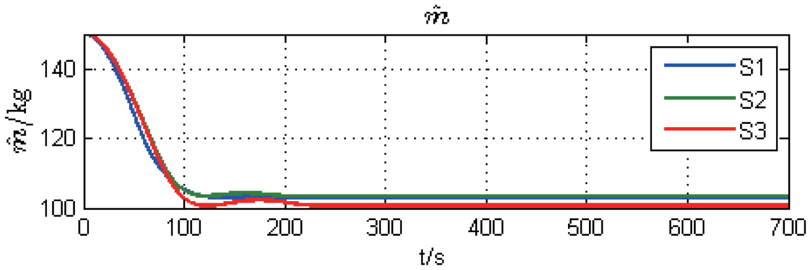

FIGURE 2. Mass estimation for spacecraft

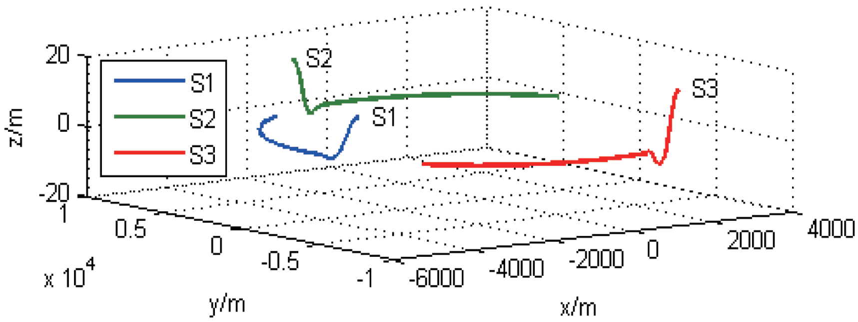

FIGURE 3. Coordination schematics

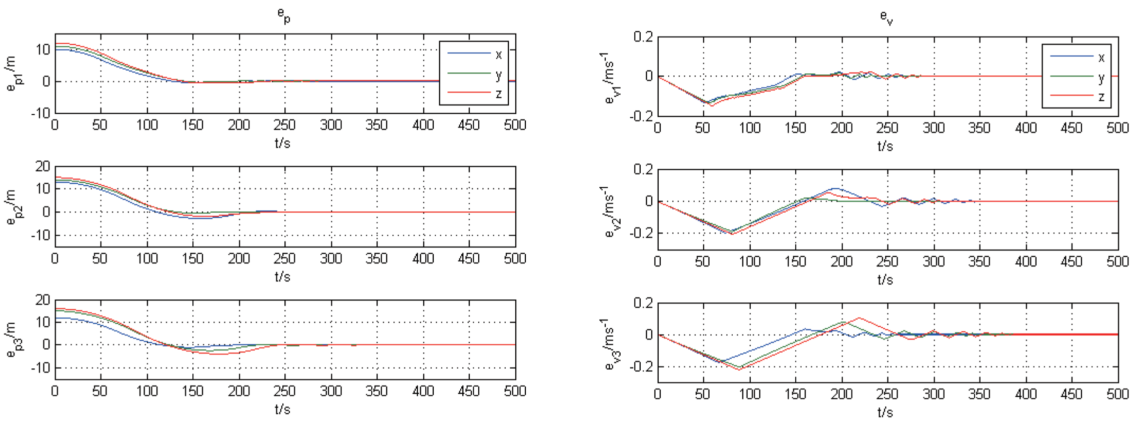

FIGURE 4. Relative position error and relative velocity error

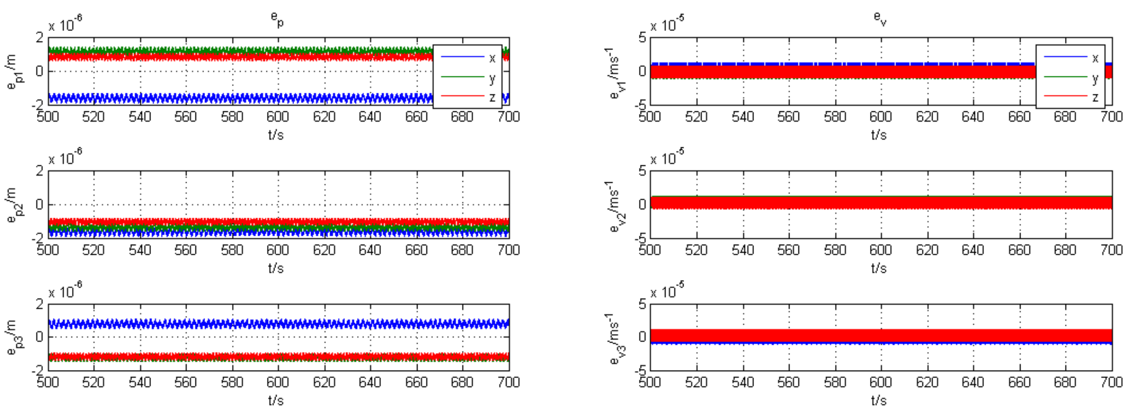

FIGURE 5. Control accuracy of modified event triggered control

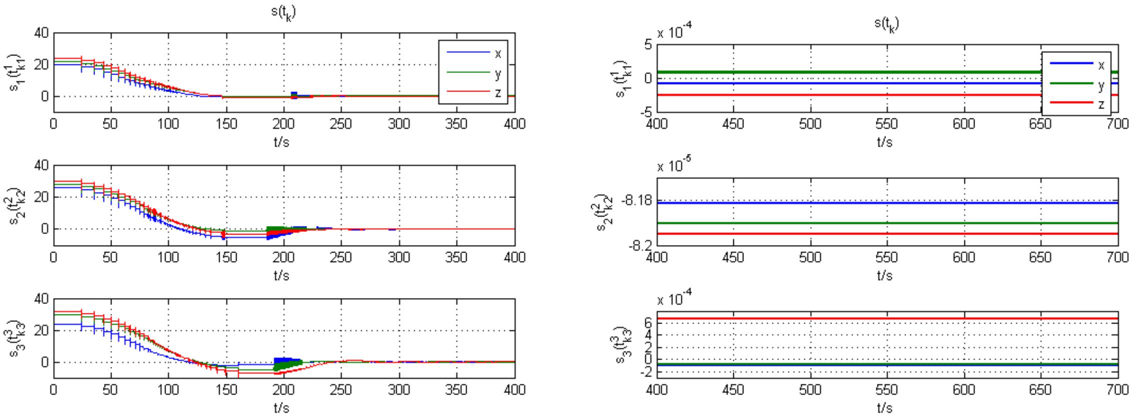

FIGURE 6. Triggered state under modified event triggered mechanism

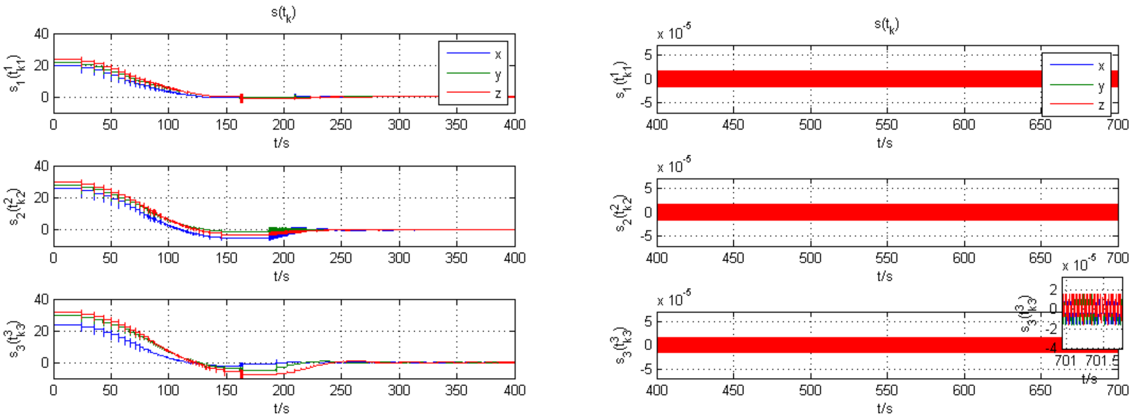

FIGURE 7. Triggered state under traditional event triggered mechanism

5. **Conclusions.** This paper considers the relative position coordinate control problem of distributed SFF system with limited communication bandwidth. A modified event triggered mechanism is proposed, and an adaptive control algorithm is designed for SFF system with external disturbance and uncertain mass parameters. Under this control algorithm spacecraft can converge to a neighbor of the origin coordinately without Zeno behavior. Both the effectiveness of the modified event triggered mechanism and the effectiveness of the control algorithm are verified by simulation examples. For future work, event trigger based control algorithm of SFF system with undirected or switching communication topology should be studied.

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