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NON-FRAGILE ASYNCHRONOUS FILTERING FOR MARKOV JUMP SYSTEMS WITH RANDOMLY OCCURRING FILTER PARAMETERS FLUCTUATION

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Abstract. *This paper investigates the problem of non-fragile l_2-l_∞ asynchronous filtering for discrete-time Markov jump systems with time-varying delays and randomly occurring filter parameters fluctuation. A non-fragile filter with randomly occurring parameters fluctuation is constructed whose modes are asynchronous with the system modes. The sufficient conditions are derived based on the Lyapunov-Krasovskii functional method, which ensure the corresponding filtering error system is stochastically stable with specified l_2-l_∞ performance index. By solving the LMIs obtained from sufficient conditions, the filter parameters can be determined. The validity of the filter design method is certified through a numerical case.*

Keywords: Markov jump systems, Non-fragile, Asynchronous filtering, Linear matrix inequalities (LMIs)

1. **Introduction.** As we all know, filtering technology plays an essential role in many situations, such as target positioning, image noise reduction, and data prediction. When it comes to Markov jump systems (MJSs), according to the degree of dependence on the system mode, the filters are roughly divided into two main types. One is the mode-dependent filter, which assumes the mode of the system is available at any time [1], but this situation is ideal and cannot be satisfied in many cases. Therefore, in order to solve this problem, scholars introduce mode-independent filters [2], which has a very good effect on the situation where the system modal information is completely inaccessible. However, there are still some problems with model-independent filters. In actual production (e.g., large-scale systems [3]), the factories, controllers, and filters are usually situated in different geographies. When information is transmitted, the system will have problems with time-delay and packet loss, resulting in errors in information transmission, which may cause asynchronous problems in MJSs. Previous mode-independent filters usually ignore all mode information of the system when the mode of the system is only partially accessible. As a consequence, it is difficult to deal with the complex asynchronous phenomenon between the filter mode and the system mode, which will lead to relatively conservative results. Considering the shortcomings of these two filters, the asynchronous filter can handle this asynchronous phenomenon [4], which makes full use of the accessible information of the system mode. On the other hand, filter gain fluctuations often occur in actual systems, which are caused by rounding errors in data calculations, the length of significant digits, and the accuracy of measuring instruments. Such small gain fluctuation may have a significant impact on the stability of the entire system, such that the designed filter should be able to tolerate the uncertainty of the coefficients. Thus, the no-fragile asynchronous filter has more practical significance, which has better resistance

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to disturbance. When filter gains fluctuate slightly, the system can still remain reliable and stable. However, up to now, the design of non-fragile l_2-l_∞ asynchronous filtering for discrete-time Markov jump systems with time-varying delays and randomly occurring filter parameters fluctuation has rarely been studied, which inspires this study.

The primary intention of this study is to develop a non-fragile asynchronous l_2-l_∞ filter for discrete-time Markov jump systems with time-varying delays and randomly occurring filter parameters fluctuation. The filtering gain fluctuation obeys the Bernoulli distribution and appears randomly. A numerical example is illustrated to prove the correctness of the design method. The simulation results prove the feasibility and effectiveness of our filter design method. The non-linear matrices are transformed into the linear matrices with the help of slack matrix. Then the coupling between the filter gain and the slack variable matrix is solved through some strategies. The filter can be determined by some sufficient conditions, which is composed of a set of LMIs.

2. **Problem Statement and Preliminaries.** Think of the following MJS with a probability space $(\Omega, \mathcal{F}, \mathcal{P})$:

$$\begin{cases} x(k+1) = A(\alpha(k))x(k) + A_d(\alpha(k))x(k-d(k)) + B(\alpha(k))\omega(k) \\ y(k) = C(\alpha(k))x(k) + C_d(\alpha(k))x(k-d(k)) + D(\alpha(k))\omega(k) \\ z(k) = L(\alpha(k))x(k) \\ x(k) = \phi(k) \quad k = -d, -d+1, \ldots, 0 \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $y(k) \in \mathbb{R}^m$ are the state variable and the measured output severally, $\omega(k) \in \mathbb{R}^q$ represents the noise signal input acquiring value in $l_2[0, +\infty)$, and $z(k) \in \mathbb{R}^p$ represents the signal to be evaluated. $d(k)$ represents the time-varying delays satisfying $d(k) \in [d_1, d_2]$, where the positive integers d_2 and d_1 are the upper and lower bounds respectively.

The parameter $\{\alpha(k)\}$ represents mode jumping process acquiring value in a limited set $\mathcal{I} = \{1, 2, \ldots, N\}$, and the corresponding transition probability matrix is $\Lambda = \{\lambda_{ij}\}$, and the mode transition probabilities are given as follows:

$$\Pr\{\alpha(k+1) = j | \alpha(k) = i\} = \lambda_{ij}$$

where $\lambda_{ij} \in [0, 1]$ for any $i, j \in \mathcal{I}$, and $\sum_{j=1}^{N} \lambda_{ij} = 1$ for any $i \in \mathcal{I}$.

In this paper, we prepare to think of the following state-space filter:

$$\begin{cases} \hat{x}(k+1) = (A_f(\beta(k)) + \mu_t \Delta A_{fs})\hat{x}(k) + (B_f(\beta(k)) + \mu_t \Delta B_{fs})y(k) \\ \hat{z}(k) = (C_f(\beta(k)) + \mu_t \Delta C_{fs})\hat{x}(k) \end{cases} \tag{2}$$

where $\hat{x} \in \mathbb{R}^n$ and $\hat{z} \in \mathbb{R}^p$ represent the filter state variable and the signal to be evaluated respectively. The parameter $\{\beta(k)\}$ represents the mode jumping process of the filter, $\{\beta(k), k \geq 0\}$ is a random jump process taking values in a limited set $\mathcal{M} = \{1, 2, \ldots, M\}$, and the corresponding conditional probability matrix is $\Pi = \{\pi_{is}\}$, and the conditional probabilities are given as follows:

$$\Pr\{\beta(k) = s | \alpha(k) = i\} = \pi_{is}$$

where $\pi_{is} \in [0, 1]$ for any $\alpha(k) = i \in \mathcal{I}$ and $s \in \mathcal{M}$, and $\sum_{s=1}^{M} \pi_{is} = 1$.

The matrices $A_f(\beta(k))$, $B_f(\beta(k))$, $C_f(\beta(k))$ represent the filter gains to be determined. In addition, ΔA_{fs}, ΔB_{fs}, ΔC_{fs} are gain variations and satisfy

$$\Delta A_{fs} = M_a E_1(t) H_a, \quad \Delta B_{fs} = M_b E_2(t) H_b, \quad \Delta C_{fs} = M_c E_3(t) H_c,$$
$$E_k^T(t) E_k(t) \leq I, \quad k = 1, 2, 3$$

where M_a, M_b, M_c, H_a, H_b, H_c are known matrices. The stochastic variable μ_t is white noise signal meeting Bernoulli-distribution, which takes values with

$$\Pr\{\mu_t = 1\} = \mu, \quad \Pr\{\mu_t = 0\} = 1 - \mu$$

where $\mu \in [0,1]$ is a known constant. Furthermore, it can be deduced that

$$\mathbb{E}\{\mu_t - \mu\} = 0, \quad \mathbb{E}\left\{(\mu_t - \mu)^2\right\} = \mu(1-\mu) = \tau^2$$

Remark 2.1. *By referring to the processing method in* [5]*, we introduce the random variable $\mu_t = \mu(t)$ to model the randomly occurring filter parameters fluctuation. The variable μ_t satisfies the Bernoulli distribution to reflect the random appearance or disappearance of the filter parameters fluctuation. When $\mu_t = 0$, the non-fragile filter* (2) *is transformed into a fragile filter, which is the special case. For the convenience of presentation, we introduce $\tilde{\mu}_t = \mu_t - \mu$.*

From (1) and (2), we can construct the following augmented filtering error system.

$$\begin{cases} \bar{x}(k+1) = \bar{A}_{is}\bar{x}(k) + \bar{A}_{dis}x(k-d(k)) + \bar{B}_{is}\omega(k) \\ \bar{z}(k) = \bar{C}_{is}\bar{x}(k) \\ \bar{x}(k) = \bar{\phi}(k) = \begin{bmatrix} \phi(k) \\ 0 \end{bmatrix} \quad k = -d, -d+1, \ldots, 0 \end{cases} \tag{3}$$

where

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}, \quad \bar{z}(k) = z(k) - \hat{z}(k)$$
$$\bar{A}_{is} = \hat{A}_{is} + \tilde{\mu}_t\Delta\tilde{A}_{fs}, \quad \bar{A}_{dis} = \hat{A}_{dis} + \tilde{\mu}_t\Delta\tilde{A}_{ds},$$
$$\bar{B}_{is} = \hat{B}_{is} + \tilde{\mu}_t\Delta\tilde{B}_{fs}, \quad \bar{C}_{is} = \hat{C}_{is} + \tilde{\mu}_t\Delta\tilde{C}_{fs}$$
$$\hat{A}_{is} = \tilde{A}_{is} + \mu\Delta\tilde{A}_{fs}, \quad \hat{A}_{dis} = \tilde{A}_{dis} + \mu\Delta\tilde{A}_{ds},$$
$$\hat{B}_{is} = \tilde{B}_{is} + \mu\Delta\tilde{B}_{fs}, \quad \hat{C}_{is} = \tilde{C}_{is} + \mu\Delta\tilde{C}_{fs}$$
$$\tilde{A}_{is} = \begin{bmatrix} A_i & 0 \\ B_{fs}C_i & A_{fs} \end{bmatrix}, \quad \Delta\tilde{A}_{fs} = \begin{bmatrix} 0 & 0 \\ \Delta B_{fs}C_i & \Delta A_{fs} \end{bmatrix},$$
$$\tilde{A}_{dis} = \begin{bmatrix} A_{di} \\ B_{fs}C_{di} \end{bmatrix}, \quad \Delta\tilde{A}_{ds} = \begin{bmatrix} 0 \\ \Delta B_{fs}C_{di} \end{bmatrix}$$
$$\tilde{B}_{is} = \begin{bmatrix} B_i \\ B_{fs}D_i \end{bmatrix}, \quad \Delta\tilde{B}_{fs} = \begin{bmatrix} 0 \\ \Delta B_{fs}D_i \end{bmatrix},$$
$$\tilde{C}_{is} = \begin{bmatrix} L_i & -C_{fs} \end{bmatrix}, \quad \Delta\tilde{C}_{fs} = \begin{bmatrix} 0 & -\Delta C_{fs} \end{bmatrix}$$

and $A_{fs} = A_f(\beta(k))$, $A_i = A(\alpha(k))$, $A_{di} = A_d(\alpha(k))$, $B_{fs} = B_f(\beta(k))$, $B_i = B(\alpha(k))$, $D_i = D(\alpha(k))$, $C_{fs} = C_f(\beta(k))$, $C_i = C(\alpha(k))$, $C_{di} = C_d(\alpha(k))$, $L_i = L(\alpha(k))$.

Definition 2.1. *The system* (3) *is stochastically stable with $\omega(k) \equiv 0$, if* (4) *holds for any initial state $\left(\bar{\phi}(k), r_0\right)$.*

$$\mathcal{E}\left[\sum_{k=0}^{\infty} \|\bar{x}(k)\|^2 | \bar{\phi}(k), r_0\right] < \infty \tag{4}$$

Definition 2.2. *For a given scalar $\gamma > 0$, the system* (3) *is stochastically stable with a given l_2-l_∞ performance γ, if* (5) *holds for any nonzero state $\omega(k) \in l_2[0, +\infty)$ under the zero initial condition.*

$$\|\bar{z}\|_{\varepsilon_\infty} < \gamma\|\omega\|_2 \tag{5}$$

where

$$\|\bar{z}\|_{\varepsilon_\infty} = \sup_k \sqrt{\mathcal{E}\left[\|\bar{z}(k)\|^2\right]}, \quad \|\omega\|_2 = \sqrt{\sum_{k=0}^{\infty} \|\omega(k)\|^2}$$

3. **Main Results.** The filtering design method of the system will be established in this part.

Theorem 3.1. *System* (3) *is stochastically stable with a given* l_2-l_∞ *performance* γ*, if there exist matrices* $P_i > 0$*,* $W_{is} > 0$*,* $R > 0$ *and a scalar* $\gamma > 0$ *for any* $i \in \mathcal{I}$ *and* $s \in \mathcal{M}$ *to guarantee the following inequalities hold.*

$$\sum_{s=1}^{t} \pi_{is} W_{is} - P_i < 0 \tag{6}$$

$$\begin{bmatrix} -P_i & \hat{C}_{is}^T & \tau\Delta\tilde{C}_{fs}^T \\ * & -\gamma^2 I & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} -\left(\bar{P}_i\right)^{-1} & 0 & \hat{A}_{is} & \hat{A}_{dis} & \hat{B}_{is} \\ * & -\left(\bar{P}_i\right)^{-1} & \tau\Delta\tilde{A}_{fs} & \tau\Delta\tilde{A}_{ds} & \tau\Delta\tilde{B}_{fs} \\ * & * & R_d - W_{is} & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

where

$$R_d = \begin{bmatrix} dR & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{P}_i = \sum_{j=1}^{n} \lambda_{ij} P_j, \quad d = \bar{d} - \underline{d} + 1$$

Proof: First, according to Schur complement, we can obtain from (8) that

$$\begin{bmatrix} \hat{A}_{is}^T & \tau\Delta\tilde{A}_{fs}^T \\ \hat{A}_{dis}^T & \tau\Delta\tilde{A}_{ds}^T \\ \hat{B}_{is}^T & \tau\Delta\tilde{B}_{fs}^T \end{bmatrix} \begin{bmatrix} \bar{P}_i & 0 \\ * & \bar{P}_i \end{bmatrix} \begin{bmatrix} \hat{A}_{is} & \hat{A}_{dis} & \hat{B}_{is} \\ \tau\Delta\tilde{A}_{fs} & \tau\Delta\tilde{A}_{ds} & \tau\Delta\tilde{B}_{fs} \end{bmatrix} + \begin{bmatrix} R_d & 0 & 0 \\ * & -R & 0 \\ * & * & -I \end{bmatrix} < \begin{bmatrix} W_{is} & 0 & 0 \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \tag{9}$$

Subsequently, we will discuss the stochastic stability of the system (3) with Lyapunov-Krasovskii functional constructed as follows:

$$V(\bar{x}(k), k, \alpha(k)) = V_1(k) + V_2(k) \tag{10}$$

where

$$V_1(k) = \bar{x}^T(k) P_{\alpha(k)} \bar{x}(k), \quad V_2(k) = \sum_{v=-\bar{d}+1}^{-\underline{d}+1} \sum_{u=k-1+v}^{k-1} x^T(u) R x(u)$$

Then, along the system (3), we consider $\Delta V_1(k)$ and $\Delta V_2(k)$ and take expectation, and according to (6) and (9), we can obtain the following inequalities under the condition $\omega(k) \equiv 0$:

$$\begin{aligned} \mathcal{E}\{\Delta V(k)\} &= \mathcal{E}\left\{\Delta V_1(k)\right\} + \mathcal{E}\left\{\Delta V_2(k)\right\} \\ &= \mathcal{E}\left\{V_1(k+1) - V_1(k) | \bar{x}(k), \alpha(k) = i\right\} + \mathcal{E}\left\{V_2(k+1) - V_2(k)\right\} \\ &\le \mathcal{E}\left\{\eta_1^T(k) \begin{bmatrix} R_d & 0 \\ 0 & -R \end{bmatrix} \eta_1(k) + \sum_{s=1}^{t} \pi_{is} \eta_1^T(k) \Omega_2^T \tilde{P}_i \Omega_2 \eta_1(k) - \bar{x}^T(k) P_i \bar{x}(k)\right\} \\ &\le \mathcal{E}\left\{\bar{x}^T(k) \left(\sum_{s=1}^{t} \pi_{is} W_{is} - P_i\right) \bar{x}(k)\right\} \\ &< -\varphi \mathcal{E}\left\{\|\bar{x}(k)\|^2\right\} \end{aligned} \tag{11}$$

where

$$\eta(k)=\begin{bmatrix}\eta_1(k)\\ \omega(k)\end{bmatrix},\eta_1(k)=\begin{bmatrix}\bar{x}(k)\\ x(k-d(k))\end{bmatrix},\tilde{P}_i=\begin{bmatrix}\bar{P}_i & 0\\ * & \bar{P}_i\end{bmatrix},\Omega_2=\begin{bmatrix}\hat{A}_{is} & \hat{A}_{dis}\\ \tau\Delta\tilde{A}_{fs} & \tau\Delta\tilde{A}_{ds}\end{bmatrix}$$

and $\varphi=\min_{i\in\mathcal{I},s\in\mathcal{M}}\left\{\lambda_{\min}\left(P_i-\sum_{s=1}^{t}\pi_{is}W_{is}\right)\right\}$. By Schur complement, we can get the following inequality from (8):

$$\begin{bmatrix}\hat{A}_{is}^T & \tau\Delta\tilde{A}_{fs}^T\\ \hat{A}_{dis}^T & \tau\Delta\tilde{A}_{ds}^T\end{bmatrix}\begin{bmatrix}\bar{P}_i & 0\\ * & \bar{P}_i\end{bmatrix}\begin{bmatrix}\hat{A}_{is} & \hat{A}_{dis}\\ \tau\Delta\tilde{A}_{fs} & \tau\Delta\tilde{A}_{ds}\end{bmatrix}+\begin{bmatrix}R_d & 0\\ * & -R\end{bmatrix}<\begin{bmatrix}W_{is} & 0\\ * & 0\end{bmatrix} \quad (12)$$

which implies that $\varphi>0$. Then we can get that the following conclusion holds under the condition $\omega(k)\equiv 0$,

$$\mathcal{E}\left\{\sum_{k=0}^{\infty}\|\bar{x}(k)\|^2\right\}<-\frac{1}{\varphi}\mathcal{E}\left\{\sum_{k=0}^{\infty}\Delta V(k)\right\}\le\frac{1}{\varphi}\mathcal{E}\{V(0)\}<\infty \quad (13)$$

which implies the system (3) is stochastically stable for $\omega(k)\equiv 0$ according to Definition 2.1.

Then, we will study whether the system (3) is stochastically stable with given l_2-l_∞ performance for any nonzero $\omega(k)$ in zero initial condition. The performance index is constructed as follows:

$$\begin{aligned}J &= \mathcal{E}\left\{V(k)-\sum_{h=1}^{k-1}\omega^T(h)\omega(h)\right\}\\ &= \mathcal{E}\left\{\sum_{h=1}^{k-1}\left(\Delta V(h)-\omega^T(h)\omega(h)\right)\right\}\\ &= \mathcal{E}\left\{\sum_{h=1}^{k-1}\left(\sum_{s=1}^{t}\pi_{is}\eta^T(k)\left(\Omega_1^T\tilde{P}_i\Omega_1+\tilde{R}\right)\eta(k)\right)-\sum_{h=1}^{k-1}\left(\sum_{s=1}^{t}\pi_{is}\bar{x}^T(k)P_i\bar{x}(k)\right)\right\}\\ &\le \mathcal{E}\left\{\sum_{h=1}^{k-1}\bar{x}^T(k)\left(\sum_{s=1}^{t}\pi_{is}W_{is}-P_i\right)\bar{x}(k)\right\}<0\end{aligned} \quad (14)$$

where

$$\tilde{R}=\begin{bmatrix}R_d & 0 & 0\\ * & -R & 0\\ * & * & -I\end{bmatrix},\quad \Omega_1=\begin{bmatrix}\hat{A}_{is} & \hat{A}_{dis} & \hat{B}_{is}\\ \tau\Delta\tilde{A}_{fs} & \tau\Delta\tilde{A}_{ds} & \tau\Delta\tilde{B}_{fs}\end{bmatrix}$$

the last two "$<$" hold as a result of (6) and (9) and the following conclusion holds:

$$\mathcal{E}\left\{\bar{x}^T(k)P_i\bar{x}(k)\right\}<\mathcal{E}\left\{V(k)\right\}<\sum_{h=1}^{k-1}\omega^T(h)\omega(h) \quad (15)$$

According to Schur complement, we can get from (7) that $\hat{C}_{is}^T\hat{C}_{is}+\tau^2\Delta\tilde{C}_{fs}^T\Delta\tilde{C}_{fs}<\gamma^2P_i$. Then we can reach that the following inequalities hold for all $k>0$:

$$\begin{aligned}\mathcal{E}\left\{\|\bar{z}(k)\|^2\right\} &= \mathcal{E}\left\{\bar{x}^T(k)\left(\hat{C}_{is}^T\hat{C}_{is}+\tau^2\Delta\tilde{C}_{fs}^T\Delta\tilde{C}_{fs}\right)\bar{x}(k)\right\}\\ &< \gamma^2\mathcal{E}\left\{\bar{x}^T(k)P_i\bar{x}(k)\right\}<\gamma^2\sum_{h=1}^{k-1}\omega^T(h)\omega(h)<\gamma^2\sum_{h=0}^{\infty}\omega^T(h)\omega(h)\end{aligned} \quad (16)$$

It implies (5) holds for any nonzero $\omega(k)\in l_2[0,+\infty)$ in zero-initial condition. This completes the proof. □

Next, we will design the filter with the help of the slack matrix method on the basis of Theorem 3.2, and we will deal with the coupling between the filter gain and the slack variable matrix and give the design method of non-fragile asynchronous filter (2). Firstly, we introduce two slack matrices as follows:

$$Q_{is} = \begin{bmatrix} Q_{is}^{(1)} & Q_s^{(2)} \\ Q_{is}^{(3)} & Q_s^{(2)} \end{bmatrix}, \quad G_{is} = \begin{bmatrix} G_{is}^{(1)} & Q_s^{(2)} \\ G_{is}^{(3)} & Q_s^{(2)} \end{bmatrix} \tag{17}$$

where $Q_{is}^{(1)} \in \mathbb{R}^n$, $Q_s^{(2)} \in \mathbb{R}^n$, $Q_{is}^{(3)} \in \mathbb{R}^n$, $G_{is}^{(1)} \in \mathbb{R}^n$, $G_{is}^{(3)} \in \mathbb{R}^n$.

Theorem 3.2. *System (3) is stochastically stable with a given l_2-l_∞ performance γ for given scalars $\gamma > 0$, $\mu > 0$, if there exist matrices $P_i = \begin{bmatrix} P_{i1} & P_{i2} \\ * & P_{i3} \end{bmatrix}$, $W_{is}, R, Q_{is}, G_{is} > 0$ and scalars $\rho > 0$, $\varepsilon > 0$ to guarantee (6), (18), and (19) hold for any $i \in \mathcal{I}$, $s \in \mathcal{M}$.*

$$\begin{bmatrix} -P_{i1} & -P_{i2} & L_i^T & 0 & 0 & 0 \\ * & -P_{i3} & -\tilde{C}_{fs}^T & 0 & 0 & \varepsilon H_c^T \\ * & * & -\gamma^2 I & 0 & -\mu M_c & 0 \\ * & * & * & -\gamma^2 I & -\tau M_c & 0 \\ * & * & * & * & -\varepsilon I & 0 \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \Sigma_1 & \bar{M} & \rho \bar{H}^T \\ * & -\rho I & 0 \\ * & * & -\rho I \end{bmatrix} < 0 \tag{19}$$

where

$$\bar{M} = \begin{bmatrix} \mu Q_s^{(2)} M_a & \mu Q_s^{(2)} M_b & \mu Q_s^{(2)} M_b & \mu Q_s^{(2)} M_b \\ \mu Q_s^{(2)} M_a & \mu Q_s^{(2)} M_b & \mu Q_s^{(2)} M_b & \mu Q_s^{(2)} M_b \\ \tau Q_s^{(2)} M_a & \tau Q_s^{(2)} M_b & \tau Q_s^{(2)} M_b & \tau Q_s^{(2)} M_b \\ \tau Q_s^{(2)} M_a & \tau Q_s^{(2)} M_b & \tau Q_s^{(2)} M_b & \tau Q_s^{(2)} M_b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} \Xi_{11} & 0 & \Xi_{12} & \Xi_{13} & \Xi_{14} \\ * & \Xi_{21} & 0 & 0 & 0 \\ * & * & R_d - W_{is} & 0 & 0 \\ * & * & * & -R & 0 \\ * & * & * & * & -I \end{bmatrix}$$

$$\bar{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & H_a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & H_b C_{di} & 0 \\ 0 & 0 & 0 & 0 & H_b C_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & H_b D_i \end{bmatrix}, \quad \Xi_{12} = \begin{bmatrix} Q_{is}^{(1)} A_i + \tilde{B}_{fs} C_i & \tilde{A}_{fs} \\ Q_{is}^{(3)} A_i + \tilde{B}_{fs} C_i & \tilde{A}_{fs} \end{bmatrix}$$

$$\Xi_{13} = \begin{bmatrix} Q_{is}^{(1)} A_{di} + \tilde{B}_{fs} C_{di} \\ Q_{is}^{(3)} A_{di} + \tilde{B}_{fs} C_{di} \end{bmatrix}, \quad \Xi_{14} = \begin{bmatrix} Q_{is}^{(1)} B_i + \tilde{B}_{fs} D_i \\ Q_{is}^{(3)} B_i + \tilde{B}_{fs} D_i \end{bmatrix}$$

$$\Xi_{11} = \bar{P}_i - Q_{is} - Q_{is}^T, \quad \Xi_{21} = \bar{P}_i - G_{is} - G_{is}^T, \quad \tilde{A}_{fs} = Q_s^{(2)} A_{fs},$$
$$\tilde{B}_{fs} = Q_s^{(2)} B_{fs}, \quad \tilde{C}_{fs} = C_{fs}$$

Then there exists a non-fragile asynchronous filter (2) such that the filtering error system (3) is stochastically stable with l_2-l_∞ performance γ. Moreover, if the LMIs (6),

(18) *and* (19) *have feasible solutions, the filter* (2) *can be determined as*

$$A_{fs} = \left(Q_s^{(2)}\right)^{-1} \tilde{A}_{fs}, \quad B_{fs} = \left(Q_s^{(2)}\right)^{-1} \tilde{B}_{fs}, \quad C_{fs} = \tilde{C}_{fs} \tag{20}$$

Proof: Based on Theorem 3.1, (7) can be rewritten as

$$\Sigma_2 + sym\left(\begin{bmatrix} 0 \\ 0 \\ -\mu M_c \\ -\tau M_c \end{bmatrix} E_3(t) \begin{bmatrix} 0 & H_c & 0 & 0 \end{bmatrix}\right) < 0 \tag{21}$$

where

$$\Sigma_2 = \begin{bmatrix} -P_{i1} & -P_{i2} & L_i^T & 0 \\ * & -P_{i3} & -\tilde{C}_{fs}^T & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix}$$

According to $E_3^T(t)E_3(t) \leq I$, (21) is obviously equivalent to (18). In the same way, according to $E_k^T(t)E_k(t) \leq I$, $k = 1, 2$, we can get that (22) is equivalent to (19).

$$\Sigma_1 + sym\left(\bar{M}\bar{E}\bar{H}\right) < 0 \tag{22}$$

where

$$\bar{E} = \begin{bmatrix} E_1(t) & 0 & 0 & 0 \\ * & E_2(t) & 0 & 0 \\ * & * & E_2(t) & 0 \\ * & * & * & E_2(t) \end{bmatrix}$$

We know that

$$\bar{P}_i - Q_{is} - Q_{is}^T \geq -Q_{is}\left(\bar{P}_i\right)^{-1} Q_{is}^T, \quad \bar{P}_i - G_{is} - G_{is}^T \geq -G_{is}\left(\bar{P}_i\right)^{-1} G_{is}^T \tag{23}$$

holds. We left-multiply and right-multiply (22) by the diagonal matrix $diag\left\{Q_{is}^{-1}, G_{is}^{-1}, I, I, I\right\}$ and its corresponding transpose, and after some proper elementary transformation with (23), we can get (8). The conditions (6), (18) and (19) are sufficient to guarantee the stochastic stability of system (3) with given l_2-l_∞ performance γ. This completes the proof. □

4. **Numerical Example.** The main goal of this part is to exemplify the validity of the filtering design method in the previous part by a numerical example. We consider the following parameters in the system we studied:

$$A_1 = \begin{bmatrix} 0.4 & 0.5 \\ 0.16 & 0.2 \end{bmatrix}, \ B_1 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \ B_2 = \begin{bmatrix} 0 \\ -0.2 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.1 & -0.2 \\ 0.1 & 0.15 \end{bmatrix}, \ A_{d2} = \begin{bmatrix} 0.1 & -0.2 \\ 0 & 0.1 \end{bmatrix}, \ C_1 = \begin{bmatrix} -0.2 & 0.1 \end{bmatrix}, \ C_2 = \begin{bmatrix} 0.4 & 0.2 \end{bmatrix}$$

$$H_a = \begin{bmatrix} 0.7 & 0.5 \end{bmatrix}, \ H_b = 0.8, \ H_c = \begin{bmatrix} 0.6 & 0.1 \end{bmatrix}, \ M_a = \begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix}, \ M_b = \begin{bmatrix} 0.8 \\ 0.4 \end{bmatrix}, \ M_c = 0.8$$

$$L_1 = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}, \ L_2 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \ D_1 = 0.5, \ D_2 = -0.1$$

The time delay varies in $\{1, 2, 3\}$ randomly. We chose the following conditional probability matrix Π and transition probability matrix Λ:

$$\Pi = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}, \quad \Lambda = \begin{bmatrix} 0.25 & 0.75 \\ 0.5 & 0.5 \end{bmatrix}$$

which implies the asynchronous filter (2) has two modes.

Next, take the above parameters and $\gamma = 0.5762$. By solving the LMIs, the filter gains can be determined as follows:

$$A_{f1} = \begin{bmatrix} -0.0049 & 0.0062 \\ 0.0013 & -0.0003 \end{bmatrix}, \quad B_{f1} = \begin{bmatrix} 0.3957 \\ -0.5583 \end{bmatrix}, \quad C_{f1} = \begin{bmatrix} 0.0242 & -0.0439 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} -0.0042 & 0.0037 \\ 0.0006 & 0.0029 \end{bmatrix}, \quad B_{f2} = \begin{bmatrix} 0.4116 \\ -0.5782 \end{bmatrix}, \quad C_{f2} = \begin{bmatrix} 0.0242 & -0.0439 \end{bmatrix}$$

Take $\mu = 0.5$, and the initial state and the noise signal input are given as follows:

$$x(0) = \begin{bmatrix} 0.2 & -0.2 \end{bmatrix}^{\mathrm{T}}, \quad w(k) = 0.9^k \sin(k)[1 \quad 1]^T$$

Based on the above parameters, we can obtain the evolution curves of $z(k)$ and $z_f(k)$ in Figure 1 and filtering errors $z(k) - z_f(k)$ in Figure 2. The results in Figure 1 and Figure 2 show that the estimation error varies in a narrow range and gradually approaches equilibrium, which illustrates that the proposed method can guaranty the stochastic stability of the system (3). Compared with fragile filters, non-fragile filters have stronger ability to resist disturbance.

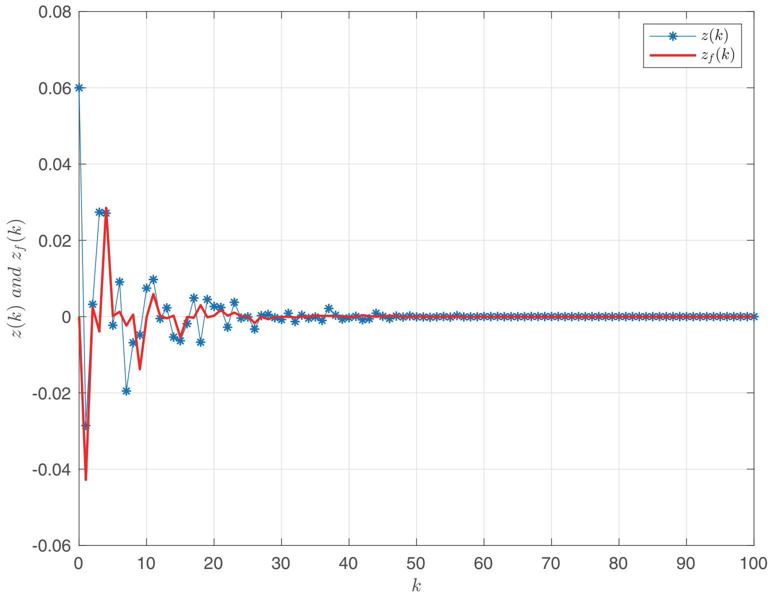

FIGURE 1. $z(k)$ and $z_f(k)$

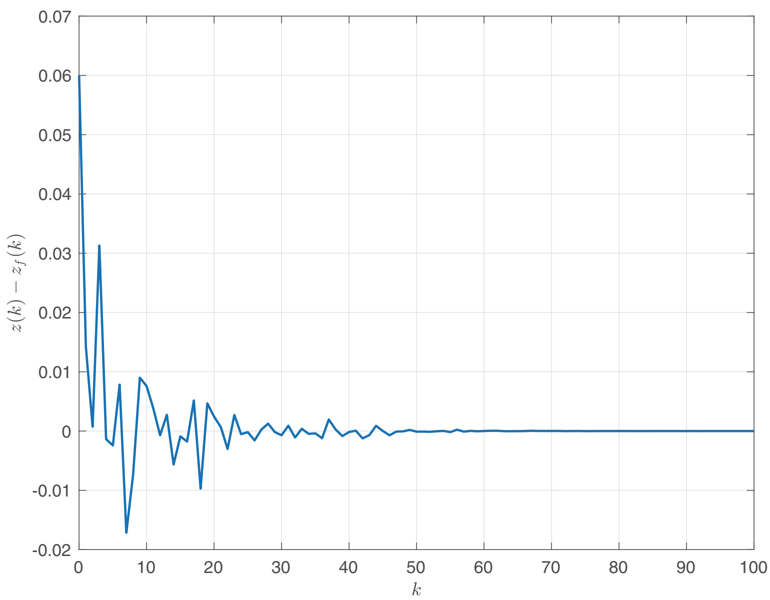

FIGURE 2. $z(k) - z_f(k)$

5. **Conclusions.** This paper has looked into the problem of non-fragile l_2-l_∞ asynchronous filtering for discrete-time MJSs with time-varying delays. Sufficient conditions have been derived from Lyapunov-Krasovskii functional method, which guarantee the stochastic stability of the filtering error system with a given l_2-l_∞ performance γ. The filtering gains have been acquired by solving a series of LMIs. The validity of the proposed filter has been certified through a numerical example. The simulation results prove the feasibility and effectiveness of our filter design method, and also provide us with ideas for further research. That is, the method we proposed has its shortcomings. It is only suitable for stable systems. Solving the asynchronous filtering problem of unstable systems will be our future work.

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