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ESSENTIAL UP-IDEALS AND t-ESSENTIAL FUZZY UP-IDEALS OF UP-ALGEBRAS

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Abstract. *In this paper, we introduce the new concepts of essential UP-subalgebras and essential UP-ideals of UP-algebras by developing from semigroups and study some properties of essential UP-subalgebras and essential UP-ideals. In addition, we extend the concept of fuzzy UP-subalgebras and fuzzy UP-ideals to t-essential fuzzy UP-subalgebras and t-essential fuzzy UP-ideals of UP-algebras and study the relations between essential UP-subalgebras (resp., essential UP-ideals) and t-essential fuzzy UP-subalgebras (resp., t-essential fuzzy UP-ideals) in UP-algebras. Moreover, we study t-essential fuzzy UP-subalgebras and t-essential fuzzy UP-ideals described by their level subsets.*
Keywords: UP-algebra, Essential UP-subalgebra, Essential UP-ideal, t-essential fuzzy UP-subalgebra, t-essential fuzzy UP-ideal

1. **Introduction.** Many algebraic structures, algebras of logic form important class of algebras. Examples of these are BCK-algebras [5], BCI-algebras [4], BCH-algebras [3], KU-algebras [11], PSRU-algebras [13], and UP-algebras [6]. They are strongly connected with logic. For example, BCI-algebras introduced by Imai and Iséki [4] in 1966 have connections with BCI-logic being the BCI-system in combinatory logic which has application in the language of functional programming. BCK and BCI-algebras are two classes of logical algebras. They were introduced by Imai and Iséki [4, 5] in 1966 and have been extensively investigated by many researchers. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. The concept of KU-algebras was introduced in 2009 by Prabpayak and Leerawat [11]. In 2017, Iampan [6] introduced the concept of UP-algebras as a generalization of KU-algebras.

The classical of fuzzy sets was proposed in 1965 by Zadeh [14]. These concepts were applied in many areas such as medical science, theoretical physics, robotics, computer science, control engineering, information science, measure theory, logic, set theory, and topology. The study of fuzzy sets is ongoing, for example, Al-Masarwah and Ahmad [1] introduced the concept of doubt bipolar fuzzy H-ideals of BCK/BCI-algebras. Essential fuzzy ideals of rings were studied by Medhi et al. in 2008 [8]. Later in 2013, Medhi and Saikia [9] studied concept of T-fuzzy essential ideals and proved properties of T-fuzzy essential ideals of rings. Recently in 2020, Baupradist et al. [2] studied essential ideals and essential fuzzy ideals in semigroups. Together, they studied 0-essential ideals and 0-essential fuzzy ideals in semigroups.

From previous reviews, we were interested in the concepts of essential ideals and essential fuzzy ideals on both rings and semigroups. For this paper, we introduce the new concepts of essential UP-subalgebras and essential UP-ideals of UP-algebras by developing

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from semigroups and study some properties of essential UP-subalgebras and essential UP-ideals. In addition, we extend the concept of fuzzy UP-subalgebras and fuzzy UP-ideals to t-essential fuzzy UP-subalgebras and t-essential fuzzy UP-ideals of UP-algebras and study the relations between essential UP-subalgebras (resp., essential UP-ideals) and t-essential fuzzy UP-subalgebras (resp., t-essential fuzzy UP-ideals) in UP-algebras. Moreover, we study t-essential fuzzy UP-subalgebras and t-essential fuzzy UP-ideals described by their level subsets.

2. **Preliminaries.** Now, we discussed the concept of UP-algebras and basic properties for the study of next sections.

Definition 2.1. [6] *An algebra $A = (A, \cdot, 0)$ of type $(2, 0)$ is called a UP-algebra, where A is a nonempty set, $\cdot$ is a binary operation on A, and 0 is a fixed element of A (i.e., a nullary operation) if it satisfies the following axioms:*

$$(\textit{for all } x, y, z \in A)((y \cdot z) \cdot ((x \cdot y) \cdot (x \cdot z)) = 0), \tag{1}$$

$$(\textit{for all } x \in A)(0 \cdot x = x), \tag{2}$$

$$(\textit{for all } x \in A)(x \cdot 0 = 0), \textit{ and} \tag{3}$$

$$(\textit{for all } x, y \in A)(x \cdot y = 0,\ y \cdot x = 0 \Rightarrow x = y). \tag{4}$$

From [6], we know that the concept of UP-algebras is a generalization of KU-algebras.
The binary relation $\leq$ on a UP-algebra $A = (A, \cdot, 0)$ is defined as follows:

$$(\text{for all } x, y \in A)(x \leq y \Leftrightarrow x \cdot y = 0) \tag{5}$$

and the following assertions are valid (see [6, 7]).

$$(\text{for all } x \in A)(x \leq x), \tag{6}$$

$$(\text{for all } x, y, z \in A)(x \leq y,\ y \leq z \Rightarrow x \leq z), \tag{7}$$

$$(\text{for all } x, y, z \in A)(x \leq y \Rightarrow z \cdot x \leq z \cdot y), \tag{8}$$

$$(\text{for all } x, y, z \in A)(x \leq y \Rightarrow y \cdot z \leq x \cdot z), \tag{9}$$

$$(\text{for all } x, y, z \in A)(x \leq y \cdot x, \text{ in particular, } y \cdot z \leq x \cdot (y \cdot z)), \tag{10}$$

$$(\text{for all } x, y \in A)(y \cdot x \leq x \Leftrightarrow x = y \cdot x), \tag{11}$$

$$(\text{for all } x, y \in A)(x \leq y \cdot y), \tag{12}$$

$$(\text{for all } a, x, y, z \in A)(x \cdot (y \cdot z) \leq x \cdot ((a \cdot y) \cdot (a \cdot z))), \tag{13}$$

$$(\text{for all } a, x, y, z \in A)(((a \cdot x) \cdot (a \cdot y)) \cdot z \leq (x \cdot y) \cdot z), \tag{14}$$

$$(\text{for all } x, y, z \in A)((x \cdot y) \cdot z \leq y \cdot z), \tag{15}$$

$$(\text{for all } x, y, z \in A)(x \leq y \Rightarrow x \leq z \cdot y), \tag{16}$$

$$(\text{for all } x, y, z \in A)((x \cdot y) \cdot z \leq x \cdot (y \cdot z)), \text{ and} \tag{17}$$

$$(\text{for all } a, x, y, z \in A)((x \cdot y) \cdot z \leq y \cdot (a \cdot z)). \tag{18}$$

Example 2.1. *Let $A = \{0, 1, 2, 3\}$ be a set with a binary operation $\cdot$ defined by the following Cayley table:*

$\cdot$	0	1	2	3
0	0	1	2	3
1	0	0	0	0
2	0	1	0	3
3	0	1	2	0

Then $(A, \cdot, 0)$ is a UP-algebra.

Definition 2.2. [6] *A nonempty subset S of a UP-algebra $A = (A, \cdot, 0)$ is called*

1) a UP-subalgebra of A if $(\forall x, y \in S)(x \cdot y \in S)$,

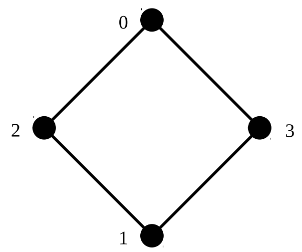

Figure 1. $(A, \leq)$

2) a UP-ideal of A if

i) the constant 0 *of* A *is in* S*, and*

ii) $(\textit{for all } x, y, z \in A)(x \cdot (y \cdot z) \in S,\ y \in S \Rightarrow x \cdot z \in S)$.

Clearly, A *and* $\{0\}$ *are UP-subalgebras and UP-ideals of* A.

We known that the concept of UP-subalgebras is a generalization of UP-ideals.

Example 2.2. *From Example 2.1, all nonempty subsets of* A *are a UP-subalgebra of* A*, and all the UP-ideals of* A *are* $\{0\}$, $\{0, 2\}$, $\{0, 3\}$, $\{0, 2, 3\}$*, and* A.

Theorem 2.1. *Let* A *be a UP-algebra and* $\{B_i\}_{i \in I}$ *a family of UP-subalgebras (resp., UP-ideals) of* A*. Then* $\bigcap_{i \in I} B_i$ *is a UP-subalgebra (resp., UP-ideal) of* A.

A *fuzzy set* ω in a nonempty set S is a function from S into the unit closed interval $[0, 1]$ of real numbers, i.e., $\omega : S \to [0, 1]$.

For any two fuzzy sets ω and μ in a nonempty set S, we define

1) $\omega \geq \mu \Leftrightarrow \omega(x) \geq \mu(x)$ for all $x \in S$.
2) $\omega = \mu \Leftrightarrow \omega \geq \mu$ and $\mu \geq \omega$.
3) $(\omega \wedge \mu)(x) = \min\{\omega(x), \mu(x)\}$ for all $x \in S$.

If $K \subseteq S$, then the characteristic function ω_K of S is a function from S into $\{0, 1\}$ defined as follows:

$$\omega_K(x) = \begin{cases} 1 & \text{if } x \in K \\ 0 & \text{otherwise.} \end{cases}$$

Definition 2.3. *A fuzzy set* ω *of a UP-algebra* $A = (A, \cdot, 0)$ *is called*

1) a fuzzy UP-subalgebra of A *if* $\omega(x \cdot y) \geq \min\{\omega(x), \omega(y)\}$ *for all* $x, y \in A$,

2) a fuzzy UP-ideal of A *if*

i) $(\textit{for all } x \in A)(\omega(0) \geq \omega(x))$*, and*

ii) $(\textit{for all } x, y, z \in A)(\omega(x \cdot z) \geq \min\{\omega(x \cdot (y \cdot z)), \omega(y)\})$.

We easily prove that if ω_1 and ω_2 are fuzzy UP-subalgebras (resp., fuzzy UP-ideals) of a UP-algebra A, then $\omega_1 \wedge \omega_2$ is also a fuzzy UP-subalgebra (resp., fuzzy UP-ideal) of A.

Example 2.3. *From Example 2.2, we have* $\{0, 1, 2\}$ *is a UP-subalgebra and* $\{0, 2, 3\}$ *is a UP-ideal of* A*. Then* $\omega_{\{0,1,2\}}$ *is a fuzzy UP-subalgebra and* $\omega_{\{0,2,3\}}$ *is a fuzzy UP-ideal of* A.

Theorem 2.2. [12] *Let* B *be a nonempty subset of a UP-algebra* A*. Then* B *is a UP-ideal (resp., UP-subalgebra) of* A *if and only if the characteristic function* ω_B *is a fuzzy UP-ideal (resp., fuzzy UP-subalgebra) of* A.

Definition 2.4. *Let* ω *be a fuzzy set in a UP-algebra* A*. For any* $t \in [0, 1]$*, the sets*

$$U(\omega; t) = \{x \in A \mid \omega(x) \geq t\} \text{ and } U^+(\omega; t) = \{x \in A \mid \omega(x) > t\}$$

are called an upper t*-level subset and an upper* t*-strong level subset of* ω*, respectively.*

Theorem 2.3. [12] *Let ω be a fuzzy set in a UP-algebra A. The following statements hold:*

i) ω is a fuzzy UP-subalgebra (resp., fuzzy UP-ideal) of A if and only if for any $t \in [0,1]$, $U(\omega; t)$ is a UP-subalgebra (resp., UP-ideal) of A if $U(\omega; t) \neq \emptyset$,

ii) ω is a fuzzy UP-subalgebra (resp., fuzzy UP-ideal) of A if and only if for any $t \in [0,1]$, $U^+(\omega; t)$ is a UP-subalgebra (resp., UP-ideal) of A if $U^+(\omega; t) \neq \emptyset$.

3. **Essential UP-Ideals of UP-Algebras.** In this section, we introduce the concepts of essential UP-ideals and essential UP-subalgebras and study properties of those.

The concept of an essential ideal of a semigroup is defined by Baupradist and Chemat as follows.

Definition 3.1. [2] *An ideal I of a semigroup S is called an essential ideal of S if $I \cap J \neq \emptyset$ for every ideal J of S.*

Based on the above concept on semigroups, we lead to the development of new concept on UP-algebras.

Definition 3.2. *A UP-ideal (resp., UP-subalgebra) B of a UP-algebra A is called an essential UP-ideal (resp., essential UP-subalgebra) of A if $B \cap C$ is a nonzero subset (actually, it is a nonzero UP-ideal) of A for every nonzero UP-ideal (resp., nonzero UP-subalgebra) C of A. Equivalently, $\{0\} \subset B \cap C$ for every nonzero UP-ideal (resp., nonzero UP-subalgebra) C of A.*

We see that every UP-algebra is an essential UP-ideal and an essential UP-subalgebra of itself.

Example 3.1. *From Example 2.2, we have $\{0, 2, 3\}$ and A are essential UP-ideals of A.*

Theorem 3.1. *Let B be an essential UP-subalgebra of a UP-algebra A. Then $B = A$.*

Proof: Let $a \in A$ with $a \neq 0$. Then $\{0, a\}$ is a nonzero UP-subalgebra of A. Since B is an essential UP-subalgebra of A, we have $B \cap \{0, a\}$ is nonzero. Thus $a \in B$, that is, $A \subseteq B$. Hence, $B = A$. □

Corollary 3.1. *A UP-subalgebra B of a UP-algebra A is an essential UP-subalgebra of A if and only if $B = A$.*

Theorem 3.2. *Let B be an essential UP-ideal of a UP-algebra A and B' be a UP-ideal of A containing B. Then B' is also an essential UP-ideal of A.*

Proof: Let C be a nonzero UP-ideal of A. Since B is an essential UP-ideal of A, we have $\{0\} \subset B \cap C \subseteq B' \cap C$. Hence, B' is an essential UP-ideal of A. □

Theorem 3.3. *Let B_1 and B_2 be essential UP-ideals of a UP-algebra A. Then $B_1 \cap B_2$ is an essential UP-ideal of A.*

Proof: Let C be a nonzero UP-ideal of A. Since B_1 and B_2 are UP-ideals of A, we have $B_1 \cap B_2$ is a UP-ideal of A. Since B_2 is an essential UP-ideal of A, we have $B_2 \cap C$ is a nonzero UP-ideal of A. Since B_1 is an essential UP-ideal of A, we have $\{0\} \subset B_1 \cap (B_2 \cap C) = (B_1 \cap B_2) \cap C$. Hence, $B_1 \cap B_2$ is an essential UP-ideal of A. □

Corollary 3.2. *Finite intersection of essential UP-ideals of a UP-algebra A is an essential UP-ideal of A.*

4. **Essential Fuzzy UP-Ideals of UP-Algebras.** In this section, we introduce the concepts of t-essential fuzzy UP-ideals and t-essential fuzzy UP-subalgebras of UP-algebras and investigate properties of those.

Definition 4.1. *Let $t \in [0, 1)$. A fuzzy UP-ideal (resp., fuzzy UP-subalgebra) ω of a nonzero UP-algebra A is called a t-essential fuzzy UP-ideal (resp., t-essential fuzzy UP-subalgebra) of A if there exists a nonzero element $x_\mu \in A$ such that $t < (\omega \wedge \mu)(x_\mu)$ for every nonzero fuzzy UP-ideal (resp., nonzero fuzzy UP-subalgebra) μ of A.*

We easily see that a fuzzy UP-ideal (resp., fuzzy UP-subalgebra) μ of a UP-algebra A is nonzero if and only if $0 < \mu(0)$.

Theorem 4.1. *Let ω be a t-essential fuzzy UP-ideal of a UP-algebra A and ω' be a fuzzy UP-ideal of A such that $\omega \leq \omega'$. Then ω' is also a t-essential fuzzy UP-ideal of A.*

Proof: Let μ be a nonzero fuzzy UP-ideal of A. Since ω is a t-essential fuzzy UP-ideal of A, there exists a nonzero element $x_\mu \in A$ such that $t < (\omega \wedge \mu)(x_\mu) = \min\{\omega(x_\mu), \mu(x_\mu)\} \leq \min\{\omega'(x_\mu), \mu(x_\mu)\} = (\omega' \wedge \mu)(x_\mu)$. Hence, ω' is a t-essential fuzzy UP-ideal of A. □

Theorem 4.2. *Let ω be a t-essential fuzzy UP-subalgebra (resp., t-essential fuzzy UP-ideal) of a UP-algebra A. Then $t < \omega(0)$.*

Proof: Let μ be a fuzzy set in A defined by $\mu(x) = 1$ for all $x \in A$. Then μ is a nonzero fuzzy UP-subalgebra (resp., nonzero fuzzy UP-ideal) of A. Since ω is a t-essential fuzzy UP-subalgebra (resp., t-essential fuzzy UP-ideal) of A, there exists a nonzero element $x_\mu \in A$ such that $t < (\omega \wedge \mu)(x_\mu) = \min\{\omega(x_\mu), \mu(x_\mu)\} = \min\{\omega(x_\mu), 1\} = \omega(x_\mu) \leq \omega(0)$. Hence, $t < \omega(0)$. □

Theorem 4.3. *Let ω_1 and ω_2 be t-essential fuzzy UP-ideals (resp., t-essential fuzzy UP-subalgebras) of a UP-algebra A. Then $\omega_1 \wedge \omega_2$ is a t-essential fuzzy UP-ideal (resp., t-essential fuzzy UP-subalgebra) of A.*

Proof: Let μ be a nonzero fuzzy UP-ideal (resp., nonzero fuzzy UP-subalgebra) of A. Since ω_1 and ω_2 are fuzzy UP-ideals (resp., fuzzy UP-subalgebras) of A, we have $\omega_1 \wedge \omega_2$ is a fuzzy UP-ideal (resp., fuzzy UP-subalgebra) of A. Since ω_2 is a t-essential fuzzy UP-ideal (resp., t-essential fuzzy UP-subalgebras) of A, we have $\omega_2 \wedge \mu$ is a nonzero fuzzy UP-ideal (resp., nonzero fuzzy UP-subalgebra) of A. Since ω_1 is a t-essential fuzzy UP-ideal (resp., t-essential fuzzy UP-subalgebras) of A, there exists a nonzero element $x_\mu \in A$ such that $t < (\omega_1 \wedge (\omega_2 \wedge \mu))(x_\mu) = ((\omega_1 \wedge \omega_2) \wedge \mu)(x_\mu)$. Hence, $\omega_1 \wedge \omega_2$ is a t-essential fuzzy UP-ideal (resp., t-essential fuzzy UP-subalgebras) of A. □

The following theorems of a relation characteristic function a 0-essential fuzzy UP-ideal and an essential UP-ideal.

Theorem 4.4. *Let B be a UP-ideal of a nonzero UP-algebra A. If B is an essential UP-ideal of A, then the characteristic function ω_B is a 0-essential fuzzy UP-ideal of A.*

Proof: Suppose that B is an essential UP-ideal of A. Then B is a UP-ideal of A. Thus by Theorem 2.2, we have ω_B is a nonzero fuzzy UP-ideal of A. Let μ be a nonzero fuzzy UP-ideal of A. Then $U^+(\mu; 0)$ is nonempty which contains a nonzero element. By Theorem 2.3 ii), we have $U^+(\mu; 0)$ is a nonzero UP-ideal of A. Since B is an essential UP-ideal of A, there exists a nonzero element $x \in B \cap U^+(\mu; 0)$. Thus, $0 < \mu(x) = \min\{1, \mu(x)\} = \min\{\omega_B(x), \mu(x)\} = (\omega_B \wedge \mu)(x)$. Hence, ω_B is a 0-essential fuzzy UP-ideal of A. □

Theorem 4.5. *Let B be a UP-ideal of a nonzero UP-algebra A. If the characteristic function ω_B is a t-essential fuzzy UP-ideal of A, then B is an essential UP-ideal of A.*

Proof: Assume that ω_B is a t-essential fuzzy UP-ideal of A. Then ω_B is a fuzzy UP-ideal of A. Thus by Theorem 2.2, we have B is a UP-ideal of A. Let C be a nonzero UP-ideal of A. Thus by Theorem 2.2, we have ω_C is a nonzero fuzzy UP-ideal of A. Since ω_B is a t-essential fuzzy UP-ideal of A, there exists a nonzero element $x_C \in A$ such that $t < (\omega_B \wedge \omega_C)(x_C) = \min\{\omega_B(x_C), \omega_C(x_C)\}$. This implies that $x_C \in B \cap C$, that is, $\{0\} \subset B \cap C$. Hence, B is an essential UP-ideal of A. □

The following two theorems show the relationship between an upper t-level subset and a t-essential fuzzy UP-subalgebra (t-essential fuzzy UP-ideal).

Theorem 4.6. *Let ω be a fuzzy set in a nonzero UP-algebra A. Then the following statements hold:*

i) if ω is a t-essential fuzzy UP-subalgebra of A, then $U(\omega;t)$ is an essential UP-subalgebra of A,
ii) if ω is a t-essential fuzzy UP-subalgebra of A, then $U^+(\omega;t)$ is an essential UP-subalgebra of A.

Proof: i) Assume that ω is a t-essential fuzzy UP-subalgebra of A. By Theorem 4.2, we have $0 \in U(\omega;t) \neq \emptyset$. Since ω is a fuzzy UP-subalgebra of A and by Theorem 2.3 i), we have $U(\omega;t)$ is a UP-subalgebra of A. Let C be a nonzero UP-subalgebra of A. By Theorem 2.2, we have ω_C is a nonzero fuzzy UP-subalgebra of A. Since ω is a t-essential fuzzy UP-subalgebra of A, there exists a nonzero element $x_C \in A$ such that $t < (\omega \wedge \omega_C)(x_C) = \min\{\omega(x_C), \omega_C(x_C)\}$. Thus, $t < \omega(x_C)$ and $\omega_C(x_C) = 1$, that is, $x_C \in U(\omega;t) \cap C$. Thus, $U(\omega;t) \cap C \neq \{0\}$. Hence, $U(\omega;t)$ is an essential UP-subalgebra of A.

ii) Assume that ω is a t-essential fuzzy UP-subalgebra of A. By Theorem 4.2, we have $0 \in U^+(\omega;t) \neq \emptyset$. Since ω is a fuzzy UP-subalgebra of A and by Theorem 2.3 ii), we have $U^+(\omega;t)$ is a UP-subalgebra of A. Let C be a nonzero UP-subalgebra of A. By Theorem 2.2, we have ω_C is a nonzero fuzzy UP-subalgebra of A. Since ω is a t-essential fuzzy UP-subalgebra of A, there exists a nonzero element $x_C \in A$ such that $t < (\omega \wedge \omega_C)(x_C) = \min\{\omega(x_C), \omega_C(x_C)\}$. Thus, $t < \omega(x_C)$ and $\omega_C(x_C) = 1$, that is, $x_C \in U^+(\omega;t) \cap C$. Thus, $U^+(\omega;t) \cap C \neq \{0\}$. Hence, $U^+(\omega;t)$ is an essential UP-subalgebra of A. □

Theorem 4.7. *Let ω be a fuzzy set in a nonzero UP-algebra A. Then the following statements hold:*

i) if ω is a t-essential fuzzy UP-ideal of A, then $U(\omega;t)$ is an essential UP-ideal of A,
ii) if ω is a t-essential fuzzy UP-ideal of A, then $U^+(\omega;t)$ is an essential UP-ideal of A.

Proof: i) Assume that ω is a t-essential fuzzy UP-ideal of A. By Theorem 4.2, we have $0 \in U(\omega;t) \neq \emptyset$. Since ω is a fuzzy UP-ideal of A and by Theorem 2.3 i), we have $U(\omega;t)$ is a UP-ideal of A. Let C be a nonzero UP-ideal of A. By Theorem 2.2, we have ω_C is a nonzero fuzzy UP-ideal of A. Since ω is a t-essential fuzzy UP-ideal of A, there exists a nonzero element $x_C \in A$ such that $t < (\omega \wedge \omega_C)(x_C) = \min\{\omega(x_C), \omega_C(x_C)\}$. Thus, $t < \omega(x_C)$ and $\omega_C(x_C) = 1$, that is, $x_C \in U(\omega;t) \cap C$. Thus, $U(\omega;t) \cap C \neq \{0\}$. Hence, $U(\omega;t)$ is an essential UP-ideal of A.

ii) Assume that ω is a t-essential fuzzy UP-ideal of A. By Theorem 4.2, we have $0 \in U^+(\omega;t) \neq \emptyset$. Since ω is a fuzzy UP-ideal of A and by Theorem 2.3 ii), we have $U^+(\omega;t)$ is a UP-ideal of A. Let C be a nonzero UP-ideal of A. By Theorem 2.2, we have ω_C is a nonzero fuzzy UP-ideal of A. Since ω is a t-essential fuzzy UP-ideal of A, there exists a nonzero element $x_C \in A$ such that $t < (\omega \wedge \omega_C)(x_C) = \min\{\omega(x_C), \omega_C(x_C)\}$. Thus, $t < \omega(x_C)$ and $\omega_C(x_C) = 1$, that is, $x_C \in U^+(\omega;t) \cap C$. Thus, $U^+(\omega;t) \cap C \neq \{0\}$. Hence, $U^+(\omega;t)$ is an essential UP-ideal of A. □

5. Conclusion.

In this paper, we have introduced the concepts of essential UP-subalgebras and essential UP-ideals in UP-algebras and provided some properties of essential UP-subalgebras and essential UP-ideals. Finally, we have studied the concepts of t-essential fuzzy UP-subalgebras and t-essential fuzzy UP-ideals in UP-algebras, which is related to the concept of essential UP-subalgebras and essential UP-ideals.

In the future work, we can study essential UP-filters and t-essential fuzzy UP-filters of UP-algebras and extend to concept of interval-valued fuzzy sets.

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