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ANALYTICAL DESIGN OF FRACTIONAL FILTER PID CONTROLLER FOR IMPROVED PERFORMANCE OF FIRST ORDER PLUS TIME DELAY PLANTS

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Abstract. *This article presents analytical design of fractional filter PID controller for first order plus time delay plants using internal model control (IMC) method. The novelty of the work lies in the use of systematic procedure for designing the controller based on the robustness (maximum sensitivity, M_s) using fractional order IMC filter structure. The closed loop performance with the proposed controller is observed for normal process conditions and for nonlinearity in the feedback path. In addition, robust stability analysis is implemented for uncertainty in the process parameters. Also, the controller fragility is studied for uncertainty in the controller parameters.*
Keywords: Internal model control, Fractional IMC filter, Maximum sensitivity, Integral absolute error, Fragility

1. **Introduction.** Fractional control has taken the lead due to the inoperability of PID controllers for plants with nonlinearity and variable operating points. The performance of such plants can be enhanced with the help of fractional controllers derived using fractional calculus [1]. The advantages of fractional controllers can be attributed to the robustness, efficient disturbance rejection and their ability to work with noisy plant environments [2].

This work is confined to the design of controller for first order plus time delay (FOPTD) plant models. The reason for choosing this model is that the dynamics of all the industrial processes with nonlinearity can be better represented as FOPTD models. There are many analytical and rule-based integer order controller design procedures [3-5] for FOPTD models. There exists a significant work on fractional order PID (FOPID) controller [6] design for FOPTD models [7,8]. However, there was complexity in tuning with more tuning parameters. An IMC-PID fractional order filter controller is designed using lower order fractional reference model for integer order systems [9]. The same procedure is followed for designing a fractional IMC-PID filter controller for non-integer order systems [10]. In both [9,10], the controller is tuned to satisfy phase margin and gain cross over frequency specification and the delay term is approximated using first order Taylor and Pade's procedure. An FOPI controller was designed whose PI settings are calculated using Zeigler-Nichols method and the integrator order was arbitrarily chosen as a fractional number from 0.8 to 1.3 [11]. They have just shown that FOPI controller improves the closed loop performance compared to PI controller in terms of less errors and control energy. However, it is also required to consider the robustness of the controller. Rahul

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and Anwar [12] proposed FOPI controller tuning rules by minimizing integral absolute error (IAE) using genetic algorithm and M_s.

In the present work, a fractional filter PID (FFPID) controller [13] design procedure is proposed for FOPTD models based on IMC scheme. The procedure uses fractional IMC filter structure and first order Pade's procedure for representing the delay term. A systematic design procedure is proposed for tuning the FFPID controller based on M_s. The advantage of the proposed controller structure is that it has only two tuning parameters. The performance of the designed controller is analyzed with the recent methods [11,12]. Also, the performance is observed for nonlinearity in the feedback path of the system. The robustness of the proposed controller is estimated for parametric perturbations and the robust stability is addressed [14].

The paper is organized as follows. Section 2 provides the controller design. The performance analysis and robustness analysis are briefly described in Section 3. The simulation results are provided in Section 4 followed by conclusion in Section 5.

2. **Controller Design Using IMC Principles.** The FOPID controller structure is

$$C(s) = K_p + \frac{K_i}{s^{\lambda}} + K_d s^{\mu} \tag{1}$$

where K_p, K_i and K_d are the proportional, integral and derivative gains; λ and μ are the fractional orders of the integrator and differentiator.

The basic IMC scheme is presented in Figure 1. In the IMC scheme, the process model is explicitly used for the controller design. The IMC controller consists of a plant model referred to as internal model. The IMC design procedure often results in a PID controller.

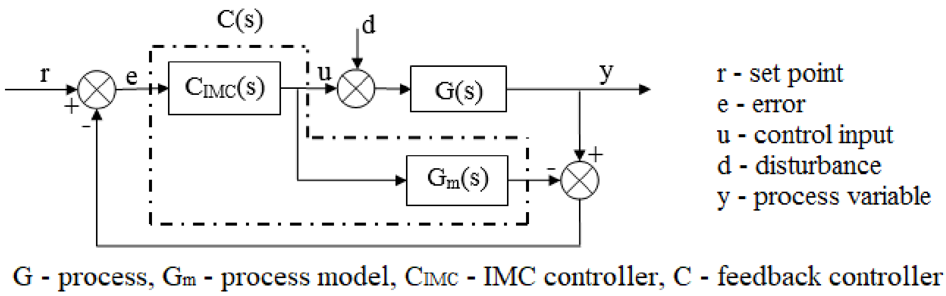

FIGURE 1. IMC based closed loop structure

The structure of the proposed FFPID controller is

$$C(s) = (\text{fractional filter term}) \left(K_p + \frac{K_i}{s} + K_d s \right) \tag{2}$$

The mathematical process model in the present work takes the form

$$G_m(s) = \frac{K e^{-Ls}}{Ts + 1} \tag{3}$$

The equivalent feedback loop controller can be computed as

$$C(s) = \frac{C_{IMC}(s)}{1 - C_{IMC}(s) G_m(s)} \tag{4}$$

where

$$C_{IMC}(s) = [G_m^-(s)]^{-1} F(s) \tag{5}$$

$G_m^-(s)$ is the invertible part of $G_m(s)$ which contains stable minimum phase elements and

$$F(s) = \frac{1}{\gamma s^p + 1} \tag{6}$$

Substituting (3), (5) and (6) into (4) the equivalent feedback controller is

$$C(s) = \frac{\frac{Ts+1}{K(\gamma s^p+1)}}{1 - \frac{Ts+1}{K(\gamma s^p+1)}\frac{Ke^{-Ls}}{Ts+1}} = \frac{Ts+1}{K[(\gamma s^p+1) - e^{-Ls}]} \tag{7}$$

Using $e^{-Ls} = \frac{1-0.5Ls}{1+0.5Ls}$ (first Pade's procedure), the controller $C(s)$ is

$$\text{Proposed, } C(s) = \left(\frac{1}{0.5\gamma Ls^p + \gamma s^{p-1} + L}\right)\left(\frac{T+0.5L}{K}\right)\left[1 + \frac{1}{(T+0.5L)s} + \left(\frac{0.5LT}{T+0.5L}\right)s\right] \tag{8}$$

where K, L and T are the process model parameters while γ is the tuning parameter for speed of response and p is the fractional order of IMC filter.

3. Closed Loop System Analysis.

3.1. **Performance analysis.** The system response is seen for a unit step change in 'u' and 'd'. The balanced and delay significant processes have been considered in the current work. Also, the closed loop performance is examined for nonlinearity [11] in the feedback path introduced by sensor output which takes the following form:

$$y_{measured}(t) = y(t) + C_1 y^3(t);\ C_1 = 0.01 \tag{9}$$

The systematic steps for tuning the FFPID controller are given as follows.

Step 1: Choose any stable FOPTD plant and derive the controller using IMC method.

Step 2: Obtain the final expression for controller in the proposed form (Equation (2)) using Pade's procedure for representing time delay.

Step 3: Start tuning the controller by selecting the unknown parameters γ and p minimizing IAE for a prefixed M_s. K_p, K_i and K_d are obtained from derived expressions using process parameters.

Step 4: Choose a basic value for p (taken as 1.01 here).

Step 5: Now, choose γ so that M_s matches the prefixed value.

Step 6: Obtain the closed loop response and record IAE for this set of γ and p.

Step 7: Change p in steps of 0.01 until it is equal to 1.1 and repeat Steps 5 and 6.

Step 8: Identify the optimum values of γ and p based on the minimum IAE.

3.2. **Robustness and fragility analysis.** The condition for robust stability of the system [14] is

$$||l_m(j\omega)T(j\omega)|| < 1 \quad \forall\omega \in (-\infty, \infty) \tag{10}$$

$T(s)_{s=j\omega} = \frac{C(s)G(s)}{1+C(s)G(s)}$ is the complementary sensitivity function and $l_m(j\omega) = \left|\frac{G(j\omega)-G_m(j\omega)}{G_m(j\omega)}\right|$ is the bound on uncertainty.

The following expression (Equation (11)) must hold good for changes in K and L

$$||T(j\omega)||_\infty < \frac{1}{\left|\left(\frac{\Delta K}{K}+1\right)e^{-\Delta L} - 1\right|} \tag{11}$$

The controller fragility [15] in terms of robustness and performance is analyzed using delta 20 fragility index ($FI_{\Delta 20}$).

The expression for calculation of robustness fragility index (RFI) is

$$RFI_{\Delta 20} = \frac{M_{s\Delta 20}}{M_s} - 1 \tag{12}$$

where the numerator is M_s for +20% uncertainty in all controller parameters and denominator is its nominal value.

Any controller is robustness resilient if $RFI_{\Delta 20} \leq 0.1$; nonfragile if $0.1 < RFI_{\Delta 20} \leq 0.5$ and fragile if $RFI_{\Delta 20} > 0.5$. The performance fragility index (PFI) is formulated as

$$PFI_{\Delta 20} = \frac{J_{E\Delta 20}}{J_E} - 1 \tag{13}$$

where the numerator is J_E for +20% uncertainty in all controller parameters and the denominator is nominal value of error. Any controller is performance resilient if $PFI_{\Delta 20} \leq 0.1$; nonfragile if $0.1 < PFI_{\Delta 20} \leq 0.5$ and fragile if $PFI_{\Delta 20} > 0.5$.

4. **Results and Discussion.** The performance is evaluated with ISE, IAE, TV, M_s and ISI [11].

$$\text{ISE (integral square error)} = \int_0^\infty e^2(t)dt \tag{14}$$

$$\text{IAE (integral absolute error)} = \int_0^\infty |e(t)|dt \tag{15}$$

$$\text{TV (total variation)} = \sum_{i=0}^{\infty} |u_{i+1} - u_i| \tag{16}$$

$$M_s = \max_{0<\omega<\infty} \left| \frac{1}{1 + C(j\omega)G(j\omega)} \right| \tag{17}$$

$$\text{ISI (integral of squared input)} = \int_0^\infty u^2(t)dt \tag{18}$$

4.1. **Example 1.** The balanced process considered for the study is

$$G_m(s) = \frac{e^{-s}}{s+1} \tag{19}$$

The proposed controller settings and the settings with Aguila-Camacho and Ponce method [11] are listed in Table 1. Though, M_s is not considered for comparison in method in [11], it is calculated, and the same M_s has been used for obtaining controller settings with the proposed methods. Using the settings in Table 1, the system response is observed for different M_s values. The servo response and the corresponding changes in error, ISE, IAE and ISI are observed. It is found that there is an offset with method in [11] for M_s of 1.905 and 1.855 whereas the proposed method is performing well with low ISE and IAE. The reason for offset is that the fractional order λ was chosen as 0.8 and 0.9. Hence, for offset free response λ should be chosen above 1. In view of this, the closed loop response is analyzed in detail for M_s of 1.762 and 1.685.

TABLE 1. Controller settings

Method	K_p	K_i	λ	K_d	γ	p	M_s
Proposed	1.5	1	–	0.5	0.61	1.02	1.762
Method in [11]	0.9	0.3	1.1	–	–	–	
Proposed	1.5	1	–	0.5	0.723	1.02	1.685
Method in [11]	0.9	0.3	1.3	–	–	–	
Proposed	2.25	1	–	1.25	4	1.02	1.4
Method in [12]	0.27	0.112	1.1	–	–	–	

The trends of error, ISE, IAE and ISI for set point tracking for M_s of 1.762 and 1.685 are presented in Figure 2. It is noticed that the error with the proposed method is quickly converging to zero for both values of M_s. The errors are low with the proposed method compared to Aguila-Camacho and Ponce method [11]. The control energy spent in terms of ISI is a little high with the proposed method in case of $M_s = 1.762$ whereas it is low for $M_s = 1.685$. The response for step disturbance of 0.1 magnitude introduced at $t = 50$s for

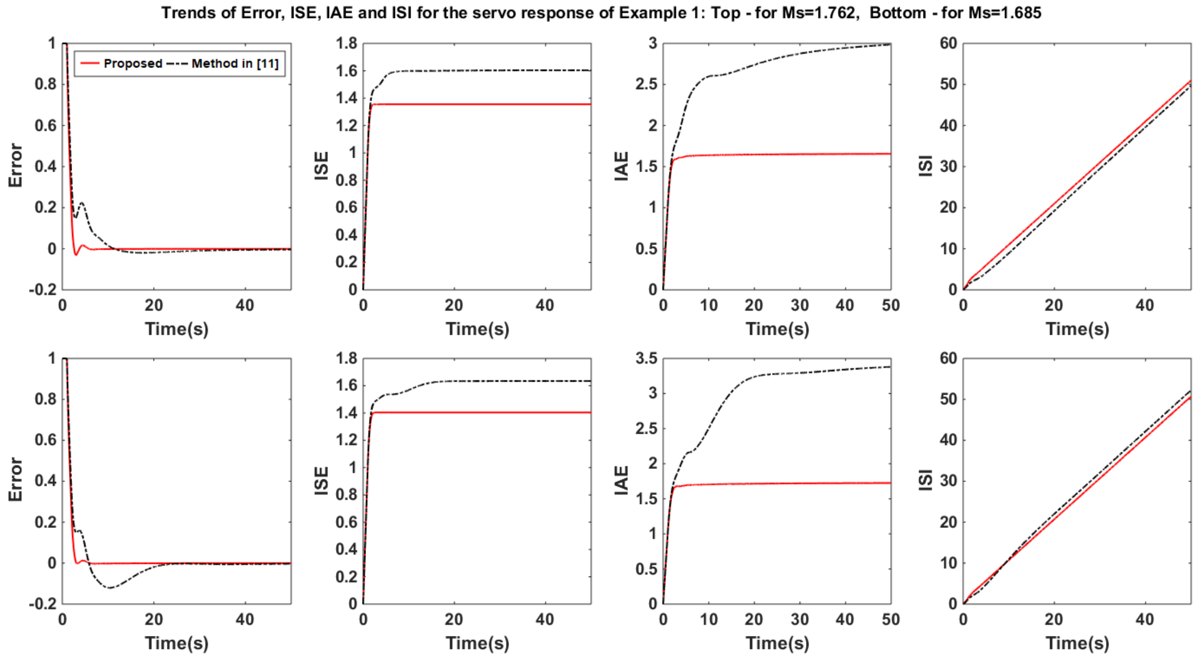

FIGURE 2. Trends of performance measures of Example 1; solid – proposed; dash dot – method in [11]

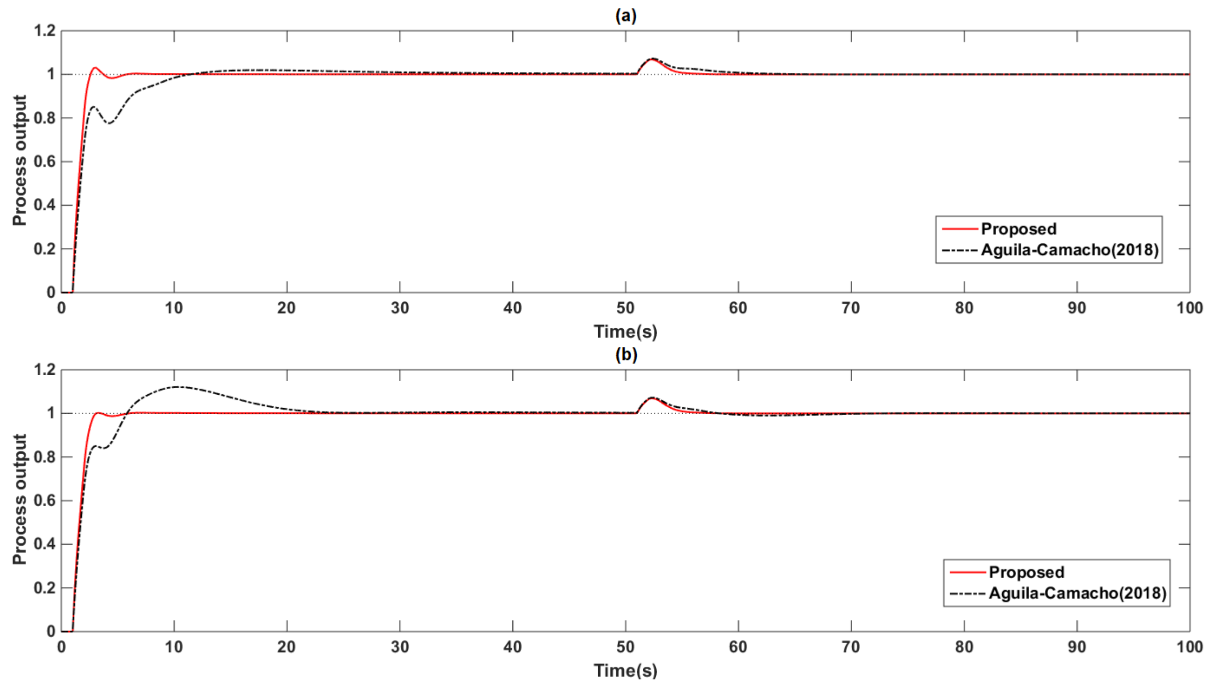

FIGURE 3. Closed loop response of Example 1 (a) for $M_s = 1.762$; (b) for $M_s = 1.685$

$M_s = 1.762$ is shown in Figure 3. The process variable settles quickly with the proposed method whereas it takes a long time to settle with method in [11] and an overshoot is also observed. This is proved with low ISE and IAE values listed in Table 2. However, the ISI is a little high with the proposed method. Also, it is discovered that the proposed FFPID controller achieves improved and faster disturbance rejection. The proposed controller continues to show performance enhancement in presence of nonlinearity and the related measures are recorded in Table 2.

TABLE 2. Closed loop performance measures of Example 1

Method	Ideal				Non ideal				M_s
	ISE	IAE	TV	ISI	ISE	IAE	TV	ISI	
Proposed	1.36	1.82	2.67	91.82	1.36	1.82	2.69	90	1.762
Method in [11]	1.62	3.29	1.18	90.75	1.61	3.27	1.19	88.95	1.762
Proposed	1.41	1.89	2.19	91.57	1.41	1.89	2.21	89.75	1.685
Method in [11]	1.65	3.71	1.15	93.08	1.64	3.69	1.14	91.17	1.685

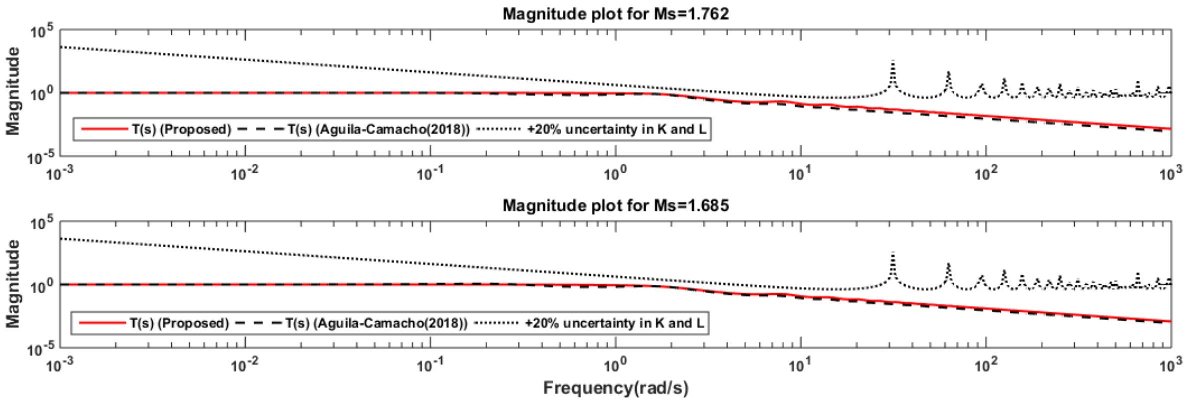

FIGURE 4. Example 1 magnitude plot

The nominal response for M_s of 1.685 is illustrated in Figure 3; the ISE, IAE, TV and ISI values are tabulated (Table 2). There is further improvement in the response with the proposed method whereas an increase in the overshoot is observed with method in [11] and settling time. The proposed FFPID controller rejects disturbance in a better way than the method used for comparison. Also, it ensures improved performance even with nonlinearity which is evident from Table 2. The robust stability for M_s of 1.762 and 1.685 is shown in Figure 4. All the methods are robust for +20% uncertainty in K and L proving the robust stability condition.

4.2. **Example 2.** The process model used for the study [12] is

$$G_m(s) = \frac{e^{-2.5s}}{s+1} \tag{20}$$

The step response when 'd' is introduced at $t = 100$s is displayed in Figure 5. The low values of errors and ISI listed in Table 3 are proof for enhanced performance with the proposed method; but the control effort is a bit high. The superiority of the proposed method is also proved for nonlinearity in the feedback signal. The closed system is stable for +20% uncertainty in K and L which is shown (Figure 6) in magnitude plot.

TABLE 3. Closed loop performance measures of Example 2

Method	Ideal				Non ideal			
	ISE	IAE	TV	ISI	ISE	IAE	TV	ISI
Proposed	8.379	12.67	2.0097	100.3	8.655	12.62	2.0004	98.3
Method in [12]	9.072	15.91	1.8832	101.8	9.353	15.94	1.8755	99.7

4.3. **Fragility.** The proposed method is robustness nonfragile for +20% of all parameter variation in the controller (Table 4) whereas the method in [11] is resilient. It is found that the proposed, Rahul and Anwar [12] methods are robustness nonfragile for +20% uncertainty (Table 4). The proposed method is performance nonfragile for $M_s = 1.762$

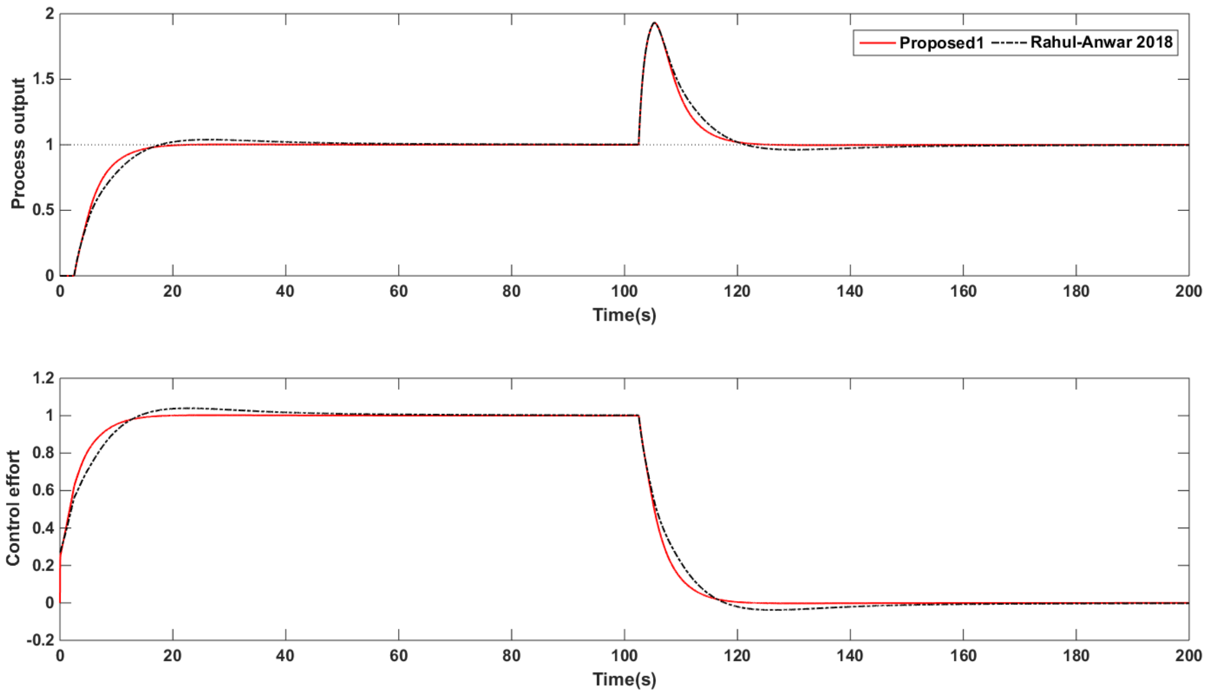

FIGURE 5. Closed loop response of Example 2 for nominal process conditions

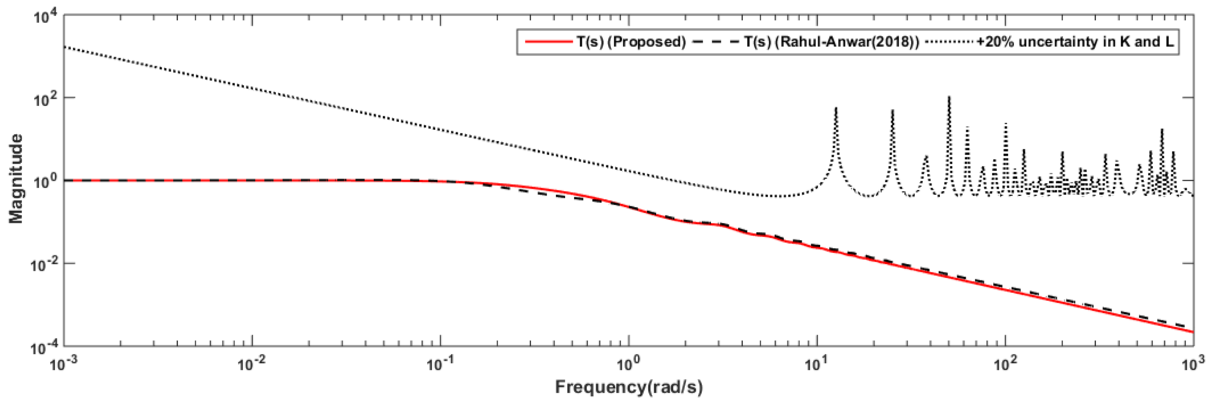

FIGURE 6. Example 2 magnitude plot

TABLE 4. Fragility index values for both the examples

Example	Method	$RFI_{\Delta 20}$	$PFI_{\Delta 20}$
Example 1	Proposed ($M_s = 1.762$)	0.231	0.259
	Method in [11]	0.098	-0.0185
Example 1	Proposed ($M_s = 1.685$)	0.21	0.233
	Method in [11]	0.096	0.502
Example 2	Proposed	0.205	-0.0063
	Method in [12]	0.335	0.219

(Table 4). Interestingly, the $PFI_{\Delta 20}$ value of method in [11] is negative meaning that there is an improvement in the performance. Similarly, the proposed method is performance nonfragile and method in [11] is fragile for $M_s = 1.685$. It is observed that there is an improvement in the performance with the proposed method as the PFI value becomes negative and the method in [12] is nonfragile.

5. **Conclusions.** An analytical design of FFPID controller is proposed for FOPTD plants using IMC method. This is achieved according to the systematic design procedure based on prefixed M_s minimizing IAE. The inclusion of M_s for controller design is justified as it ensures robust performance even with parametric uncertainties. An important observation is that p should be chosen as a value close to 1. As it moves away from 1, the overshoot increases in the response and the settling time. The proposed controller gives robust performance even under nonlinearity in the feedback path and for parametric uncertainties. The proposed controllers are both robustness nonfragile and performance nonfragile for controller parameter uncertainties.

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