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COMMAND FILTERED ADAPTIVE FUZZY BACKSTEPPING FAULT-TOLERANT CONTROL AGAINST ACTUATOR FAULT

Nanzhu Lu, Xun Sun, Xiaoxiao Zheng and Qikun Shen*

College of Information Engineering
Yangzhou University
No. 198, Huayang West Road, Yangzhou 225127, P. R. China
{ 462369405; 374193993 }@qq.com; zxx101404@126.com
*Corresponding author: qkshen@yzu.edu.cn

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Abstract. *This paper investigates the problem of fuzzy adaptive tracking control for a class of uncertain nonlinear strict-feedback systems with actuator fault. The actuator fault is assumed to have not only time-varying gain fault but also time-varying bias fault. Combining command filtered backstepping design with the integral-type Lyapunov function and utilizing Nussbaum-type gain technique, an adaptive fuzzy fault-tolerant control scheme is proposed to guarantee that the resulting closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin. The control scheme requires only virtual control and its first one derivative instead of the higher derivatives in backstepping design procedures. Simulation results demonstrate the effectiveness of the proposed techniques.*

Keywords: Fuzzy adaptive control, Command filtered backstepping, Fault-tolerant control

1. **Introduction.** Fuzzy control has found extensive applications for modeling nonlinear systems in the past ten years. According to the fuzzy approximation theorem of the fuzzy logic systems (FLSs) [1], researchers proposed many approximation-based adaptive fuzzy control design methods for nonlinear systems (see, e.g., [2], and the references therein).

It has been proved that adaptive backstepping technique is a powerful tool to propose an adaptive fuzzy output-feedback control for a class of pure-feedback uncertain nonlinear systems [3]. For such systems, many adaptive fuzzy backstepping controllers have been developed (see, e.g., [4] and the references therein), where an observer-based adaptive fuzzy control method is developed to solve the problem of stochastic nonlinear system with unknown-delay. It is well known that, however, in standard backstepping design procedure, analytic computation of the first derivatives of virtual control signals α_i ($i = 1, 2, \ldots, n-1$), i.e., $\dot{\alpha}_i$, is necessary. Note that, the computation of $\dot{\alpha}_i$ requires the higher derivatives of $\dot{\alpha}_j$, $j = 0, 1, \ldots, i-1$. Obviously, as system dimension, i.e., n, increases, the computation of $\dot{\alpha}_i$ becomes increasingly complicated. This limits the theoretical results' field of practical applications. Hence, how to reduce the computation of $\dot{\alpha}_i$ is a crucial issue in controller design, which is a motivation of this paper.

On the other hand, actuators, sensors or other system components in practical engineering fail frequently, which can cause system performance deterioration and lead to instability that can further produce catastrophic accidents. Thus, many effective fault tolerant control (FTC) approaches have been proposed to improve system reliability and to guarantee system stability in all situations [2,5]. An integrated fault estimation (FE) and fault-tolerant control design for a rigid spacecraft attitude system with inertia uncertainties, external disturbances, input saturation, and different type multiple actuator

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faults was proposed in [5]. However, most of the results concerning actuator faults reported in the literature only considered bias faults. Gain faults did not attract enough attention, which motivates this work, again.

In this paper, a bank of command filters (see, e.g., [6] and the references therein) are proposed to respectively generate the first derivations of the desired trajectory and virtual control signals. Then, by using backstepping technique, a robust adaptive fuzzy controller is proposed to guarantee that the tracking error converges to a neighborhood of the origin, where FLSs are utilized to approximate the unknown functions. The contributions of our work are generalized in the following aspects:

1) In contrast with the existing results such as [7,8] where the desired trajectory and its first n derivatives, i.e., $y_d^{(i)}(t)$, $i = 0, 1, \ldots, n$ should be available, the desired trajectory and only its first derivative are necessary for the control scheme presented in this paper, which is more reasonable in practical applications. The theoretic results of this paper are thus valuable in a wide field of practical applications;

2) Compared with the existing literature concerning the standard backstepping design, the control scheme presented in this paper does not need to compute the higher derivatives of virtual control signals in backstepping design procedures, which decreases the computation complexity;

3) Different from [9,10] where all system functions are known, the system functions considered in this paper are unknown. In particular, the signs of control gain functions are also unknown.

4) The actuator fault model that is presented in this paper integrates not only unknown gain faults, but also unknown bias faults, where both faults are dependent on the system state and will be approximated by FLSs.

2. Problem Statement and Preliminaries.

2.1. Problem statement.

Consider the following uncertain nonlinear systems:

$$\begin{cases} \dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)\, x_{i+1} + d_i(\bar{x}_{i+1}, t), \quad i = 1, 2, \ldots, n-1; \\ \dot{x}_n = f_n(\bar{x}_n) + g_n(\bar{x}_n)\, u(t) + d_n(\bar{x}_n, t); \\ y = x_1 \end{cases} \tag{1}$$

where $\bar{x}_i = (x_1, \ldots, x_i)^T \in R^i$, $i = 1, \ldots, n$ is the state; y denotes the output; $u \in R$ is the input; $f_i(\cdot) \in R$ and $g_i(\cdot) \in R$, $i = 1, \ldots, n$ are the unknown smooth functions; $d_i(\cdot, t)$, $i = 1, \ldots, n$, denote the unknown dynamic disturbances.

In practical applications, actuators may fail. The fault model considered in this paper can be described as follows:

$$u^f = g_f(\bar{x}_n)\, u + b_f(\bar{x}_n), \quad t > t_F \tag{2}$$

where $g_f(\bar{x}_n)$ and $b_f(\bar{x}_n)$ are smooth functions, which denote unknown gain fault and bias fault, respectively; t_F is an unknown fault occurrence time.

Control objective is to design an adaptive fuzzy controller by backstepping with command filter for system (1) such that output y can track accurately the desired trajectory y_d as possible regardless of actuator fault and unknown dynamic disturbances.

To design appropriate controller, the following lemma and some assumptions are given.

Lemma 2.1. *For $\forall x \in R$, $|x| - \tanh(x/\delta)x \le 0.2785\delta$, where $\delta > 0 \in R$.*

Assumption 2.1. *There exist known constants $g_{i0} > 0 \in R$ and $g_{i1} > 0 \in R$ such that $g_{i1} \ge |g_i(\bar{x}_i)| \ge g_{i0} > 0$, $\forall \bar{x}_i \in R^i$, $i = 1, 2, \ldots, n$.*

Assumption 2.2. *There exist unknown constant p_i^* and known smooth positive function $\phi_i(\bar{x}_i)$ such that $|d_i(\cdot, t)| \le p_i^* \phi_i(\bar{x}_i)$.*

Assumption 2.3. *The desired trajectory $y_d(t)$ and its first derivative are bounded and available.*

Assumption 2.4. *$g_f(\bar{x}_n)$ is bounded, i.e., there exist known constants $g_{f0} > 0 \in R$ and $g_1 > 0 \in R$ such that $g_{f1} \geq |g(\bar{x}_n)| \geq g_{f0}$.*

2.2. **Nussbaum type gain.** Any continuous function $N(s)$: $R \to R$ is a function of Nussbaum type if it has the following properties:

1) $\lim_{s\to+\infty} \sup \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = +\infty$;
2) $\lim_{s\to-\infty} \inf \frac{1}{s}\int_0^s N(\varsigma)d\varsigma = -\infty$.

For example, the continuous functions $\varsigma^2\cos(\varsigma)$, $\varsigma^2\sin(\varsigma)$, and $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ verify the above properties and are thus Nussbaum-type functions. The even Nussbaum function $e^{\varsigma^2}\cos((\pi/2)\varsigma)$ is used throughout this paper.

Lemma 2.2. [11] *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0,t_f)$ with $V(t) \geq 0$, $\forall t \in [0,t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. If the following inequality holds:*

$$V(t) \leq c_0 + \int_0^t \left(\underline{g}N(\varsigma) + 1\right)\dot{\varsigma}d\tau, \quad \forall t \in [0,t_f)$$

where $\underline{g} \neq 0$ is a constant, and c_0 represents a suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0,t_f)$.

Lemma 2.3. [12] *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0,t_f)$ with $V(t) \geq 0$, $\forall t \in [0,t_f)$, and $N(\cdot)$ be an even smooth Nussbaum-type function. For $\forall t \in [0,t_f)$, if the following inequality holds,*

$$V(t) \leq c_0 + e^{-c_1 t}\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}e^{c_1\tau}d\tau + e^{-c_1 t}\int_0^t \dot{\varsigma}e^{c_1\tau}d\tau$$

where constant $c_1 > 0$, $\underline{g}(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I := \left[l^{-1}, l^{+1}\right]$ with $0 \notin I$, and c_0 represents some suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t \underline{g}(\tau)N(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0,t_f)$.

2.3. **Mathematical description of fuzzy logic systems.** A fuzzy logic system consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules, and the defuzzifier. The knowledge base for FLS comprises a collection of fuzzy if-then rules of the following form:

$$\begin{aligned} R^l:\ & \text{if } x_1 \text{ is } A_1^l \text{ and } x_2 \text{ is } A_2^l \cdots \text{ and } x_n \text{ is } A_n^l, \\ & \text{then } y \text{ is } B^l, \quad l = 1,2,\ldots,M \end{aligned}$$

where $\underline{x} = [x_1,\ldots,x_n]^T \subset R^n$ and y are the FLS input and output, respectively. Fuzzy sets A_i^l and B^l are associated with the fuzzy functions $\mu_{A_i^l}(x_i) = \exp\left(-\left(\frac{x_i - a_i^l}{b_i^l}\right)^2\right)$ and $\mu_{B^l}\left(y^l\right) = 1$, respectively. M is the rules number. Through singleton function, center average defuzzification and product inference, the FLS can be expressed as:

$$y(x) = \sum_{l=1}^M \bar{y}^l\left(\prod_{i=1}^n \mu_{A_i^l}(x_i)\right) \Big/ \sum_{l=1}^M\left(\prod_{i=1}^n \mu_{A_i^l}(x_i)\right)$$

where $\bar{y}^l = \max_{y\in R}\mu_{B^l}$. Define the fuzzy basis functions as:

$$\xi_l(x) = \prod_{i=1}^n \mu_{A_i^l}(x_i) \sum_{l=1}^M \left(\prod_{i=1}^n \Big/ \mu_{A_i^l}(x_i)\right)$$

and define $\theta^T = \left[\bar{y}^1, \bar{y}^2, \dots, \bar{y}^M\right] = [\theta_1, \theta_2, \dots, \theta_M]$ and $\xi(x) = [\xi_1(x), \dots, \xi_M(x)]^T$, and then the above FLS can be rewritten as:

$$y(x) = \theta^T \xi(x)$$

Lemma 2.4. *Let $f(x)$ be a continuous function defined on a compact set Ω. Then for any constant $\varepsilon > 0$, there exists an FLS such as*

$$\sup_{x\in\Omega} \left| f(x) - \theta^T \xi(x) \right| \le \varepsilon$$

By Lemma 2.4, we know, FLS can approximate any smooth function on a compact space. Due to this approximation capability, we can assume that the nonlinear function $f(x)$ can be approximated as

$$f(x, \theta) = \theta^T \xi(x)$$

Define the optimal parameter vector θ^* as

$$\theta^* = \arg\min_{\theta\in\Omega} \left[\sup_{x\in U} |f(x) - f(x, \theta^*)| \right]$$

where Ω and U are compact regions for θ and x, respectively. Also the FLS minimum approximation error is defined as:

$$\varepsilon = f(x) - \theta^{*T} \xi(x)$$

Assumption 2.5. *There exists an unknown real bounded constant $\varepsilon^* > 0$ such that $|\varepsilon| \le \varepsilon^*$ on compact sets Ω and U.*

In this paper, we use the above FLS to approximate the unknown function $h_i(z_i)$, $(i = 1, \dots, n)$ which will be defined later, namely, there exist θ_i^* and ε_i such that

$$h_i(z_i) = \theta_i^{*T} \xi_i(z_i) + \varepsilon_i$$

3. **Design of Adaptive Fuzzy Controller and Stability Analysis.** Define

$$z_i = x_i - \alpha_{i-1}, \quad i = 1, 2, \dots, n \tag{3}$$

where $\alpha_0 = y_d$, α_{i-1} $(i = 2, \dots, n)$ is virtual control which will be designed at each step, $\alpha_n = u$ is actual control input. The recursive design procedure contains n steps. From Step 1 to Step $n-1$, α_i $(i = 1, \dots, n-1)$ is designed at each step. Finally an overall control law $u(\alpha_n)$ is constructed at Step n.

In order to estimate the virtual control α_{i-1}, $i = 2, \dots, n$, define the following command filter

$$\dot{\omega}_i = -\eta_\omega(\omega_i - \alpha_{i-1}), \quad i = 2, \dots, n \tag{4}$$

where $\eta_\omega > 0$ is a design parameter. Let us define the estimation error signal v_i as

$$v_i = \omega_i - \alpha_{i-1}, \quad i = 2, \dots, n$$

Remark 3.1. *The command filter (4) is constructed to avoid the computation of the higher derivatives of α_{i-1}, $i = 2, \dots, n$. It should be pointed out that the error v_i will be compensated at Step n in this paper.*

Step 1: Define the following function

$$V_{z_1} = \int_0^{z_1} \frac{\sigma}{|g_1(\sigma + y_d)|} d\sigma \tag{5}$$

Virtual control α_1 is defined as follows:

$$\alpha_1 = N(\varsigma_1) \left[k_1 z_1 + h_1\left(z_1, \hat{\theta}_1\right) + \hat{b}_1 \bar{\varphi}_1(x_1) \tanh\left(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1} \right) \right] \tag{6}$$

$$\dot{\varsigma}_1 = k_1 z_1^2 + h_1\left(z_1, \hat{\theta}_1\right) z_1 + \hat{b}_1 \bar{\varphi}_1(x_1) z_1 \tanh\left(\frac{z_1 \bar{\varphi}_1(\bar{x}_1)}{\eta_1} \right) \tag{7}$$

where $k_1 > 1$ is a design parameter; $h_1\left(z_1, \hat{\theta}_1\right) = \hat{\theta}_1^T \xi_1\left(\bar{z}_1\right)$ and $\hat{\theta}_1$ are estimates of $\theta_1^{*T} \xi_1\left(\bar{z}_1\right)$ and θ_1^*, respectively; $\hat{b}_1$ is an estimate of $b_1^* = \max\left\{\varepsilon_1^*, \frac{p_1^*}{g_{10}}\right\}$, $\bar{\varphi}_1\left(\bar{x}_1\right) = 1 + \varphi_1\left(\bar{x}_1\right)$.

Consider the following function

$$V_1(t) = V_{z_1} + \frac{1}{2}\left[\tilde{\theta}_1^T \Gamma_1^{-1} \tilde{\theta}_1\right] + \frac{1}{2\lambda_1}\tilde{b}_1^2 \tag{8}$$

Adaptive laws are defined as follows:

$$\dot{\hat{\theta}}_1 = \Gamma_1\left[z_1 \xi_1\left(\bar{z}_1\right) - \sigma_1 \hat{\theta}_1\right] \tag{9}$$

$$\dot{\hat{b}}_1 = \lambda_1\left[z_1 \bar{\varphi}_1\left(\bar{x}_1\right) \tanh\left(\frac{z_1 \bar{\varphi}_1\left(\bar{x}_1\right)}{\eta_1}\right) - \sigma_{b_1} \hat{b}_1\right] \tag{10}$$

where Γ_1 is a positive matrix with appropriate dimensions, $\sigma_1 > 0$, $\sigma_{b_1} > 0$, $\eta_1 > 0$ and $\lambda_1 > 0$ are design parameters.

Differentiating V_1 with respect to time t and considering (7)-(10), we have

$$\dot{V}_1 \leq -(k_1 - 1)z_1^2 + \frac{1}{4}z_2^2 + \frac{g_1\left(\bar{x}_1\right)}{\left|g_1(x_1)\right|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1 + 0.2785\eta_1 b_1^* - \sigma_1 \tilde{\theta}_1^T \hat{\theta}_1 - \sigma_{b_1} \tilde{b}_1 \hat{b}_1 + \Delta_1 \tag{11}$$

where Lemma 2.1 is used, namely, $0 \leq |x| - x \tanh\left(\frac{x}{\varepsilon}\right) \leq 0.2785\varepsilon$, $\forall \varepsilon > 0$, $\forall x \in R$.

Since

$$\sigma_1 \tilde{\theta}_1^{\mathrm{T}} \hat{\theta}_1 \leq -\frac{\sigma_1 \left\|\tilde{\theta}_1\right\|^2}{2} + \frac{\sigma_1 \left\|\theta_1^*\right\|^2}{2}, \quad \sigma_{b_1} \tilde{b}_1 \hat{b}_1 \leq -\frac{\sigma_{b_1} \tilde{b}_1^2}{2} + \frac{\sigma_{b_1} b_1^{*2}}{2} \tag{12}$$

then (11) can be derived as

$$\dot{V}_1 \leq -c_1 V_1 + \frac{1}{4}z_2^2 + \frac{g_1(\bar{x}_1)}{\left|g_1(x_1)\right|} z_1 N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma} + c_{\varepsilon 1} + \Delta_1 \tag{13}$$

where

$$c_{\varepsilon 1} = 0.2785\eta_1 b_1^* + \frac{\sigma_1 \left\|\theta_1^*\right\|^2}{2} + \frac{\sigma_{b_1} b_1^{*2}}{2}$$

$$c_1 = \min\left\{2(k_1 - 1)g_{10}, \frac{\sigma_1}{\lambda_{\min}\left(\Gamma_1^{-1}\right)}, \frac{\sigma_{b_1}}{\lambda_1}\right\}$$

Notice that, the boundedness of z_2 will be considered in the next step, and the error $e^{-c_1 t}\int_0^t e^{c_1 t}\Delta_1 d\tau$ will be compensated in Step n.

Step n: Define the following Lyapunov function

$$V_{z_n} = \int_0^{z_n} \frac{\sigma}{\left|\bar{g}_n\left(\bar{x}_{n-1}, \sigma + \alpha_{n-1}\right)\right|} d\sigma \tag{14}$$

From the analysis in the previous step, V_{z_n} is a positive definite function of z_n.

Similar to the previous steps, differentiating V_{z_n} with respect to time t, one has

$$\dot{V}_{z_n} \leq \frac{z_n}{\left|\bar{g}_n\left(\bar{x}_n\right)\right|}\left(\bar{g}_n\left(\bar{x}_n\right) u + d_n\left(\bar{x}_n, t\right)\right) + h_n'\left(\bar{z}_n\right) z_n + \Delta_n \tag{15}$$

where

$$\begin{aligned} h_n'\left(\bar{z}_n\right) = {} & \frac{\bar{f}_n\left(\bar{x}_n\right)}{\left|\bar{g}_n\left(\bar{x}_n\right)\right|} + \frac{1}{z_n}\int_0^{z_n} \sigma\left[\frac{\partial\left|\bar{g}_n^{-1}\left(\bar{x}_n, \sigma + \omega_n\right)\right|}{\partial \bar{x}_n}\dot{\bar{x}}_n d\sigma\right] \\ & + \frac{\dot{\omega}_n}{z_n}\int_0^{z_n} \frac{1}{\left|\bar{g}_n^{-1}\left(\bar{x}_n, \sigma + \omega_n\right)\right|} d\sigma \\ \Delta_n = {} & \int_0^{z_n} \sigma\left[\frac{\partial\left|\bar{g}_n^{-1}\left(\bar{x}_n, \sigma + \alpha_{n-1}\right)\right|}{\partial \bar{x}_n}\dot{\bar{x}}_n d\sigma\right] + \dot{\alpha}_{n-1}\int_0^{z_i} \frac{1}{\left|\bar{g}_n^{-1}\left(\bar{x}_{n-1}, \sigma + \alpha_{n-1}\right)\right|} d\sigma \end{aligned} \tag{16}$$

$$-\frac{1}{z_n}\int_0^{z_i}\sigma\left[\frac{\partial\left|\bar{g}_n^{-1}(\bar{x}_n,\sigma+\omega_n)\right|}{\partial\bar{x}_n}\dot{\bar{x}}_n d\sigma\right]-\frac{\dot{\omega}_n}{z_n}\int_0^{z_n}\frac{1}{\left|\bar{g}_n^{-1}\left(\bar{x}_n,\sigma+\omega_n\right)\right|}d\sigma \tag{17}$$

Let

$$h\left(\bar{Z}_n\right)=h'\left(\bar{Z}_n\right)+\sum_{j=1}^{n-1}\Delta_j$$

where $\bar{Z}_n=\left(\bar{x}_n^T,\bar{z}_n^T,\bar{\alpha}_n^T,\dot{\bar{\alpha}}_n^T,\bar{\omega}_n^T,\dot{\bar{\omega}}_n^T\right)^T$.

The actual control is defined as follows:

$$u=N(\varsigma_n)\left[k_nz_n+h_n\left(\bar{Z}_n,\hat{\theta}_n\right)+\hat{b}_n\bar{\varphi}\left(\bar{x}_n\right)\tanh\left(\frac{z_n\bar{\varphi}\left(\bar{x}_n\right)}{\eta_n}\right)\right] \tag{18}$$

$$\dot{\varsigma}_n=k_nz_n^2+h_n\left(\bar{Z}_n,\hat{\theta}_n\right)z_n+\hat{b}_n\bar{\varphi}\left(\bar{x}_n\right)z_n\tanh\left(\frac{z_n\bar{\varphi}\left(\bar{x}_n\right)}{\eta_n}\right) \tag{19}$$

where $k_n>\frac{1}{4}$ is a design parameter; $h_n\left(\bar{Z}_n,\hat{\theta}_n\right)=\hat{\theta}_n^T\xi_n\left(\bar{Z}_n\right)$ is an estimate of $\theta_n^{*T}\xi_n\left(\bar{Z}_n\right)$; $\hat{b}_n$ is an estimate of $b_n^*=\max\left\{\varepsilon_n^*,\frac{p_n^*}{g_{10}}\right\}$; $\bar{\varphi}_n\left(\bar{x}_n\right)=1+\varphi_n\left(\bar{x}_n\right)$.

Define the following Lyapunov function

$$V_n(t)=V_{n-1}+V_{z_n}+\frac{1}{2}\left[\tilde{\theta}_n^T\Gamma_n^{-1}\tilde{\theta}_n\right]+\frac{1}{2\lambda_n}\tilde{b}_n^2 \tag{20}$$

The following adaptive laws are defined as:

$$\dot{\hat{\theta}}_n=\Gamma_n\left[z_n\xi_n\left(\bar{Z}_n\right)-\sigma_n\hat{\theta}_n\right] \tag{21}$$

$$\dot{\hat{b}}_n=\lambda_n\left[z_n\bar{\varphi}_n\left(\bar{x}_n\right)\tanh\left(\frac{z_n\bar{\varphi}_n\left(\bar{x}_n\right)}{\eta_n}\right)-\sigma_{b_n}\hat{b}_n\right] \tag{22}$$

where Γ_n is a positive definite matrix, $\eta_n>0$, $\sigma_n>0$, $\sigma_{b_n}>0$ and $\lambda_n>0$ are design parameters.

Differentiating V_n with respect to time t and considering (21), (22) and Lemma 2.1, similar to the previous steps, one has

$$\dot{V}_n\leq\dot{V}_{n-1}-k_nz_n^2+\frac{\bar{g}_n\left(\bar{x}_n\right)}{\left|\bar{g}_n\left(\bar{x}_n\right)\right|}N(\varsigma_n)\dot{\varsigma}_n+\dot{\varsigma}_n+0.2785\eta_nb_n^*-\sigma_n\tilde{\theta}_n^T\hat{\theta}_n-\sigma_{b_n}\tilde{b}_n\hat{b}_n \tag{23}$$

Let $c_{\varepsilon n}=0.2785\eta_nb_n^*+\frac{\sigma_n\|\theta_n^*\|^2}{2}+\frac{\sigma_{b_n}b_n^{*\,2}}{2}$, and then (23) can be derived as

$$\dot{V}_n\leq\dot{V}_{n-1}-2k_n\left|\bar{g}_n\left(\bar{x}_n\right)\right|V_n+\frac{\bar{g}_n\left(\bar{x}_n\right)}{\left|\bar{g}_n\left(\bar{x}_n\right)\right|}mv(t)N(\varsigma_n)\dot{\varsigma}_n+\dot{\varsigma}_n+c_{\varepsilon n}$$
$$-\frac{\sigma_n\left\|\tilde{\theta}_n\right\|^2}{2}-\frac{\sigma_{b_n}\left\|\tilde{b}_n\right\|^2}{2} \tag{24}$$

Let

$$c_n=\min\left\{2k_n\bar{g}_{n0},\frac{\sigma_n}{\lambda_{\min}(\Gamma_n^{-1})},\frac{\sigma_{b_n}}{\lambda_n}\right\}$$

From the analysis in the previous steps, then (24) can be further developed as follows:

$$\dot{V}_n\leq\sum_{i=1}^n\left[\frac{\bar{g}_i\left(\bar{x}_i\right)}{\left|\bar{g}_i\left(\bar{x}_i\right)\right|}N(\varsigma_i)\dot{\varsigma}_i+\dot{\varsigma}_i+c_{\varepsilon i}\right] \tag{25}$$

Further, we have

$$\frac{d}{dt}\left(V_n(t)e^{c_nt}\right)\leq e^{c_nt}\sum_{i=1}^n\left[\frac{\bar{g}_i\left(\bar{x}_i\right)}{\left|\bar{g}_i\left(\bar{x}_i\right)\right|}N(\varsigma_i)\dot{\varsigma}_i+\dot{\varsigma}_i+c_{\varepsilon i}\right] \tag{26}$$

where $\bar{g}_i(\cdot)=g_i(\cdot)$, $i=1,\ldots,n-1$.

Let $\rho_n = \frac{\sum_{j=1}^{n} c_{\varepsilon j}}{c_n}$. Similar to the previous steps, integrating both the sides of the above inequality, we have

$$\begin{aligned} V_n(t) &\le \rho_n + [V_n(0) - \rho_n] e^{-c_n t} + e^{-c_n t} \int_0^t \left[e^{c_n t} \sum_{i=1}^{n} \left(\frac{\bar{g}_i(\bar{x}_i)}{|\bar{g}_i(\bar{x}_i)|} N(\varsigma_i) + 1 \right) \dot{\varsigma}_i \right] d\tau \\ &\le \rho_n + V_n(0) + e^{-c_n t} \int_0^t \left[e^{c_n t} \sum_{i=1}^{n} \left(\frac{\bar{g}_i(\bar{x}_i)}{|\bar{g}_i(\bar{x}_i)|} N(\varsigma_i) + 1 \right) \dot{\varsigma}_i \right] d\tau \end{aligned} \tag{27}$$

From Lemmas 2.2 and 2.3, it is easily seen that $V_n(t)$, ς_n, $\hat{\theta}_n$, $\hat{b}_n$ are bounded in $[0, t_f)$. From [13], the same results can be obtained in $[0, +\infty)$. Thus, it can be obtained that z_n is bounded in $[0, +\infty)$, which means that z_{n-1} in $(n-1)$th step is bounded. Doing the same reasoning, we finally obtained that all $z_i(t)$, $i = 1, 2, \ldots, n$ are bounded.

From the definitions of V_{z_i} and V_i, $i = 1, \ldots, n$, which are defined by (5), (8), (14) and (20), we know

$$V_n(t) = \sum_{i=1}^{n} \left[V_{z_i} + \frac{1}{2} \left(\tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i + \frac{1}{2\lambda_i} \tilde{b}_i^2 \right) \right] \tag{28}$$

From the previous analysis, we have

$$\frac{z_i^2}{2g_{i1}} \le V_{z_i} = \int_0^{z_i} \frac{\sigma}{|g_i(\bar{x}_{i-1}, \sigma + \alpha_{i-1})|} d\sigma \le \frac{z_i^2}{2g_{i0}} \tag{29}$$

Hence, from (27), (28) and (29), we have

$$|\bar{z}_i| \le \sqrt{\mu}, \quad \|\theta_i\|^2 \le \frac{\mu}{\lambda_{\min}\left(\Gamma_i^{-1}\right)}, \quad b_i^2 \le \lambda_i \mu^2, \quad i = 1, 2, \ldots, n, \quad \forall t \ge 0$$

where $\mu = 2\bar{g}_{\max}(\rho_n + V_n(0) + N_n)$, $\tilde{g}_{\max} = \max_{1 \le i \le n} \bar{g}_{i1} > 0$, $\bar{g}_{i1} = g_{i1}$, $i = 1, \ldots, n-1$, $\bar{g}_{n1} = g_{n1} g_{f1}$,

$$N_n = \lim_{t \to +\infty} \sum_{i=1}^{n} \left[e^{-c_n t} \int_0^t \left(\frac{\bar{g}_i(\bar{x}_i)}{|\bar{g}_i(\bar{x}_i)|} N(\varsigma_i) + 1 \right) e^{c_n t} \dot{\varsigma}_n d\tau \right] \tag{30}$$

The above design procedures and analysis are summarized in the following theorem.

Theorem 3.1. *Consider system (1) and fault (2). If Assumptions 2.1-2.5 hold, command filter (4), actual control defined by (18) and (19), and the adaptation laws (9), (10), (21) and (22) are employed, and then the closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin.*

Proof: From the aforementioned analysis, it is easy to obtain the conclusion. The detailed proof is omitted here.

4. **Illustrative Example.** In this example, a class of nonlinear systems are described as follows:

$$\begin{cases} \dot{x}_1 = x_1 + \left(1 + 0.5\sin\left(x_1^2\right)\right)x_2 + 0.2x_1\sin(x_2 t) \\ \dot{x}_2 = x_1 x_2 + (3 - \cos(x_1 x_2))u + 0.1\cos(0.5x_2 t) \\ y = x_1 \end{cases} \tag{31}$$

From (31), it is easily seen that $g_{10} = 0.5$, $g_{11} = 1.5$, $g_{20} = 2$, $g_{21} = 4$, $p_1^* = 0.2$, $\varphi_1 = x_1$, $p_2^* = 0.1$ and $\varphi_2 = 1$, which means that Assumptions 2.1 and 2.2 hold. In this work, the desired trajectory $y_d = 0.1\sin(t)$. Obviously, Assumption 2.3 holds. The actuator fault considered in this simulation research is described as follows:

$$u^f = (1 - 0.5\sin(x_2))u + \cos(x_1 x_2)$$

Obviously, $g_{f0} = 0.5$ and $g_{f1} = 1.5$, which means that Assumption 2.4 holds.

For this work, the following parameters are given: $k_1 = k_2 = 3$, $\Gamma_1 = \Gamma_2 = diag(1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$, $\lambda_1 = \lambda_2 = 1$, $\eta_1 = \eta_2 = 0.01$, $\sigma_{b_1} = \sigma_{b_2} = 0.1$, $\theta_i \in R^{10}$, $i = 1, 2$ are taken randomly in interval $(0, 1]$. Initial state $x(0)$ is set as $(0.2, 0.1)^T$. The sample time is 0.08s. From Figure 1, we can find that system (1) has good tracking performance.

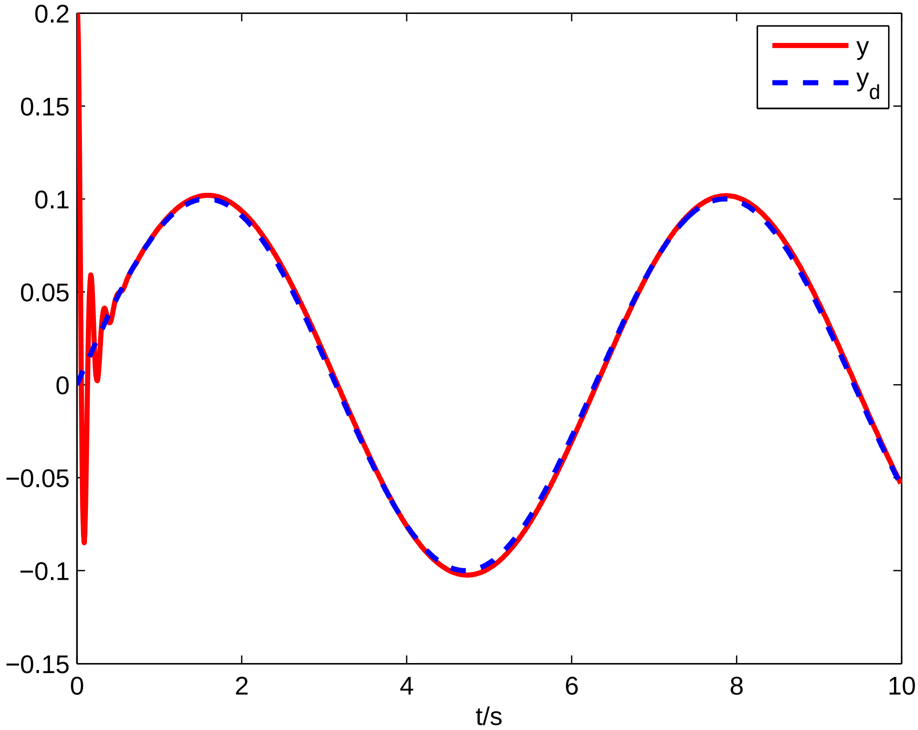

FIGURE 1. The time profiles of system output y and desired signal y_d

5. **Conclusions.** In this paper, an adaptive fuzzy tracking fault-tolerant control problem of a class of uncertain strict-feedback nonlinear systems with actuator fault has been investigated. FLSs are used to approximate the unknown nonlinear functions. By applying adaptive command filtered backstepping recursive design, integral-type Lyapunov function method and Nussbaum-type gain technique, an adaptive fuzzy control scheme is proposed to guarantee that the closed-loop system is asymptotically bounded with the tracking error converging to a neighborhood of the origin. In this paper, a certain relation should be satisfied among $d_i\left(\bar{x}_{i+1}, t\right)$ and $y_d(t)$ is available. However, without these restriction, how to achieve the practical tracking goal will be the future work.

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