ICIC Express Letters
Volume **15**, Number **4**, April **2021**

ICIC International ©2021 ISSN 1881-803X
pp. 383–387

ROUND-TRIP TIME CALCULATION OF BOTH FULL-FLOOR AND HIGH-RISE ELEVATORS CONSIDERING PASSENGERS' ARRIVAL RATES

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Received September 2020; accepted November 2020

Abstract. *The round-trip time of elevator varies according to its specifications such as speed, size, number of elevators, and building height. In addition to these elements, we newly introduce the passengers' arrival rates and their waiting behavior, and mathematically formulate the round-trip time of both full-floor and high-rise elevators. The results of the paper can be utilized in computing the service rate that is essentially necessary to calculate the passengers' waiting time for the elevator, based on which an effective elevator operating policy can be established.*
Keywords: Round-Trip Time (RTT), Elevator operating policy, Waiting time

1. **Introduction.** Traditionally, when designing a building, the number of elevators, its speed (acceleration) and capacity, etc. are determined considering the passengers' arrival rate, the purpose of the building, the number of floors, and facilities (dining rooms, meeting rooms) [1,2]. While the elevator's specifications are usually fixed when the building is constructed, the elevator operating policy would usually vary according to the passengers' arrival rates, which cannot be controlled by building manager after the building is completed. Once the number and specifications of elevators are determined and installed, the operation policy can be controlled by various methods such as full-floor operation, high-rise operation, and odd/even floor operation by the building manager, hence the traffic patterns are also very various such as random, up-peak, and down-peak [3,4].

Consider the office building with two elevators controlled by either operation of the full-floor or high-rise. In rush hour it might not be efficient to operate elevator as full-floor rather than the operating method that some are full-floor and the others are high-rise. If one of the elevators operates for high-rise only, the waiting time of high-rise visitors might decrease because the number of stops (visiting floors) is reduced. However, the total average waiting time of all visitors might increase in some cases because the other full-floor elevator becomes more crowded. This can be said that high-rise elevator operation itself cannot be optimal. So it is important to determine that how many floors the high-rise elevator should visit to minimize all passengers' waiting time for the elevators. In addition, passengers' waiting time may depend on their waiting behavior that in front of which elevator they should make line to get in the elevator on the first (original) floor. Although operating high-rise elevator can reduce the waiting time, if lots of the high-rise visitors move to the full-floor elevator, the efficiency of high-rise elevator decreases, which leads to the increment of vistors' total waiting time.

This paper provides a basic study on calculation of elevator round-trip time that is used to compute the elevator service rates, from which passengers' waiting time can be

DOI: 10.24507/icicel.15.04.383

derived [5-7]. The round-trip time calculation for only full-floor elevator was proposed by [8]. With the same approach, we newly consider the passengers' arrival rates and their waiting behavior on the first floor, and calculate the round-trip time for the following two types of elevator operating systems: full-floor elevator and high-rise elevator, which were not introduced in [8].

2. **Notations.** To begin, we define the round-trip time, shortly RTT, that refers to the time it takes for the elevator to depart from the first floor and return to the origin floor after all passengers on board get off at their destination. As shown in the left side of Figure 1, RTT can be given by the total sum of passengers' departure time, the time taken to open and close the elevator doors, the travel time to the arrival floor of the last passenger, and the descending time. In addition, it is necessary to figure out the average number of stops, denoted by S, to the highest floor, and the height of which is represented by H, where the elevator turns its direction to the downward (the right side of Figure 1).

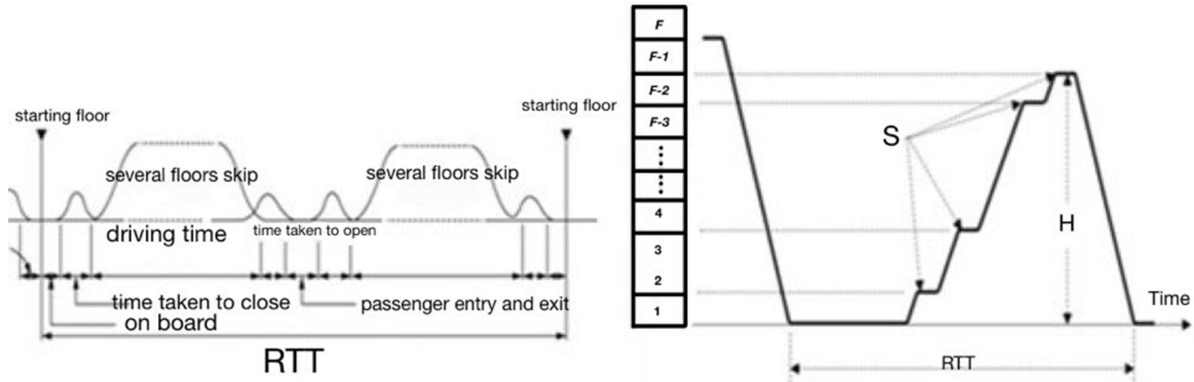

FIGURE 1. Elevator Round-Trip Time (RTT)

Our research is a basic study to calculate the RTT of elevator, so we deal with a simple elevator operating system that has only two types of elevators; one is full-floor elevator (denoted by EVa) and the other one is only high-rise elevator (denoted by EVh). In this case, RTT depends on not only the elevator specifications that is already determined when the building is designed but also passengers' arrival pattern and waiting behavior of which elevator to take. Considering the latter element that have not introduced so far, we formulate RTT for both full-floor and high-rise elevators.

The elevator-related terms in the paper are stated in accordance with the Korea Construction Standards Center (KCSC) [1], and the related notations are defined as follows:

B_h: Floor-to-floor height of a building (m).
v: Elevator maximum speed (m/sec).
a_v: Elevator acceleration (m/sec^2).
d_a: Acceleration distance to the maximum speed (m).
t_0: Operation time taken to move to the next stop (sec).
t_c: Door opening and closing time (sec).
t_d: Time taken for one passenger to get in and out (sec).
t_r: Travel time to the next stop (sec), $t_r = t_0 * H/S$.
F: The number of floors in a building.
h $(= 1, 2, \ldots, F-1)$: Number of visiting floors of a high-rise elevator. h counts from F (the highest floor) downwards. Therefore, the lowest floor that the high-rise elevator visits is $F - h + 1$.
λ_f: Passenger arrival rate to the fth floor $(1 \le f \le F)$.
Λ $(= \sum \lambda_f)$: Total passenger arrival rates.
λ_h: Passenger arrival rate to high-rise floors, $\lambda_h = \sum_{i=F-h+1}^{F} \lambda_i$.

λ_l: Passenger arrival rate to lower floors, $\lambda_l = \sum_{f=2}^{F-h} \lambda_f$.
q $(0 \leq q < 1)$: Probability of high-rise floor visiting passengers' using full-floor elevator.
n: Average number of riding passengers. Approximately 70% of the boarding capacity.
p_f: Probability that EVa visits the floor f $(2 \leq f \leq F)$, and it is defined as follows.

$$p_f = \begin{cases} \dfrac{\lambda_f}{\lambda_l + q\lambda_h}, & 2 \leq f < F - h + 1 \\ \dfrac{q\lambda_f}{\lambda_l + q\lambda_h}, & F - h + 1 \leq f \leq F \end{cases} \tag{1}$$

$\widetilde{p_f}$: Probability that EVh visits the floor f $(F - h + 1 \leq f \leq F)$, and it is defined as follows.

$$\widetilde{p_f} = \frac{\lambda_f}{\lambda_h}, \quad F - h + 1 \leq f \leq F \tag{2}$$

3. Round-Trip Time Calculation.

3.1. **Case of full-floor elevator.** First, for the convenience of proof, let us define the following random variable S_f that follows the binomial distribution:

$$S_f \sim B(n, p_f), \quad f = 2, 3, \ldots, F$$

Since S_f represents the number of passengers who get off at the fth floor when there are n passengers on EVa, the probability that EVa stops at floor f can be given by the probability that at least one passenger gets off, which means S_f is greater than or equal to one $(= 1)$. Hence,

$$\Pr(S_f \geq 1) = 1 - \Pr(S_f = 0) = 1 - (1 - p_f)^n$$

Therefore, the average number of stops before the passenger leaves the elevator, S_a is given as follows.

$$S_a = 1 + \sum_{f=2}^{F} (1 - (1 - p_f)^n) = F - \sum_{f=2}^{F} (1 - p_f)^n \tag{3}$$

Next, we calculate H_a that is the height where the elevator turns its direction to the downward after all passengers get off at their destination. The probability that EVa will not rise above the $(f+1)$th floor is equal to the sum of the probability that each floor from the second floor to the fth floor will be the highest one. That is $\Pr(H = 2) + \Pr(H = 3) + \cdots + \Pr(H = f)$.

In the same way, the probability that EVa will not rise above fth floor is given as follows.

$$\Pr(H = 2) + \Pr(H = 3) + \cdots + \Pr(H = f - 1)$$

Therefore, we have $\Pr(H = f) = \Pr(\text{Eva does not rise above } (f + 1)\text{th floor}) - \Pr(\text{Eva does not rise above } f\text{th floor})$. In this equation, we can obtain $\Pr(\text{EVa does not rise more than } (f + 1)\text{th floor})$ as follows.

$$\begin{aligned} &\Pr(\text{EVa does not rise more than } (f + 1)\text{th floor}) \\ &= \Pr(\text{no passenger visits higher floor than } (f + 1)\text{th floor}) \\ &= \Pr(S_{f+1} = 0)\Pr(S_{f+2} = 0|S_{f+1} = 0) \cdots \Pr(S_F = 0|S_{f+1} = 0, S_{f+2} = 0, \ldots, S_{F-1} = 0) \end{aligned}$$

Now, since the total number of passengers using EVa is given by $\Lambda_a = \lambda_l + q\lambda_h$, the above expression can be arranged as follows.

$$\left(1 - \frac{\lambda_{f+1}}{\Lambda_a}\right)^n \left(1 - \frac{\lambda_{f+2}}{\Lambda_a - \lambda_{f+1}}\right)^n \cdots \left(1 - \frac{q\lambda_{F-h+1}}{\Lambda_a - \lambda_{f+1} - \cdots - \lambda_{F-h+2}}\right)^n \cdots \left(1 - \frac{q\lambda_F}{\Lambda_a - \lambda_{f+1} - \cdots - q\lambda_{F-1}}\right)^n$$

$$= \left(\frac{\Lambda_a - \lambda_{f+1}}{\Lambda_a}\right)^n \left(\frac{\Lambda_a - \lambda_{f+1} - \lambda_{f+2}}{\Lambda_a - \lambda_{f+1}}\right)^n \cdots \left(\frac{\Lambda_a - \lambda_{f+1} - \cdots - q\lambda_F}{\Lambda_a - \lambda_{f+1} - \cdots - q\lambda_{F-1}}\right)^n$$
$$= \left(\frac{\Lambda_a - \lambda_{f+1} - \cdots - q\lambda_F}{\Lambda_a}\right)^n$$
$$= \left(\frac{\sum_{i=2}^{f} \lambda_i}{\Lambda_a}\right)^n \quad \therefore \Lambda_a = \lambda_2 + \lambda_3 + \cdots + q\lambda_F$$

In the same way we can obtain the following.

$$\Pr(\text{EVa does not rise above } f\text{th floor}) = \left(\frac{\sum_{i=2}^{f-1} \lambda_i}{\Lambda_a}\right)^n$$

Therefore, we have

$$\Pr(H = f) = \left(\frac{\sum_{i=2}^{f} \lambda_i}{\Lambda_a}\right)^n - \left(\frac{\sum_{i=2}^{f-1} \lambda_i}{\Lambda_a}\right)^n$$

Finally we have the average height of H_a as follows:

$$H_a = E(H) = \sum_{f=2}^{F} f\Pr(H = f) = \sum f \left(\left(\frac{\sum_{i=2}^{f} \lambda_i}{\Lambda_a}\right)^n - \left(\frac{\sum_{i=2}^{f-1} \lambda_i}{\Lambda_a}\right)^n\right)$$
$$= \left(\frac{1}{\Lambda_a}\right)^n \left(2\left(0 + \lambda_2^n\right) + 3\left(-\lambda_2^n + (\lambda_2 + \lambda_3)^n\right) + \cdots + F\left(-\left(\lambda_2 + \lambda_3 + \cdots + q\lambda_{F-1}\right)^n\right.\right.$$
$$\left.\left. + \left(\lambda_2 + \lambda_3 + \cdots + q\lambda_F\right)^n\right)\right)$$
$$= \left(\frac{1}{\Lambda_a}\right)^n \left(-\lambda_2^n - (\lambda_2 + \lambda_3)^n - \cdots - (\lambda_2 + \lambda_3 + \cdots + q\lambda_{F-1})^n\right.$$
$$\left. + F\left(\lambda_2 + \lambda_3 + \cdots + q\lambda_F\right)^n\right)$$
$$= \left(\frac{1}{\Lambda_a}\right)^n \left(-\lambda_2^n - (\lambda_2 + \lambda_3)^n - \cdots - (\lambda_2 + \lambda_3 + \cdots + q\lambda_{F-1})^n + F\Lambda_a^n\right)$$
$$= F + \left(-p_2^n - (p_2 + p_3)^n - \cdots - (p_2 + p_3 + \cdots + p_{F-1})^n\right) \quad (\therefore \text{Eq. (1)})$$
$$= F - \sum_{g=2}^{F-1} \left(\sum_{f=2}^{g} p_f\right)^n \tag{4}$$

Using the average number of stops (S_a) and the height of the highest floor (H_a) obtained above, we can calculate RTT of full-floor elevator as follows:

$$RTT_a = nt_d + S_a(t_c + t_r) + (3H_a - d_a)/v + 2v/a_v \tag{5}$$

3.2. **Case of high-rise elevator.** In the same way as Section 3.1 we can obtain the average number of stops (S_h) and the height of the highest floor (H_h) for EVh as follows:

$$S_h = \sum_{f=F-h+1}^{F} \left(1 - (1 - \widetilde{p_f})^n\right) = h - \sum_{f=F-h+1}^{F} (1 - \widetilde{p_f})^n$$

$$H_h = \begin{cases} F - \sum_{g=F-h+1}^{F-1} \left(\sum_{f=F-h+1}^{g} \widetilde{p_f}\right)^n, & h \geq 2 \\ F, & h = 1 \end{cases}$$

Therefore, RTT of high-rise elevator is expressed as follows:

$$RTT_h = \begin{cases} nt_d + S_h t_c + (S_h - 1)t_r + \dfrac{3(F - h + 1 + H_h/S_h) - d_a}{v} + \dfrac{3(F - h + 1 + H_h) - d_a}{v} + 2(2v/a_v), & F - h \geq d_a/B_h \\ nt_d + S_h(t_c + t_r) + (3H_a - d_a)/v + 2v/a_v, & F - h < d_a/B_h \end{cases} \tag{6}$$

4. **Examples of Calculation.** We present some numerical examples of the round-trip time using Equations (5) and (6) obtained in the previous section. The numerical experiments are implemented under the following conditions:

$B_h = 3$ m, $v = 3.5$ m/sec, $a_v = 1.6$ m/sec^2, $d_a = 7.65$ m, $t_0 = 4.2$ sec, $t_c = 4.2$ sec, $t_d = 2.5$ sec, $F = 15$, $n = 10$, λ_f $(2 \leq f \leq 15) \sim uni(0.15, 0.25)$.

Table 1 shows calculations of RTT_a and RTT_h with respect to the high-rise elevator's total visiting floors, h. From this we can see: 1) in both cases RTT_a is convex in h while RTT_h in concave in h, 2) RTT_a of case I is greater than that of case II for all h whereas RTT_h is the same in both cases (RTT_h is independent of q), which means that the more the passengers change their waiting line (i.e., the greater q becomes) the longer the full-floor elevator's travel time takes.

TABLE 1. Examples of round-trip time calculation (unit: sec)

h		1	2	3	4	5	6	7	8	9	10	11	12	13	14
Case I	RTT_a	128.9	125.0	120.6	117.3	114.3	111.8	110.0	109.1	109.3	111.2	113.2	117.5	124.4	132.8
$(q = 0.1)$	RTT_h	85.0	112.5	122.5	127.7	130.5	132.1	132.8	132.9	132.6	131.9	131.2	130.1	128.2	128.2
Case II	RTT_a	131.4	130.4	129.9	129.7	129.7	129.9	130.1	130.4	130.8	131.3	131.6	132.1	132.5	132.8
$(q = 0.6)$	RTT_h	85.0	112.5	122.5	127.7	130.5	132.1	132.8	132.9	132.6	131.9	131.2	130.1	128.2	128.2

5. **Conclusions.** We have successfully formulated the equation of elevator round-trip time for both full-floor and high-rise elevators taking into account not only various elevator specifications but also passengers' arrival rates and their waiting behavior that have not been introduced so far. We expect that our results can be utilized in calculating passenger's waiting time for elevator easily, based on which the optimal elevator operating policy can be derived by applying the queueing theory. To make our results more useful and realistic, we need to additionally consider the following factors in the future: 1) more than two elevators, 2) more than two types of elevator operation policies such as even floors, odd floors, and sky lobby, 3) external call type during elevator operation.

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