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RESEARCH ON THE KNOWLEDGE SHARING STRATEGY BETWEEN GOVERNMENT AND UNIVERSITIES BASED ON DIFFERENTIAL GAME

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Abstract. *Aiming at the collaborative innovation system formed by the government and universities, the differential game model is used to explore the knowledge sharing strategy between the two. The decision-making process and optimal strategy of both parties are discussed in three situations: Nash non-cooperative game, Stackelberg master-slave game, and cooperative game. The results show that the cost-sharing of government to universities can increase the optimal benefits of both parties; in the case of collaborative game, the overall benefit of the system reaches the Pareto optimal.*

Keywords: Knowledge sharing, Differential game, Pareto optimal, HJB equation, Optimal strategy, Collaborative innovation system

1. **Introduction.** In the era of knowledge economy, any organization, team, and individual cannot do without knowledge sharing. The main purpose of participating in knowledge sharing is to develop rapidly. While universities will actively participate in the knowledge activities they need, and may choose a passive participation attitude for the knowledge they do not need. At this time, the government needs to implement a knowledge sharing subsidy strategy for universities to encourage universities to actively participate in knowledge sharing activities.

For the knowledge and understanding of knowledge sharing, in recent years, many scholars have conducted research by establishing relevant game models. Koessler [1] constructed a Bayesian game model to study knowledge sharing strategies. Luo and Yin [2] analyzed the problem of knowledge sharing using an interest game model and proposed the incentives for knowledge sharing by factors such as knowledge absorptive capacity, benefit distribution, and moral hazard prevention. Li and Li [3] used game theory models to analyze the incentive strategies for knowledge sharing among different types of practice subjects. Hou and Lin [4] constructed a complete information game model to study knowledge sharing within organizations. Zhu et al. [5] and Qi et al. [6] discussed the optimal strategy of knowledge sharing among innovative subjects using game models. Jiang and Hu [7] used a master-slave game model to study the impact of knowledge transfer decisions between an enterprise and multiple partners on technological innovation, and pointed out that the enterprise's knowledge marginal revenue is large enough, and the partners are transferred according to the proportional structure of their respective knowledge marginal revenues. Zhu and Shi [8] used the knowledge transfer decision model to study the game Nash equilibrium of tacit knowledge sharing decision-making, and revealed the source of value added in the process of tacit knowledge transfer and sharing. Zang and Ma [9] used the evolutionary game model to analyze the cooperative innovation problems between enterprises and universities, and obtained evolutionary stability strategies. Wang

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and Bao [10] used evolutionary game theory to establish an evolutionary game model of knowledge sharing behavior among horizontal enterprises in a cluster supply chain, and analyzed the evolution path of knowledge sharing behavior in a cluster supply chain and the characteristics of enterprises that affect knowledge sharing behavior. Yao et al. [11] analyzed the encourage impact on knowledge sharing behaviors of participation users by using virtual community platform. They introduced the platform rewards, knowledge sharing income and knowledge sharing income as the impact factors. The virtual community knowledge sharing evolutionary game payment matrix they constructed could obtain the evolutionary equilibrium solution.

Therefore, this article uses the government to share a certain percentage of the cost of knowledge sharing for colleges and universities, so as to promote the active participation of colleges and universities in knowledge sharing activities, and the government can obtain corresponding social benefits. Make the two achieve a win-win situation. At the same time, this paper analyzes the problem of knowledge sharing between the two game subjects of the government and universities, and analyzes the Nash non-cooperative game, the Stackleberg master-slave game, and the balanced knowledge sharing strategy under the cooperative game. The research results show the situation of the cooperative game under the circumstance, and the overall income of the two reaches the Pareto optimal.

2. **Model Analysis.**

For the convenience of analysis, this paper only examines the collaborative innovation system composed of the government (S) and a single university (P). Assuming that the participants are completely rational, they aim at maximizing their own profits and have complete information. The efforts of government and university in knowledge sharing are $A_S(t)$ and $A_P(t)$ respectively. The cost function expressions of government and university on knowledge sharing are

$$C_S(t) = \frac{1}{2}k_S \cdot A_S^2(t); \quad C_P(t) = \frac{1}{2}k_P \cdot A_P^2(t)$$

where k_S and k_P respectively represent the knowledge sharing cost coefficients of government and university. $C_S(t)$ and $C_P(t)$ respectively represent the knowledge sharing cost of government and university at time t.

Assuming that $X(t)$ represents the knowledge level of cooperative innovation system at time t, which is determined by the degree of effort of the two in knowledge sharing. The knowledge level of the innovation system changes with time as follows:

$$X'(t) = \frac{dX(t)}{dt} = \beta_S A_S(t) + \beta_P A_P(t) - \delta X(t) \tag{1}$$

where the initial state of the knowledge level of the system $X(0) = X_0 \geq 0$; β_S and β_P respectively represent the degree of influence of the efforts made by government and university on knowledge sharing on the knowledge level of the innovation system. $\delta > 0$ represents the knowledge depreciation rate of the system when the effort of government and university on knowledge sharing is zero. The total revenue $W(t)$ of the innovation system at time t can be expressed as:

$$W(t) = \xi_S \cdot A_S(t) + \xi_P \cdot A_P(t) + \theta \cdot X(t) \tag{2}$$

where ξ_S and ξ_P respectively represent the impact of the efforts of government and university on knowledge sharing on the total revenue of the cooperative innovation system. θ is the influence coefficient of the new knowledge created by both parties in the knowledge sharing behavior on the system's knowledge sharing benefits.

Assuming that the benefits obtained by the collaborative innovation system of government and university are only distributed between the two participants, and university gets λ, government gets $1 - \lambda$. λ is a constant between $(0, 1)$, which is called the distribution coefficient. In order to encourage university to share knowledge, government will take the initiative to bear the knowledge sharing cost of $\psi(t)$ for it. $\psi(t) \in (0,1)$ is the subsidy factor of government to university. Suppose that both participants are rational decision-makers, with complete information, and the discount rate ρ ($\rho > 0$) at any time is the same.

The model established in this paper contains control variables $C_S(t)$, $C_P(t)$ and $\psi(t)$, and state variable $W(t)$. Because it is very difficult to solve under the condition of dynamic parameters, it is assumed that all other parameters are constants greater than zero and are not related to time. For convenience, $C_S(t)$, $C_P(t)$, $\psi(t)$, and $W(t)$ are abbreviated as C_S, C_P, ψ, and W.

2.1. **Nash non-cooperative game.** In this case, the government and university are independent and equal in status. Both parties will simultaneously, independently and rationally make their own optimal strategies in order to achieve their respective goals of maximizing profits. The combination of strategies constitutes a Nash equilibrium solution.

Hamilton-Jacobi-Bellman equation (HJB equation for short) is a partial differential equation that is the core of optimal control. The solution of the HJB equation is a real-valued function with the smallest cost for a specific dynamic system and related cost functions.

Assuming that government and university have optimal knowledge sharing income functions $V_S(X)$ and $V_P(X)$, which are continuously differentiable, HJB equation must be satisfied for all $X \geq 0$:

$$\begin{aligned}\rho V_S(X) = \max_{A_S \geq 0} \Bigg\{ &(1-\lambda)\left(\xi_S A_S + \xi_P A_P + \theta X\right) - \frac{K_S}{2} A_S^2 \\ &+ V_S'(X)(\beta_S A_S + \beta_P A_P - \delta X) \Bigg\}\end{aligned} \tag{3}$$

$$\rho V_P(X) = \max_{A_{P\geq 0}} \left\{ \lambda\left(\xi_S A_S + \xi_P A_P + \theta X\right) - \frac{k_P}{2} A_P^2 + V_P'(X)\left(\beta_S A_S + \beta_P A_P - \delta X\right) \right\} \tag{4}$$

The condition for solving the right part of the HJB equation and maximizing it is that Equations (3) and (4) respectively calculate the first-order partial derivatives of A_S and A_P, and make them equal to zero. Solve the values of A_S and A_P. Substitute A_S and A_P into (3) and (4) to get the following formulas.

$$\begin{aligned}\rho V_S(X) = &\left[(1-\lambda)\theta - \delta V_S'(X)\right] X + \frac{\left[(1-\lambda)\xi_S + \left(\beta_S V_S'(X)\right)\right]^2}{2k_S} \\ &+ \frac{\left[(1-\lambda)\xi_P + \xi_P V_S'(X)\right]\left(\lambda\xi_P + \beta_P V_P'(X)\right)}{k_P}\end{aligned} \tag{5}$$

$$\begin{aligned}\rho V_P(X) = &\left(\lambda\theta - \delta V_P'(X)\right) X + \frac{\left[(1-\lambda)\xi_S + \beta_S V_S'(X)\right]\left(\lambda\xi_S + \beta_S V_P'(X)\right)}{k_S} \\ &+ \frac{\left(\lambda\xi_P + \beta_P V_P'(X)\right)^2}{2k_P}\end{aligned} \tag{6}$$

It can be seen from the structure of Formulae (5) and (6) that the one-dimensional linear function formula with X as the independent variable is the solution of the HJB equation, so let (where m_1, m_2, n_1 and n_2 are constants)

$$V_S(X) = m_1 X + m_2; \quad V_P(X) = n_1 X + n_2 \tag{7}$$

$$V_S'(X) = dV_S(X)/dX = m_1; \quad V_P'(X) = dV_P(X)/dX = n_1 \tag{8}$$

From Equations (5)-(8), m_1, m_2, n_1, n_2 can be obtained. Then substituting the values of m_1 and n_1 into A_S and A_P, the optimal strategy for government and university can be obtained.

$$A_S^* = \frac{(1-\lambda)\left[(\rho+\delta)\xi_S + \theta\beta_S\right]}{(\rho+\delta)k_S} \tag{9}$$

$$A_P^* = \frac{\lambda \cdot \left[(\rho+\delta)\cdot\xi_P + \theta\cdot\beta_P\right]}{(\rho+\delta)\cdot k_P} \tag{10}$$

Substituting m_1, m_2, n_1, and n_2 into Formula (7), the expressions of the optimal knowledge sharing income functions $V_S(X)$ and $V_P(X)$ can be obtained respectively.

$$V_S^*(X) = \frac{(1-\lambda)\,\theta}{\rho+\delta}X + \frac{(1-\lambda)^2\left[(\rho+\delta)\,\xi_S + \theta\beta_S\right]^2}{2\rho k_S\,(\rho+\delta)^2} + \frac{\lambda\,(1-\lambda)\left[(\rho+\delta)\,\xi_P + \theta\beta_P\right]^2}{\rho k_P(\rho+\delta)^2} \tag{11}$$

$$V_P^* = \frac{\lambda\theta}{\rho+\delta}X + \frac{\lambda\,(1-\lambda)\left[\xi_S\,(\rho+\delta) + \theta\beta_S\right]^2}{\rho(\rho+\delta)^2 k_S} + \frac{\lambda^2\left[\xi_P\,(\rho+\delta) + \theta\beta_P\right]^2}{2\rho(\rho+\delta)^2 k_P} \tag{12}$$

$$\begin{aligned} V^*(X) &= V_S^*(X) + V_P^*(X) \\ &= \frac{\theta}{\rho+\delta}X + \frac{(1-\lambda^2)\left[(\rho+\delta)\,\xi_S + \theta\beta_S\right]^2}{2\rho(\rho+\delta)^2 k_S} + \frac{\lambda\,(2-\lambda)\left[(\rho+\delta)\,\xi_P + \theta\beta_P\right]^2}{2\rho(\rho+\delta)^2 k_P} \end{aligned} \tag{13}$$

2.2. **Stackelberg master-slave game.** In order to encourage university to carry out collaborative innovation, government will bear a certain percentage of knowledge sharing costs for university. Therefore, in this collaborative innovation system, the government can be regarded as a leader and the university as a follower of Stackelberg game. The government will determine in advance the proportion of subsidies for the cost of knowledge sharing in university and determine its own optimal knowledge sharing effort. After the university observes the decision of the government, it then makes the best knowledge sharing effort decision to maximize its own benefits.

Assuming that government and university have optimal knowledge sharing income functions $V_S(X)$ and $V_P(X)$, and they are continuously bounded and differentiable, the HJB equation must be satisfied for all $X \geq 0$. Solve unilateral optimal control problems in university.

$$\begin{aligned} \rho V_P(X) = \max_{A_P \geq 0} \Big\{ &\lambda\left(\xi_S A_S + \xi_P A_P + \theta X\right) - \frac{k_P}{2}\left(1-\psi\right) A_P^2 \\ &+ V_P'\left(\beta_S A_S + \beta_P A_P - \delta X\right) \Big\} \end{aligned} \tag{14}$$

The condition for solving the right part of the HJB equation to maximize it is that Equation (14) takes the first derivative of A_P and makes it zero to obtain the AP expression. The government will rationally predict that university will determine their optimal knowledge sharing effort strategy A_S based on A_P. At this time, the optimal control problem is

$$\begin{aligned} \rho V_S(X) = \max_{A_S \geq 0} \Big\{ &(1-\lambda)\left(\xi_S A_S + \xi_P A_P + \theta X\right) - \frac{k_S}{2}A_S^2 - \frac{k_P}{2}\psi A_P^2 \\ &+ V_S'(X)\left(\beta_S A_S + \beta_P A_P - \delta X\right) \Big\} \end{aligned} \tag{15}$$

Substituting A_P into Equation (15), and solving the right part of the equation, the condition to maximize it is that Equation (15) obtains the first-order partial derivatives of A_S and ψ, and makes them all zero. The A_S and ψ expressions are available. Substituting formulas A_P, A_S and ψ into Formulas (14) and (15), simplify and sort out the following formulas.

$$\begin{aligned}\rho V_S(X) = {} & \Big[(1-\lambda)\,\theta - \delta V_S'(X)\Big] X + \frac{\left[(1-\lambda)\,\xi_S + V_S'(X)\beta_S\right]^2}{2k_S} \\ & + \frac{\left[(2-\lambda)\,\xi_S + \left(2V_S'(X)+V_P'(X)\right)\beta_P\right]^2}{8k_P}\end{aligned} \tag{16}$$

$$\begin{aligned}\rho V_P(X) = {} & \Big(\lambda\theta - \delta V_P'(X)\Big) X + \frac{\left[(1-\lambda)\,\xi_S + V_S'(X)\beta_S\right]\left(\lambda\xi_S + \beta_S V_P'(X)\right)}{k_S} \\ & + \frac{\left(\lambda\xi_P + \beta_P V_P'(X)\right)\left[(2-\lambda)\,\xi_P + \left(2V_S'(X)+V_P'(X)\right)\beta_P\right]}{4k_P}\end{aligned} \tag{17}$$

It can be seen from the structure of Formulae (16) and (17) that the one-dimensional linear function formula with X as the independent variable is the solution of the HJB equation, so let (where m_1, m_2, n_1 and n_2 are constants)

$$V_S(X) = m_1 \cdot X + m_2; \quad V_P(X) = n_1 \cdot X + n_2 \tag{18}$$

$$V_S'(X) = dV_S(X)/dX = m_1; \quad V_P'(X) = dV_S(X)/dX = n_1 \tag{19}$$

We can know from the previous assumptions, formulas $V_P(X)$ and $V_S(X)$ should satisfy all $X \geq 0$, so m_1, m_2, n_1, and n_2 can be obtained. Then substituting m_1 and n_1 into equations A_S, A_P, ψ, the optimal knowledge sharing effort strategies $A_S(X)$ and $A_P(X)$ of government and university can be obtained, as well as the optimal government to universitiy the subsidy factor ψ is

$$A_S^{**} = \frac{(1-\lambda)\cdot\left[\xi_S\cdot(\rho+\delta)+\theta\cdot\beta_S\right]}{(\rho+\delta)\cdot k_S} \tag{20}$$

$$A_P^{**} = \frac{(2-\lambda)\cdot\left[(\rho+\delta)\cdot\xi_P+\theta\cdot\beta_P\right]}{2k_P\cdot(\rho+\delta)} \tag{21}$$

$$\psi^{**} = \begin{cases} \dfrac{2-3\lambda}{2-\lambda}, & 0<\lambda<\dfrac{2}{3} \\[2ex] 0, & \dfrac{2}{3}<\lambda<1 \end{cases} \tag{22}$$

where since $0<\psi<1$ and $0<\lambda<1$, we can find $0<\lambda<\frac{2}{3}$.

Substituting m_1, m_2, n_1, and n_2 into Equation (18), the optimal knowledge sharing income function $V_S(X)$ and $V_P(X)$ can be obtained.

$$V_P^{**}(X) = \frac{\lambda\theta}{\rho+\delta}X + \frac{\lambda\,(1-\lambda)\left[\xi_S\,(\rho+\delta)+\theta\beta_S\right]^2}{\rho(\rho+\delta)^2k_S} + \frac{\lambda\,(2-\lambda)\left[\xi_S\,(\rho+\delta)+\theta\beta_P\right]^2}{4\rho(\rho+\delta)^2k_P} \tag{23}$$

$$V_S^{**}(X) = \frac{(1-\lambda)\,\theta}{\rho+\delta}X + \frac{(1-\lambda)^2\left[\xi_S\,(\rho+\delta)+\theta\beta_S\right]^2}{2\rho(\rho+\delta)^2k_S} + \frac{(2-\lambda)^2\left[\xi_P\,(\rho+\delta)+\theta\beta_P\right]^2}{8\rho(\rho+\delta)^2k_P} \tag{24}$$

$$\begin{aligned}V^{**}(X) &= V_P^{**}(X)+V_S^{**}(X) \\ &= \frac{\theta}{\rho+\delta}X + \frac{(1-\lambda^2)\left[\xi_S\,(\rho+\delta)+\theta\beta_S\right]^2}{2\rho(\rho+\delta)^2\cdot k_S} + \frac{(4-\lambda^2)\left[\xi_P\,(\rho+\delta)+\theta\beta_P\right]^2}{8\rho(\rho+\delta)^2k_P}\end{aligned} \tag{25}$$

2.3. **Cooperative game.** As an organic whole, the government and university will aim to maximize the overall benefits of the cooperation system.

Assuming that the collaborative innovation system has an optimal profit function, and it is continuous, bounded and differentiable, the HJB equation must be satisfied for all $X \geq 0$:

$$\rho V(X) = \max_{A_S\geq 0;A_P\geq 0}\left\{\xi_S A_S + \xi_P A_P + \theta X - \frac{k_S}{2}A_S^2 - \frac{k_P}{2}A_P^2 + V'(X)\left(\beta_S A_S + \beta_P A_P - \delta X\right)\right\} \tag{26}$$

The condition for solving the right part of the HJB equation and maximizing it is that Equation (26) calculates the first-order partial derivative of A_S and A_P respectively, and sets it equal to zero to obtain the expressions of A_S and A_P. Substitute A_S and A_P into Equation (26) to get the following equation.

$$\rho V(X) = (\theta - \delta V'(X))\, X + \frac{(\xi_S + \beta_S V'(X))^2}{2k_S} + \frac{(\xi_P + \beta_P V'(X))^2}{2k_P} \tag{27}$$

It can be seen from the structure of Formula (27) that the one-dimensional linear function formula with X as the independent variable is the solution of the HJB equation, so let (where g_1 and g_2 are constants)

$$V(X) = g_1 \cdot X + g_2 \tag{28}$$

$$V'(X) = dV(X)/dX = g_1 \tag{29}$$

Substituting (28) and (29) into (27), we know from the previous assumption that: $V(X)$ is satisfied for all $X \geq 0$, so the values of g_1 and g_2 can be obtained, and g_1 is substituted into A_S, A_P can obtain the best knowledge sharing effort strategy A_S and A_P for government and university respectively.

$$A_S^{***} = \frac{\xi_S(\rho + \delta) + \theta\beta_S}{k_S(\rho + \delta)} \tag{30}$$

$$A_P^{***} = \frac{\xi_P(\rho + \delta) + \theta\beta_P}{k_P(\rho + \delta)} \tag{31}$$

Substituting g_1 and g_2 into Formula (28), the optimal profit function expression of the collaborative innovation system can be obtained.

$$V^{***}(X) = g_1 X + g_2 = \frac{\theta}{\rho + \delta} X + \frac{[\xi_S(\rho + \delta) + \beta_S\theta]^2}{2k_S(\rho + \delta)^2\rho} + \frac{[\xi_P(\rho + \delta) + \beta_P\theta]^2}{2k_P(\rho + \delta)^2\rho} \tag{32}$$

In this case, the optimal knowledge sharing revenue functions $V_S(X)$ and $V_P(X)$ can be obtained.

$$V_S^{***} = \frac{(1 - \lambda)\theta}{\rho + \delta} X + \frac{(1 - \lambda)[\xi_S(\rho + \delta) + \beta_S\theta]^2}{2k_S(\rho + \delta)^2\rho} + \frac{(1 - \lambda)[\xi_P(\rho + \delta) + \beta_P\theta]^2}{2k_P(\rho + \delta)^2\rho} \tag{33}$$

$$V_P^{***} = \frac{\lambda\theta}{\rho + \delta} X + \frac{\lambda[\xi_S(\rho + \delta) + \beta_S\theta]^2}{2k_S(\rho + \delta)^2\rho} + \frac{\lambda[\xi_P(\rho + \delta) + \beta_P\theta]^2}{2k_P(\rho + \delta)^2\rho} \tag{34}$$

3. **Analysis of Balanced Results.** In the three game situations, the optimal knowledge sharing effort strategy and optimal income among the government, university and the entire collaborative innovation system are different. By comparing the differences, we could obtain the conclusions.

From Formulas (9), (10), (20), (21), (22), (30), (31), we can get (where $0 < \lambda < \frac{2}{3}$)

$$A_S^{**} - A_S^{*} = 0; \quad A_S^{***} - A_S^{**} > 0; \quad A_P^{***} - A_P^{**} > 0; \quad A_P^{**} - A_P^{*} = A_P^{**} \cdot \psi^{**} > 0$$

Proposition 3.1. *The profit coefficient of the collaborative innovation system composed of government and university is $\lambda \in \left(0, \frac{2}{3}\right)$. The knowledge sharing effort level of university under the Stackelberg master-slave game situation is significantly improved compared to the Nash non-cooperative game situation. And the improvement intensity is equal to the proportion of government sharing the cost of knowledge sharing. It also shows the cost of knowledge sharing subsidy as an incentive mechanism, which encourages university to make more efforts to share knowledge than without subsidies. In both cases, the knowledge sharing efforts of government remain unchanged. When government and university conduct collaborative games, the optimal knowledge sharing effort of both parties reaches the* *maximum, and it is better than the Nash non-cooperative game situation. That is when* $\lambda \in \left(0, \frac{2}{3}\right)$*, we can get some conclusions.*

$$A_S^* = A_S^{**} < A_S^{***}; \quad A_P^* < A_P^{**} < A_P^{***}, \ \frac{A_P^{**} - A_P^*}{A_P^{**}} = \psi^{**}.$$

From Equations (11), (12), (23), (24), we can get some conclusions.

$$V_P^{**} - V_P^* > 0; \quad V_S^{**} - V_S^* > 0.$$

Proposition 3.2. *The profit coefficient of the collaborative innovation system composed of government and university is* $\lambda \in \left(0, \frac{2}{3}\right)$*. In the case of the Stackelberg master-slave game, the optimal knowledge sharing benefits of government and university are higher than those of the Nash non-cooperative game, and government and university prefer the Stackelberg master-slave game. Immediately* $0 < \lambda < \frac{2}{3}$*, we can get some conclusions.*

$$V_P^*(X) < V_P^{**}(X); \quad V_S^*(X) < V_S^{**}(X).$$

From Equations (13), (25), (32), we can get

$$V^{**}(X) - V^*(X) > 0; \quad V^{***}(X) - V^{**}(X) > 0.$$

Proposition 3.3. *The value range of the income distribution coefficient of the collaborative innovation system is* $\lambda \in \left(0, \frac{2}{3}\right)$*. The optimal benefit of the system is that it reaches the highest under the cooperative game, followed by the Stackelberg master-slave game, and the Nash non-cooperative game is the lowest. For the entire innovation system, the optimal return under the Stackelberg master-slave game situation is better than the optimal return under the Nash non-cooperative game. And when the two parties engage in a cooperative game, the optimal return of the system reaches the highest, better than non-cooperative game situation. Immediately* $0 < \lambda < \frac{2}{3}$*, we can get*

$$V^*(X) < V^{**}(X) < V^{***}(X).$$

4. **Analysis of Coordination Mechanism of Knowledge Sharing Behavior.** From Proposition 3.3, it can be seen that the total benefit of the collaborative innovation system under the cooperative game situation is the highest. If the income distribution of the government and the university is reasonable and feasible, the optimal knowledge sharing income of both parties in the cooperative game situation is higher than that in the non-cooperative situation. For both parties, the cooperative game is Pareto optimal. To coordinate the knowledge sharing behavior of both parties, the following conditions should be met.

$$V_S^{***}(X) - V_S^*(X) \geq 0; \quad V_P^{***}(X) - V_P^*(X) \geq 0 \tag{35}$$

$$V_S^{***}(X) - V_S^{**}(X) \geq 0; \quad V_P^{***}(X) - V_P^{**}(X) \geq 0 \tag{36}$$

It can be seen from Proposition 3.2, $V_P^*(X) < V_P^{**}(X)$ and $V_S^*(X) < V_S^{**}(X)$, only Formula (36) needs to be established. According to Formula (36), we can get the following results (Let $\pi_1 = \left[\xi_S (\rho + \delta) + \theta\beta_S\right]^2$; $\pi_2 = \left[\xi_P (\rho + \delta) + \theta\beta_P\right]^2$)

$$\text{Simplified: } \frac{2\pi_1 \cdot k_P}{4\pi_1 \cdot k_P + \pi_2 \cdot k_S} \leq \lambda \leq \frac{4\pi_1 \cdot k_P}{4\pi_1 \cdot k_P + \pi_2 \cdot k_S} \tag{37}$$

Because $0 < \lambda < \frac{2}{3}$, and obviously $0 < \frac{2\pi_1 \cdot k_P}{4\pi_1 \cdot k_P + \pi_2 \cdot k_S} < \frac{2\pi_1 \cdot k_P}{4\pi_1 \cdot k_P} = \frac{1}{2} < \frac{2}{3}$, now it only needs to discuss the size of $\frac{2}{3}$ and $\frac{4\pi_1 \cdot k_P}{4\pi_1 \cdot k_P + \pi_2 \cdot k_P}$.

$$\text{When } \frac{4\pi_1 k_P}{4\pi_1 k_P + \pi_2 k_S} \geq \frac{2}{3}, \text{ available: } \frac{\pi_1 k_P}{\pi_2 k_S} > \frac{1}{2}, \ \frac{2\pi_1 k_P}{4\pi_1 k_P + \pi_2 k_S} \leq \lambda < \frac{2}{3} \tag{38}$$

$$\text{When } \frac{4\pi_1 k_P}{4\pi_1 k_P + \pi_2 k_S} < \frac{2}{3}, \text{ available: } 0 < \frac{\pi_1 k_P}{\pi_2 k_S} < \frac{1}{2}, \ \frac{2\pi_1 k_P}{4\pi_1 k_P + \pi_2 k_S} \leq \lambda \leq \frac{4\pi_1 \cdot k_P}{4\pi_1 \cdot k_P + \pi_2 \cdot k_S} \tag{39}$$

Proposition 4.1. *In order to coordinate the knowledge sharing behavior of government and university, and realize that both parties can achieve the Pareto optimal under the highest overall income of the system, the optimal income distribution coefficient* λ *of the collaborative innovation system of government and university is set as Formulas (38) and (39).*

5. **Conclusion.** This paper uses differential game theory to study the problem of knowledge sharing under the innovation system formed by government and university, and uses dynamic programming equations to find their optimal knowledge sharing effort strategy in the three situations of Nash non-cooperative game, Stackelberg master-slave game, and cooperative cooperative game. The results show: 1) The knowledge sharing effort level of university in the Stackelberg master-slave game situation is significantly improved compared to the Nash non-cooperative game situation. The improvement intensity is equal to the proportion of government to the cost of knowledge sharing. It also shows that the cost of knowledge sharing subsidy as an incentive mechanism urges university to make more efforts in knowledge sharing behavior than without subsidy. However, the knowledge sharing effort of government and enterprises remains unchanged in these two situations. 2) When the income distribution of the collaborative innovation system is reasonable, the optimal income of the system is the highest under the cooperative game, the Stackelberg master-slave game is the second, and the Nash non-cooperative game is the lowest. 3) In order to coordinate the knowledge sharing behavior, and realize that both parties can achieve Pareto optimal under the situation of the highest overall benefit of the system, the value range of the benefit distribution coefficient λ is determined.

This article can be expanded from two aspects. 1) Consider extending the model of this article to the dynamic decision-making problem of cooperative innovation among multiple universities and multiple government. 2) Taking time parameters into consideration, study the model numerical solution under dynamic conditions.

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