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RELIABILITY IMPORTANCE MEASUREMENT BY CROSS-EVALUATION-BASED WEIGHTED LINEAR OPTIMIZATION

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Abstract. *For more realistic and accurate measurement of reliability importance, it is necessary to use a method by which the reliability importance for given multiple criteria such as quantitative and qualitative factors can be comprehensively evaluated. This paper introduces a method of comprehensive reliability importance measurement for a complex non-repairable system considering qualitative and quantitative criteria such as frequency of failure, repair time, repair cost, failure effect, and safety. The purpose of the current study was to determine the rank of each component in the system based on reliability importance measures that are obtained according to the cross-evaluation concept of DEA (Data Envelopment Analysis). As a demonstration of the proposed approach, we applied it to an example system consisting of 59 components in a series structure. Through a comparison with an interview-based approach, namely AHP (Analytic Hierarchy Process), the proposed approach showed highly reliable reliability importance ranking results.*
Keywords: Qualitative data, Reliability importance, Cross-evaluation, Data envelopment analysis

1. **Introduction.** Nowadays, modern systems have been becoming more complicated in response to the increasing demands of customers for higher reliability. For that reason, many researchers have studied, devised, and proposed methods that meet reliability (i.e., RAM: Reliability, Availability, and Maintainability) requirements such as redundancy allocation, hot-swappability, on-line repairability, and multi-stage interconnection design. In order to meet the reliability requirements of complex systems efficiently and effectively, it is necessary to first determine which components are more important to system reliability improvement or more critical to system failure. This attribute, called "reliability importance", is evaluated based on operation scenarios, system reliability structures (series, parallel, stand-based, etc.), and the lifetime distribution and maintenance characteristics of each component in the system.

Reliability importance indices are calculated through a combinatorial approach such as reliability block diagram, fault tree analysis, structure function, or Markov modeling. For this reason, many developers have often used reliability importance indices to improve system reliability and establish maintenance characteristics [1]. In the reliability literature, many well-known reliability importance measures have been proposed, such as Birnbaum importance, Barlow-Proschan importance, Fussell-Vesely importance, Natvig

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importance, importance for multistate systems, structure importance, and joint importance [2]. Birnbaum [3] first proposed the concept of reliability importance, classifying the importance measures into three classes (structure importance, reliability importance, and lifetime importance measures). These three important measures can be related to the effects of components' failures on system reliability. Thus, system design engineers have considered them when evaluating RAM for satisfaction of customer demands in the system design and development phase [4]. To evaluate each component's reliability contribution to system reliability, reliability block diagram often has been used. In the past, some researchers have considered series functional configuration with reliability block diagram, because it can easily obtain the reliability importance measures. Although there are mathematical difficulties in the case of a complex system that cannot be repaired, system reliability can be calculated, based on which, the importance of components can be evaluated. These results, however, will not be applied to the determination of the effects of components' reliability in a complex system, because the importance of each component depends on various factors such as failure rate, repair time, repair cost, criticality, and safety. Given these component characteristics, it has proved difficult to obtain system reliability for a complex repairable system via a mathematical model. Alternatively, simulation methods have been widely used to numerically obtain system reliability for such a system. Simulation can provide system-related statistics as well as components' event histories. Wang et al. [1] proposed a reliability importance measure FCI (Failure Criticality Index), which is the percentage of times that a system failure event is caused by a failure of a given component over the course of the simulation time.

However, it is not easy to collect complete information that is needed for simulation implementation in the initial system design and development phase. Moreover, simulation-based methods consider only quantitative factors (or criteria) in measuring the reliability importance of components in a complicated system, not qualitative factors such as criticality or maintenance difficulty. For more realistic and accurate measurement of reliability importance, it is necessary to derive a method that can comprehensively evaluate the importance of reliability when multiple criteria such as quantitative and qualitative factors are given. In the past, some studies have used quantitative information such as component's unreliability, reliability structure, and maintenance time. Also, the aspect of qualitative analysis for system reliability needs to be considered for evaluating system reliability effectively. However, the proposed method in the reliability area is used to calculate system reliability quantitatively. Thus, this paper introduces a comprehensive scheme of reliability importance measurement for a complex non-repairable system considering quantitative and qualitative criteria such as frequency of failure, repair time, repair cost, failure effect, and safety. The purpose of this study was to determine the rank of each component in the system based on reliability importance measures that are obtained by the cross-evaluation concept of DEA (Data Envelopment Analysis) that enables objective weight assignment and multi-criteria decision-making. The paper is structured as follows. Section 2 reviews the existing studies related to reliability importance, and Section 3 explains the cross-evaluation DEA procedure. Section 4 provides an example of cross-evaluation-based DEA experiment performed for one complex system. Finally, Section 5 concludes the paper.

2. **Literature Review.** Reliability importance measures have often been used to evaluate the effect of component reliability on system reliability and to assess system risk in the system design and development phase. Birnbaum's measure for independent components in a system is still one of the most popular importance measures [3]. It focuses on system reliability for small-scale systems with a series or parallel structure due to mathematical complexity. Also, various other importance measures such as criticality importance and redundant importance have been proposed to attain better solutions for reliability

optimization problems [5]. Kuo and Zhu [6] made a comprehensive contribution to importance measures in reliability engineering and explained reliability optimization problems in terms of importance measures such as redundancy allocation, improved system reliability, and component assignment.

Recently, simulation methods such as Monte-Carlo and discrete-event simulation have been used to solve reliability optimization problems of complex systems. Yun et al. [7] proposed a discrete-event simulation method that uses an object-oriented model to estimate RAM for a multi-unit system of a complex reliability structure. Later, Chung [8] developed a complex simulation model for evaluating the RAM of Multi-Indenture Multi-Echelon (MIME) systems. A simulation method can provide approximations of system performance measure and is useful for support of near-optimal solution discovery in reliability optimization problems. Han and Yun [9] used the simulation model proposed by Yun et al. [7] to find the weakest importance among components in a system based on cost-effectiveness. Han et al. [10] studied an optimal maintenance strategy for 145 kV gas-insulated switchgear and determined the weakest importance of a component that should be maintained more frequently, by a commercial simulation s/w, AvSim. In order to obtain accurate statistics related to system reliability through simulation, all information (failure rate, repair time, repair cost, etc.) required for the simulation should be given. However, it is difficult to obtain information in the initial system design and development phase, and thus a study evaluating the importance of components in a system through a qualitative method was also necessary.

3. **Methodology.** Since this proposed approach considers multiple criteria (i.e., five criteria: frequency of failure, repair time, repair cost, failure effect, and safety) in measuring the reliability importance of the components in a system, the problem to be solved can be considered to be of the Multi-Criteria Decision-Making (MCDM) type. The most important issue in MCDM problems is how to aggregate multiple criteria into a single measurement score in a proper manner by choosing a set of reasonable weights for each criterion. For assignment of reasonable weights to criteria, Data Envelopment Analysis (DEA) has been utilized in many areas. DEA provides a way to make systematic choices of the most favorable weights on multiple criteria for the evaluated Decision-Making Units (DMUs)' optimal scores, the weights being determined by solving mathematical programs. A DEA determines the optimal score for a DMU, and it can rank DMUs according to their scores. In terms of applying DEA to an MCDM problem, a DMU in DEA corresponds to multiple alternatives in MCDM, and the input and output factors in DEA correspond to multiple criteria in MCDM. Project selection, supplier selection, and ABC inventory classification are some popular application areas wherein DEA is used as a multi-factor performance measurement model [11].

To measure the reliability importance index in a multi-component system in terms of multiple criteria such as frequency of failure, repair time, repair cost, failure effect, and safety, we utilized the cross-evaluation concept of DEA. Conventional DEA has a shortcoming in that it is too flexible in choosing weights for input and output factors. This shortcoming is caused by the fact that a DMU can attain a full optimal score by choosing extremely high weights on some input or output factors and extremely low weights on other factors. This drawback may cause serious problems, especially when DEA is used in the MCDM context, since it may prevent a reasonably acceptable choice of weights for aggregating multiple criteria. This problem, in turn, leads to an unacceptable ranking of items in the reliability importance index. Several approaches have been proposed to address this problem, such as the restricted weighted model, the super-efficiency model, and the cross-efficiency model [12,13].

The main idea of the cross-evaluation concept is that it is a peer-evaluation approach, not self-evaluation in a conventional DEA model. The cross-evaluation concept has three advantages: first, it provides a unique ordering of DMUs; second, it eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from application-area experts; third, it allows for effective differentiation between good and poor performers. For these reasons, the cross-evaluation concept has been considered a powerful extension of DEA. Although the cross-evaluation concept of DEA has proven effective in ranking DMUs, there still exist some problems that limit its use. One such problem is the non-uniqueness of the calculated result (i.e., optimal score). Specifically, optimal scores obtained from the original DEA model are generally not unique, and depend on which of the alternative optimal solutions to the DEA linear programs is used. Several approaches have been developed to alleviate this problem such as aggressive models, benevolent models, the game cross-efficiency model, and the units-invariant multiplicative model. Especially, we apply the PEG (Pairwise Efficiency Game) model proposed by Talluri [14], which is one of the aggressive models. The reliability importance index of each component in a system is obtained by the cross-evaluation concept of DEA as follows.

Algorithm of cross-evaluation concept of DEA

Index:
r: Index of criterion ($r = 1, \ldots, s$)
k: Index of evaluated component
j: Index of component s

Parameters:
u_{rk}: Weight given to the r-th criterion of the k-th component
l_k: Reliability score of the k-th component in terms of different criteria
C_{pk}: Cross-evaluation score for the p-th component cross-evaluated by the k-th component

Method:

Step 1: Calculate the l_k of each component by following model (1)

$$\begin{aligned} l_k &= Max \sum_{r=1}^{s} u_{rk} y_{rk} \\ \text{s.t.} &\sum_{r=1}^{s} u_{rj} y_{rj} \leq 1, \quad j = 1, \ldots, n \\ &u_{rk} \geq 0, \quad \forall r \end{aligned} \tag{1}$$

Step 2: Calculate the cross-reliability importance score of each component by following model, where l_k^* is the optimal score of the k-th component as determined by model (1). If p and k are the same, the optimal cross-evaluation score will be the same as the relative score

$$\begin{aligned} C_{pk} &= Min \sum_{r=1}^{s} u_{rk} y_{rp} \\ \text{s.t.} &\sum_{r=1}^{s} u_{rj} y_{rj} = l_k^* \\ &\sum_{r=1}^{s} u_{rj} y_{rj} \leq 1, \quad j = 1, \ldots, n, \, j \neq k \\ &u_{rk} \geq 0, \quad \forall r \end{aligned} \tag{2}$$

Step 3: Calculate the reliability importance indices (I_p) of the component s by model (3), where C_{pk}^* is the optimal cross-evaluation score for the p-th component by the k-th component.

$$I_p = \frac{\sum_{k=1}^{n} C_{pk}^*}{n}, \quad p = 1, \ldots, n \tag{3}$$

Figure 1 shows an illustrative example of the cross-evaluation concept of the DEA algorithm for the five components with five criteria. The relative scores (l_k^*) for the five components are calculated.

Step 1: Reliability importance score of the five components

Component(k)	1	2	3	4	5
l_k^*	1.00	1.00	1.00	0.34	1.00

Step 2: Cross-reliability importance score of the five items

	Target component	Competing component 1	2	3	4	5
C_{pk}^*	1	1.00	0.01	0.59	0.35	0.06
	2	0.12	1.00	0.37	0.11	0.30
	3	0.12	$\boxed{0.93}$	1.00	0.11	0.70
	4	0.22	0.01	0.57	0.34	0.51
	5	0.12	0.93	0.55	0.11	1.00

Step 3: Reliability importance indices of the five components

Component(k)	1	2	3	4	5
I_p^*	0.32	0.58	0.62	0.20	0.51

Figure 1. Illustrative example of cross-evaluation

The scores of components 1, 2, 3, and 5 are the same, which means that the ranking is not precise. The cross-evaluation score table illustrates that each cell shows the cross-evaluation score of the competing components in the corresponding cell evaluated by the target component. For example, the cross-evaluation score of competing component 2 is 0.93, which is evaluated by target component 3. Based on the cross-reliability importance score of the five items, the reliability importance indices for the five components are calculated by model (3).

4. **Numerical Results.** As a demonstration of the proposed approach, we applied it to an example system consisting of 59 components in a series structure. The reliability importance of each of the units was evaluated in terms of five criteria: frequency of failure, repair time (minutes), repair cost (10,000 won), failure effect (%), and safety (%). All of these criteria were assumed to be positively related to the reliability importance of the components. The descriptive statistics for the five criteria of the components in the system are listed in Table 1. l_k^* and I_p^* for the reliability importance of all of the components and their ranking are presented in Figure 2, in which all of the DMUs are sorted in descending order of reliability importance index (I_p^*). According to the l_k^* score, 26 components (5, 26, 6, 27, ..., 22, and 9) have the same score 1, and thus are deemed to be of greater importance. However, the same score among many components possibly incurs

Table 1. Descriptive statistics for components' five criteria

Statistics	**Criteria**				
	Frequency of failure	**Repair time**	**Repair cost**	**Failure effect**	**Safety**
Avg	49.3	24.2	78.6	64.2	31.1
Max	50.0	75.0	600.0	90.0	60.0
Min	10.0	2.0	10.0	10.0	27.0
StDev	5.2	18.4	146.0	26.5	9.0

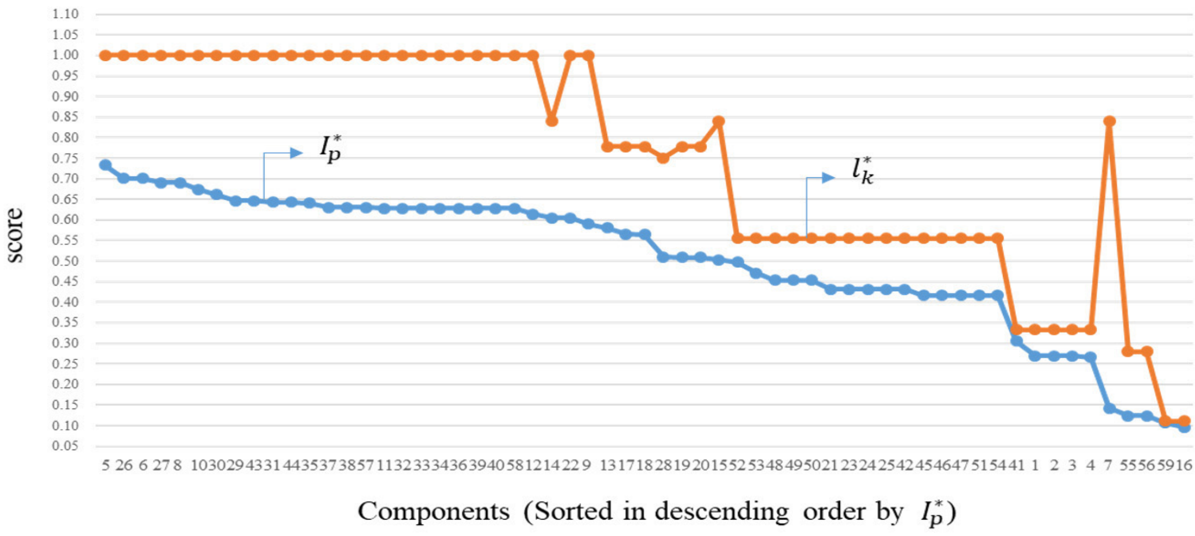

FIGURE 2. Reliability importance results for 59 components

the problem of insufficient differentiation among components and inaccurate ordering. In contrast, the reliability importance index (I_p^*) of the components yields a unique ordering. For example, the reliability importance of components is the highest in the order of components 5, 26, and 6, even if they have the same score of l_k^*.

In order to evaluate the effectiveness of the results of the proposed approach, the reliability importance ranking results were compared with engineer-interview-based approach, namely AHP (Analytic Hierarchy Process). As a result of AHP, the weights for each criterion of the system were calculated as 0.172 for frequency of failure, 0.213 for repair time, 0.201 for repair cost, 0.317 for failure effect, and 0.097 for safety. In the case of the system used in this numerical experiment, the expert engineers gave quantitative scores for each criterion by referring to Tables 2 and 3 in measuring the reliability importance of the components. Figure 3 shows the comparison results for the reliability importance index as obtained by cross-evaluation and interview, and it can be seen that the patterns for the two sets of rankings are very similar. In addition, the correlation coefficient for the ranking of the two approaches was 0.7283, indicating a high correlation. Therefore,

TABLE 2. Summarization of continuous data as categorical data

Grade	**Repair time**	**Repair cost**	**Effect of failure**	**Safety**	**Frequency of failure**
1	∼ 1 day	∼ 10	No effect	No effect	Occurs frequently
2	1 day ∼ 3 days	10 ∼ 50	Running but no monitoring	Recurring	Rarely occurs
3	3 days ∼ 1 week	50 ∼ 200	Running but no protection	Dying	Never occurs
4	1 week ∼ 2 weeks	200 ∼ 300	Running but degradation		
5	2 weeks ∼ 4 weeks	300 ∼ 600	Main functions are down		
6	4 weeks ∼ 6 weeks	600 ∼ 700			
7	6 weeks ∼ 8 weeks				
8	8 weeks ∼ 10 weeks				
9	10 weeks ∼ 12 weeks				

Table 3. Score of each criterion according to grade

	Grade	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Criteria	**Repair time**	1	2	6	11	21	35	49	63	75
	Repair cost	10	30	140	250	450	600			
	Failure effect	10	30	50	70	90				
	Safety	30	60	90						
	Frequency of failure	90	50	10						

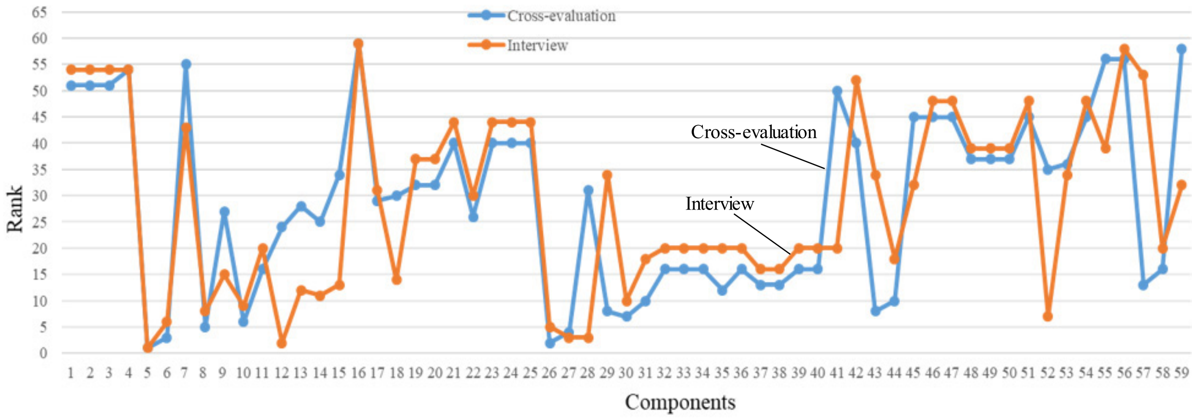

Figure 3. Ranking results for the proposed approach and interview-based approach

we may claim that the proposed approach shows highly reliable results comparable to those of the interview-based approach, and that it can be considered to be an alternative method to the existing subjective weight-assignment methods such as AHP.

5. **Conclusion.** In this paper, we introduced a comprehensive scheme of reliability importance measurement for a complex non-repairable system considering multiple qualitative and quantitative criteria. To achieve this, we utilized the cross-evaluation concept of DEA (Data Envelopment Analysis), and ranked all of the components in a system based on reliability importance indices. As a demonstration of the proposed approach, we applied it to an example system consisting of 59 components in a series structure. We showed that the reliability importance index of the components as obtained by the proposed approach yielded a unique ordering and a highly reliable result in its reliability importance ranking by comparison with the interview-based approach. However, in order to more accurately evaluate system importance, a method combining qualitative and quantitative methods is needed. In a further study, we will propose a new methodology to evaluate the reliability importance by given quantitative and qualitative multi-criteria.

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