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MULTI CRITERIA GROUP DECISION MAKING BASED ON VIKOR AND TOPSIS METHODS FOR FERMATEAN FUZZY SOFT WITH AGGREGATION OPERATORS

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Abstract. *Spherical fermatean fuzzy soft set is a generalization of fermatean fuzzy soft set and spherical fuzzy soft set. Now, we talk about the aggregated operation for aggregating spherical fermatean fuzzy soft decision matrix. Technique for Order of Preference by Similarity to Ideal Solution and VIekriterijumsko KOmpromisno Rangiranje approaches are strong point of view for multi criteria group decision making, which is a various generalization of fuzzy soft sets. We talk through a score function based on aggregating these two approaches to the spherical fermatean fuzzy soft positive ideal solution and the spherical fermatean fuzzy soft negative ideal solution. Also these two approaches are provided the weights of decision makings. To find out the optimal alternative under nearness is introduced. A medical company plans to invest some medicine in the stock exchange by purchasing some shares of the best five medical companies. In order to minimize the factor, they establish to invest their medicine various percentage.*

Keywords: Spherical fermatean fuzzy soft set, Technique for Order of Preference by Similarity to Ideal Solution, VIekriterijumsko KOmpromisno Rangiranje, Aggregation operator

1. **Introduction.** Multi criteria group decision making (MCGDM) refers to the problem of classifying or ranking the alternatives based on the opinions provided by multiple experts concerning multiple criteria [10, 26], which is a valuable research topic with extensive theoretical and practical backgrounds [9, 11]. Decision making problem indicates the finding of best optional alternatives. Hwang and Yoon [6] discussed multiple attributes decision making approach. Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) and VIekriterijumsko KOmpromisno Rangiranje (VIKOR) methods for decision making problems have been studied by Adeel et al. [1], Akram and Arshad [2], Boran et al. [4], Eraslan and Karaaslan [5], Peng and Dai [19], Xu and Zhang [24] and Zhang and Xu [29]. In 2021, Zulqarnain et al. discussed the TOPSIS extends to interval

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valued intuitionistic fuzzy soft set (IVIFSS). They also discussed a new type of correlation coefficient under IVIFSS [30]. TOPSIS approach consists of distances to positive ideal solution (PIS) and negative ideal solution (NIS), and calculating a preference order is ranked under relative nearness, and finding a combination of these two distance measures. VIKOR approach concept is based on ranking and selecting from a set of alternatives, and computes compromise solutions for a problem [12, 13]. Opricovic and Tzeng [14] discussed VIKOR approach using fuzzy logic. Tzeng et al. [22] discussed about comparison of VIKOR with TOPSIS approach using public transportation problem. A fuzzy set was introduced by Zadeh [28] and it suggests that decision-makers are to be solving uncertain problems by considering membership degrees. The concept of an intuitionistic fuzzy set was introduced by Atanassov and it is characterized by a degree of membership and non-membership satisfying the condition that sum of its membership degree and non-membership degree is not exceeding unity [3]. However, we may interact a problem in decision making (DM) events where the sum of the degree of membership and non-membership of a particular attribute is exceeding unity. So Yager [25] introduced the concept of Pythagorean fuzzy sets which is characterized by the condition that the square sum of its degree of membership and non-membership is not to exceed unity. In 2019, spherical fuzzy sets were introduced by Gündoğdu and Kahraman [31] as an extension of Pythagorean, neutrosophic and picture fuzzy sets. Ashraf et al. [32] discussed spherical fuzzy sets which is an advanced tool of the fuzzy sets and intuitionistic fuzzy sets. In decision making problems, times squares of sum of its degree of membership and non membership exceed unity. So Senapati and Yager introduced fermatean fuzzy set [21]. Also, fermatean fuzzy soft set is a generalization of the Pythagorean fuzzy soft set and it is characterized by the condition that the cubes of sum of its degree of membership and non membership are not to exceed unity.

The idea of a spherical fermatean fuzzy soft set based on TOPSIS and VIKOR is presented in this study, and some of its attributes are derived using the MCGDM technique. The paper is organized of six sections as follows. Section 1 is called an introduction. In Section 2, brief descriptions of spherical fermatean fuzzy soft sets are given. Section 3 talks through MCGDM based on spherical fermatean fuzzy soft-TOPSIS aggregating operator. Section 4 talks about MCGDM based on spherical fermatean fuzzy soft-VIKOR aggregating operator with real life example. Section 5 discusses about the comparison and discussion for the spherical fermatean fuzzy soft TOPSIS approach and spherical fermatean fuzzy soft VIKOR approach. Finally, the conclusion is provided in Section 6.

2. **Preliminaries.**

Definition 2.1. *Let $\mathbb{U}$ be a non-empty set of the universe, and spherical fermatean fuzzy set X in $\mathbb{U}$ is of the following form: $X = \{u, (\alpha_X(u), \beta_X(u), \gamma_X(u)) : u \in \mathbb{U}\}$, where $\alpha_X(u)$, $\beta_X(u)$ and $\gamma_X(u)$ represent the degree of positive, neutral and negative-membership of X, respectively. Consider the mapping $\alpha_X : \mathbb{U} \to [0,1]$, $\beta_X : \mathbb{U} \to [0,1]$, $\gamma_X : \mathbb{U} \to [0,1]$ and $0 \leq (\alpha_X(u))^3 + (\beta_X(u))^3 + (\gamma_X(u))^3 \leq 1$. Then $X = (\alpha_X, \beta_X, \gamma_X)$ is called a spherical fermatean fuzzy number (SFFN).*

Definition 2.2. *Let $\mathbb{U}$ and E be the universe and set of parameters, respectively. The pair (Υ, X) or Υ_X is called a spherical fermatean fuzzy soft (SFFS) set on $\mathbb{U}$ if $X \sqsubseteq E$ and $\Upsilon : X \to SFF^{\mathbb{U}}$, where $SFF^{\mathbb{U}}$ is denoted the set of all spherical fermatean fuzzy subsets of $\mathbb{U}$. That is, $\Upsilon_X = \left\{ \left(e, \left\{ \frac{u}{(\alpha_{\Upsilon_X}(u), \beta_{\Upsilon_X}(u), \gamma_{\Upsilon_X}(u))} \right\} \right) : e \in X, u \in \mathbb{U} \right\}$.*

Remark 2.1. *Let $p_{ij} = \alpha_{\Upsilon_X}(e_j)(u_i)$, $q_{ij} = \beta_{\Upsilon_X}(e_j)(u_i)$ and $r_{ij} = \gamma_{\Upsilon_X}(e_j)(u_i)$, where $1 \leq i \leq m$ and $1 \leq j \leq n$. Then the SFFS set Υ_X is defined in matrix form:*

$$\Upsilon_X = [(p_{ij}, q_{ij}, r_{ij})]_{m\times n} = \begin{bmatrix} (p_{11}, q_{11}, r_{11}) & (p_{12}, q_{12}, r_{12}) & \cdots & (p_{1n}, q_{1n}, r_{1n}) \\ (p_{21}, q_{21}, r_{21}) & (p_{22}, q_{22}, r_{22}) & \cdots & (p_{2n}, q_{2n}, r_{2n}) \\ \vdots & \vdots & \ddots & \vdots \\ (p_{m1}, q_{m1}, r_{m1}) & (p_{m2}, q_{m2}, r_{m2}) & \cdots & (p_{mn}, q_{mn}, r_{mn}) \end{bmatrix}.$$

This matrix is called a spherical fermatean fuzzy soft matrix (SFFSM).

Definition 2.3. *The cardinal set of the SFFS set Υ_X over $\mathbb{U}$ is an SFFS set over E and is defined as* $c\Upsilon_X = \left\{ \frac{e}{\left(\alpha_{c\theta_X}(e), \beta_{c\xi_X}(e), \gamma_{c\varphi_X}(e)\right)} : e \in E \right\}$, *where $\alpha_{c\theta_X}$, $\beta_{c\xi_X}$ and $\gamma_{c\varphi_X} : E \to [0,1]$ are mappings, respectively, where $\alpha_{c\theta_X}(e) = \frac{|\theta_X(e)|}{|\mathbb{U}|}$, $\beta_{c\xi_X}(e) = \frac{|\xi_X(e)|}{|\mathbb{U}|}$ and $\gamma_{c\varphi_X}(e) = \frac{|\varphi_X(e)|}{|\mathbb{U}|}$, where $|\theta_X(e)|$, $|\xi_X(e)|$ and $|\varphi_X(e)|$ denote the scalar cardinalities of the SFFS sets $\theta_X(e)$, $\xi_X(e)$ and $\varphi_X(e)$, respectively, and $|\mathbb{U}|$ represents cardinality of the universe $\mathbb{U}$. The collection of all cardinal sets of SFFS sets of $\mathbb{U}$ is represented as $cSFF^{\mathbb{U}}$. If $X \subseteq E = \{e_i : i = 1, 2, \ldots, n\}$, then $c\Upsilon_X \in cSFF^{\mathbb{U}}$ may be represented in matrix form as $[(p_{1j}, q_{1j}, r_{1j})]_{1\times n} = [(p_{11}, q_{11}, r_{11}), (p_{12}, q_{12}, r_{12}), \ldots, (p_{1n}, q_{1n}, r_{1n})]$, where $(p_{1j}, q_{1j}, r_{1j}) = \mu_{r\Upsilon_X}(e_j)$, $\forall j = 1, 2, \ldots, n$. This matrix is called as cardinal matrix of $c\Upsilon_X$ of E.*

Definition 2.4. *Let $\Upsilon_X \in SFF^{\mathbb{U}}$ and $c\Upsilon_X \in cSFF^{\mathbb{U}}$. The SFFS set aggregation operator $SFFS_{agg} : cSFF^{\mathbb{U}} \times SFF^{\mathbb{U}} \to SFFS(\mathbb{U}, E)$ is defined as* $SFFS_{agg}(c\Upsilon_X, \Upsilon_X) = \left\{ \frac{u}{\mu_{\Upsilon^*_X}(u)} : u \in \mathbb{U} \right\} = \left\{ \frac{u}{\left(\alpha_{\theta^*_X}(u), \beta_{\xi^*_X}(u), \gamma_{\varphi^*_X}(u)\right)} : u \in \mathbb{U} \right\}$. *This collection is called aggregate spherical fermatean fuzzy set of SFFS set Υ_X. The positive membership function $\alpha_{\theta^*_X}(u) : \mathbb{U} \to [0,1]$ by $\alpha_{\theta^*_X}(u) = \frac{1}{|E|}\sum_{e\in E} \alpha_{c\theta_X}(e)$, neutral membership function $\beta_{\xi^*_X}(u) : \mathbb{U} \to [0,1]$ by $\beta_{\xi^*_X}(u) = \frac{1}{|E|}\sum_{e\in E} \beta_{c\xi_X}(e)$ and negative membership function $\gamma_{\varphi^*_X}(u) : \mathbb{U} \to [0,1]$ by $\gamma_{\varphi^*_X}(u) = \frac{1}{|E|}\sum_{e\in E} \gamma_{c\varphi_X}(e)$. The set $SFFS_{agg}(c\Upsilon_X, \Upsilon_X)$ is expressed in matrix form as*

$$[(p_{i1}, q_{i1}, r_{i1})]_{m\times 1} = \begin{bmatrix} (p_{11}, q_{11}, r_{11}) \\ (p_{21}, q_{21}, r_{21}) \\ \vdots \\ (p_{m1}, q_{m1}, r_{m1}) \end{bmatrix}$$

where $[(p_{i1}, q_{i1}, r_{i1})] = \mu_{\Upsilon^*_X}(u_i)$, $\forall i = 1, 2, \ldots, m$. This matrix is called an SFFS aggregate matrix of $SFFS_{agg}(c\Upsilon_X, \Upsilon_X)$ over $\mathbb{U}$.

3. **SFFS under TOPSIS Aggregating Operator.** We can make an MCGDM based on SFFS-TOPSIS by the following flowchart and algorithm.

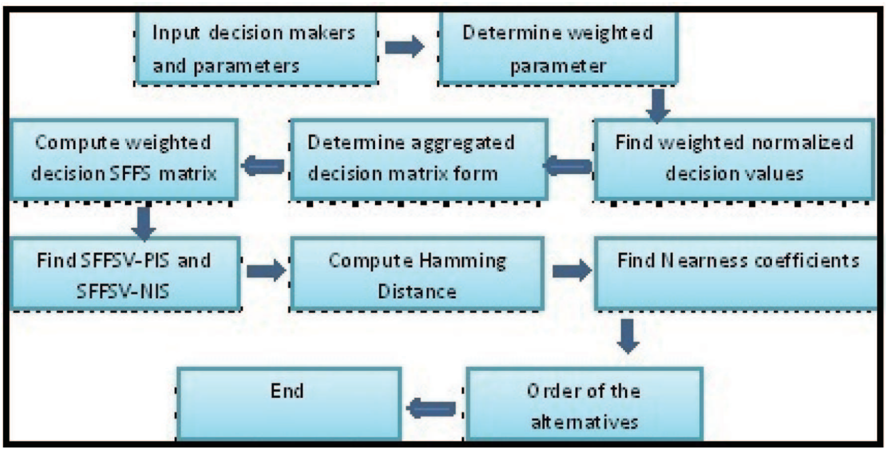

Figure 1. Flowchart representation using MCGDM based on TOPSIS

Step-1: Suppose that the finite number of decision makers $\mathscr{D} = \{\mathscr{D}_i : i \in \mathbb{N}\}$, the finite collection of alternatives $\mathscr{C} = \{\ddot{z}_i : i \in \mathbb{N}\}$ and finite family of parameters $\mathcal{D} = \{e_i : i \in \mathbb{N}\}$.

Step-2: Form a linguistic variable with weighted parameter matrix defined as $\mathcal{P} = (p_{ij})_{n\times m}$, where p_{ij} denotes $\mathcal{D}_i$ to $\mathcal{P}_j$ by considering linguistic variables.

Step-3: Determine weighted normalized decision matrix as $\widehat{\mathcal{N}} = (\widehat{n}_{ij})_{n\times m}$, where $\widehat{n}_{ij} = \frac{p_{ij}}{\sqrt[3]{\sum_{i=1}^{n} p_{ij}^3}}$ is called the normalized parameter and weighted vector $\mathcal{W} = (m_1, m_2, \ldots, m_m)$, where $m_i = \frac{p_i}{\sqrt[3]{\sum_{l=1}^{n} p_{li}}}$ is the weight of the jth parameter and $p_j = \frac{\sum_{i=1}^{n} \widehat{n}_{ij}}{n}$.

Step-4: Form the SFFS decision matrix $\mathscr{D}_i = \left(z_{jk}^i\right)_{l\times m}$, where z_{jk}^i is an SFFS element for the ith decision maker $\mathscr{D}_i$ for each i. Determine the aggregating matrix $\mathcal{Y} = \frac{\mathscr{D}_1+\mathscr{D}_2+\cdots+\mathscr{D}_n}{n} = (\dot{z}_{jk})_{l\times m}$.

Step-5: Determine the decision weighted SFFS matrix $\mathcal{Z} = (\ddot{z}_{jk})_{l\times m}$, where $\ddot{z}_{jk} = m_k \times \dot{z}_{jk}$.

Step-6: Calculate SFFS-PIS and SFFS-NIS. Now,
SFFS-PIS $= \left(\ddot{z}_1^+, \ddot{z}_2^+, \ldots, \ddot{z}_l^+\right) = \left\{\left(\vee_k \ddot{z}_{jk}, \wedge_k \ddot{z}_{jk}, \wedge_k \ddot{z}_{jk}\right) : k = 1, 2, \ldots, m\right\}$ and
SFFS-NIS $= \left(\ddot{z}_1^-, \ddot{z}_2^-, \ldots, \ddot{z}_l^-\right) = \left\{\left(\wedge_k \ddot{z}_{jk}, \vee_k \ddot{z}_{jk}, \vee_k \ddot{z}_{jk}\right) : k = 1, 2, \ldots, m\right\}$, where $\vee$ represents SFFS union and $\wedge$ represents SFFS intersection.

Step-7: Determine the SFFS Hamming distances from SFFSV-PIS and SFFSV-NIS. Since $d_j^+ = \left|\sum_{k=1}^{m}\left\{\left(\alpha_{jk}-\alpha_j^+\right)^3 + \left(\beta_{jk}-\beta_j^+\right)^3 + \left(\gamma_{jk}-\gamma_j^+\right)^3\right\}\right|$ and $d_j^- = \left|\sum_{k=1}^{m}\left\{\left(\alpha_{jk}-\alpha_j^-\right)^3 + \left(\beta_{jk}-\beta_j^-\right)^3 + \left(\gamma_{jk}-\gamma_j^-\right)^3\right\}\right|$, where $j = 1, 2, \ldots, n$.

Step-8: Compute the values for nearness with respect to ideal solution $C^*(\ddot{z}_j) = \frac{d_j^-}{d_j^+ + d_j^-} \in [0, 1]$.

Step-9: Illustrate the rank of alternatives using nearness coefficients based on decreasing (or) increasing order.

Step-10: Finally, output for the optimal alternative.

Example 3.1. *A medical company plans to invest some medicine in stock exchange by purchasing some shares of best five medical companies. In order to minimize the factor, they establish to invest their medicine percentage of 30, 25, 20, 15 and 10. Find the top five ranked companies.*

Step-1: A finite number of decision makers $\mathscr{D} = \{\mathscr{D} : i = 1, 2, 3, 4, 5\}$, the collection of medical companies/alternatives $\mathscr{C} = \{\ddot{z}_i : i = 1, 2, \ldots, 10\}$ and finite family of parameters $\mathcal{D} = \{e_i : i = 1, 2, 3, 4, 5\}$, put e_1 = Momentum, e_2 = Value, e_3 = Growth, e_4 = Volatility, e_5 = Quality.

Step-2: Determine weighted parameter matrix based on the linguistic variables.

Linguistic variables	Fuzzy weights
Very Good Settle (VGS)	0.95
Good Settle (GS)	0.9
Average Settle (AS)	0.8
Poor Settle (PS)	0.65
Very Poor Settle (VPS)	0.5

Determine the weighted parameter matrix (p_{ij} means weight of the $\mathcal{D}_i$ to $\mathcal{P}_j$).

$$\mathcal{P} = (p_{ij})_{5\times 5} = \begin{bmatrix} AS & GS & VGS & VPS & PS \\ VPS & VGS & GS & PS & AS \\ VGS & PS & VPS & AS & GS \\ AS & VPS & PS & VGS & VPS \\ PS & AS & VGS & GS & PS \end{bmatrix} = \begin{bmatrix} 0.8 & 0.9 & 0.95 & 0.5 & 0.65 \\ 0.5 & 0.95 & 0.9 & 0.65 & 0.8 \\ 0.95 & 0.65 & 0.5 & 0.8 & 0.9 \\ 0.8 & 0.5 & 0.65 & 0.95 & 0.5 \\ 0.65 & 0.8 & 0.95 & 0.9 & 0.65 \end{bmatrix}$$

Step-3: The weighted normalized decision matrix is

$$\widehat{\mathcal{N}} = (\widehat{n}_{ij})_{5\times 5} = \begin{bmatrix} 0.6077 & 0.6633 & 0.6706 & 0.3685 & 0.5234 \\ 0.3798 & 0.7002 & 0.6353 & 0.4791 & 0.6442 \\ 0.7217 & 0.4791 & 0.3529 & 0.5896 & 0.7247 \\ 0.6077 & 0.3685 & 0.4588 & 0.7002 & 0.4026 \\ 0.4938 & 0.5896 & 0.6706 & 0.6633 & 0.5234 \end{bmatrix}$$

and $\mathcal{W} = 0.1519, 0.1474, 0.1412, 0.1474, 0.161.$

Step-4: The aggregated decision matrix is

$$\mathcal{Y} = \frac{\mathscr{D}_1 + \mathscr{D}_2 + \mathscr{D}_3 + \mathscr{D}_4 + \mathscr{D}_5}{5}$$

$$= \begin{bmatrix} (0.5, 0.7, 0.65) & (0.85, 0.6, 0.7) & (0.8, 0.65, 0.75) & (0.65, 0.55, 0.65) & (0.7, 0.6, 0.5) \\ (0.6, 0.8, 0.5) & (0.6, 0.7, 0.5) & (0.6, 0.5, 0.8) & (0.6, 0.8, 0.6) & (0.6, 0.7, 0.55) \\ (0.5, 0.65, 0.7) & (0.5, 0.75, 0.65) & (0.5, 0.8, 0.7) & (0.65, 0.5, 0.7) & (0.55, 0.45, 0.75) \\ (0.5, 0.4, 0.8) & (0.65, 0.6, 0.7) & (0.7, 0.75, 0.6) & (0.6, 0.65, 0.75) & (0.5, 0.45, 0.8) \\ (0.7, 0.5, 0.75) & (0.75, 0.55, 0.65) & (0.8, 0.5, 0.55) & (0.5, 0.6, 0.65) & (0.65, 0.6, 0.7) \\ (0.6, 0.75, 0.65) & (0.9, 0.7, 0.5) & (0.85, 0.4, 0.6) & (0.8, 0.65, 0.45) & (0.8, 0.5, 0.55) \\ (0.4, 0.7, 0.6) & (0.5, 0.65, 0.75) & (0.5, 0.75, 0.65) & (0.75, 0.5, 0.6) & (0.6, 0.8, 0.55) \\ (0.5, 0.6, 0.7) & (0.5, 0.7, 0.85) & (0.65, 0.5, 0.75) & (0.75, 0.5, 0.65) & (0.55, 0.7, 0.5) \\ (0.8, 0.5, 0.6) & (0.45, 0.8, 0.6) & (0.75, 0.7, 0.8) & (0.6, 0.45, 0.7) & (0.6, 0.8, 0.4) \\ (0.65, 0.5, 0.75) & (0.55, 0.65, 0.7) & (0.6, 0.65, 0.5) & (0.65, 0.7, 0.5) & (0.7, 0.65, 0.8) \end{bmatrix}$$

$= (\dot{z}_{jk})_{10\times 5}.$

Step-5: The weighted decision SFFS matrix is

$$\mathcal{Z} = m_k \times \dot{z}_{jk}$$

$$= \begin{bmatrix} (0.076, 0.1064, 0.0988) & (0.1253, 0.0884, 0.1032) & (0.1129, 0.0918, 0.1059) \\ (0.0912, 0.1215, 0.076) & (0.0884, 0.1032, 0.0737) & (0.0847, 0.0706, 0.1129) \\ (0.076, 0.0988, 0.1064) & (0.0737, 0.1106, 0.0958) & (0.0706, 0.1129, 0.0988) \\ (0.076, 0.0608, 0.1215) & (0.0958, 0.0884, 0.1032) & (0.0988, 0.1059, 0.0847) \\ (0.1064, 0.076, 0.114) & (0.1106, 0.0811, 0.0958) & (0.1129, 0.0706, 0.0776) \\ (0.0912, 0.114, 0.0988) & (0.1327, 0.1032, 0.0737) & (0.12, 0.0565, 0.0847) \\ (0.0608, 0.1064, 0.0912) & (0.0737, 0.0958, 0.1106) & (0.0706, 0.1059, 0.0918) \\ (0.076, 0.0912, 0.1064) & (0.0737, 0.1032, 0.1253) & (0.0918, 0.0706, 0.1059) \\ (0.1215, 0.076, 0.0912) & (0.0663, 0.1179, 0.0884) & (0.1059, 0.0988, 0.1129) \\ (0.0988, 0.076, 0.114) & (0.0811, 0.0958, 0.1032) & (0.0847, 0.0918, 0.0706) \\ \\ (0.0958, 0.0811, 0.0958) & (0.1127, 0.0966, 0.0805) & \\ (0.0884, 0.1179, 0.0884) & (0.0966, 0.1127, 0.0886) & \\ (0.0958, 0.0737, 0.1032) & (0.0886, 0.0725, 0.1208) & \\ (0.0884, 0.0958, 0.1106) & (0.0805, 0.0725, 0.1288) & \\ (0.0737, 0.0884, 0.0958) & (0.1047, 0.0966, 0.1127) & \\ (0.1179, 0.0958, 0.0663) & (0.1288, 0.0805, 0.0886) & \\ (0.1106, 0.0737, 0.0884) & (0.0966, 0.1288, 0.0886) & \\ (0.1106, 0.0737, 0.0958) & (0.0886, 0.1127, 0.0805) & \\ (0.0884, 0.0663, 0.1032) & (0.0966, 0.1288, 0.0644) & \\ (0.0958, 0.1032, 0.0737) & (0.1127, 0.1047, 0.1288) & \end{bmatrix}$$

$= (\ddot{z}_{jk})_{10\times 5}.$

Step-6: The values for SFFSV-PIS and SFFSV-NIS can be calculated as

$\ddot{z}^+$	SFFSV-PIS
$\ddot{z}_1^+$	$(0.1253, 0.0811, 0.0805)$
$\ddot{z}_2^+$	$(0.0966, 0.0706, 0.0737)$
$\ddot{z}_3^+$	$(0.0958, 0.0725, 0.0958)$
$\ddot{z}_4^+$	$(0.0988, 0.0608, 0.0847)$
$\ddot{z}_5^+$	$(0.1129, 0.0706, 0.0776)$
$\ddot{z}_6^+$	$(0.1327, 0.0565, 0.0663)$
$\ddot{z}_7^+$	$(0.1106, 0.0737, 0.0884)$
$\ddot{z}_8^+$	$(0.1106, 0.0706, 0.0805)$
$\ddot{z}_9^+$	$(0.1215, 0.0663, 0.0644)$
$\ddot{z}_{10}^+$	$(0.1127, 0.076, 0.0706)$

$\ddot{z}^-$	SFFSV-NIS
$\ddot{z}_1^-$	$(0.076, 0.1064, 0.1059)$
$\ddot{z}_2^-$	$(0.0847, 0.1215, 0.1129)$
$\ddot{z}_3^-$	$(0.0706, 0.1129, 0.1208)$
$\ddot{z}_4^-$	$(0.076, 0.1059, 0.1288)$
$\ddot{z}_5^-$	$(0.0737, 0.0966, 0.114)$
$\ddot{z}_6^-$	$(0.0912, 0.114, 0.0988)$
$\ddot{z}_7^-$	$(0.0608, 0.1288, 0.1106)$
$\ddot{z}_8^-$	$(0.0737, 0.1127, 0.1253)$
$\ddot{z}_9^-$	$(0.0663, 0.1288, 0.1129)$
$\ddot{z}_{10}^-$	$(0.0811, 0.1047, 0.1288)$

Step-7: We find SFFS Hamming distances from SFFSV-PIS and SFFSV-NIS.

$\ddot{z}_i$	$\ddot{z}_1$	$\ddot{z}_2$	$\ddot{z}_3$	$\ddot{z}_4$	$\ddot{z}_5$	$\ddot{z}_6$	$\ddot{z}_7$	$\ddot{z}_8$	$\ddot{z}_9$	$\ddot{z}_{10}$
d_i^+	0.00009	0.0004	0.00010	0.00030	0.00007	0.0003	0.00002	0.0001	0.0004	0.00030
d_i^-	0.0002	0.0003	0.0001	0.0002	0.00008	0.0001	0.00009	0.0002	0.0003	0.0004

Step-8: The nearness coefficients from SFFSV-PIS and SFFSV-NIS.

$\ddot{z}_i$	$\ddot{z}_1$	$\ddot{z}_2$	$\ddot{z}_3$	$\ddot{z}_4$	$\ddot{z}_5$	$\ddot{z}_6$	$\ddot{z}_7$	$\ddot{z}_8$	$\ddot{z}_9$	$\ddot{z}_{10}$
C_i^*	0.6711	0.4024	0.5386	0.4281	0.5596	0.26	0.8368	0.6123	0.4228	0.5466

Step-9: The order of the alternatives C_i^* is $\ddot{z}_7 \geq \ddot{z}_1 \geq \ddot{z}_8 \geq \ddot{z}_5 \geq \ddot{z}_{10} \geq \ddot{z}_3 \geq \ddot{z}_4 \geq \ddot{z}_9 \geq \ddot{z}_2 \geq \ddot{z}_6$.

From the aforementioned information, a graph shows how much each medical firm invests according to MCGDM and TOPSIS approaches.

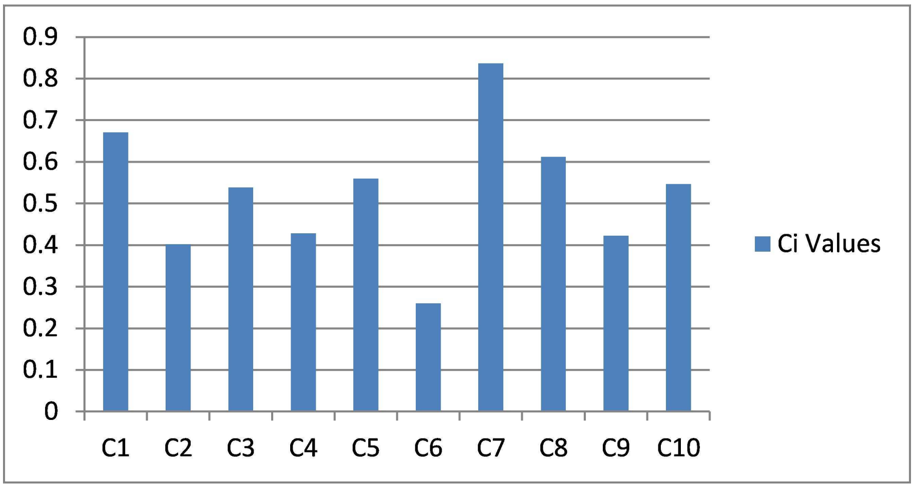

FIGURE 2. Graphical representation using MCGDM based on TOPSIS

Step-10: We conclude that the medical company $\ddot{z}_9$ invests the medicine 30%, $\ddot{z}_6$ invests the medicine 25%, $\ddot{z}_3$ invests the medicine 20%, $\ddot{z}_7$ invests the medicine 15% and $\ddot{z}_4$ invests the medicine 10%.

4. **SFFS-VIKOR Aggregating Operator.** We can make an MCGDM based on SFFS-VIKOR by the following flowchart and algorithm.

Step-1: Suppose that the finite number of decision makers $\mathscr{D} = \{\mathscr{D}_i : i \in \mathbb{N}\}$, the finite collection of alternatives $\mathscr{C} = \{\ddot{z}_i : i \in \mathbb{N}\}$ and finite family of parameters $\mathcal{D} = \{e_i : i \in \mathbb{N}\}$.

Step-2: Form a linguistic variable with weighted parameter matrix $\mathcal{P} = (p_{ij})_{n\times m}$, where p_{ij} denotes $\mathcal{D}_i$ to $\mathcal{P}_j$ by considering linguistic variables.

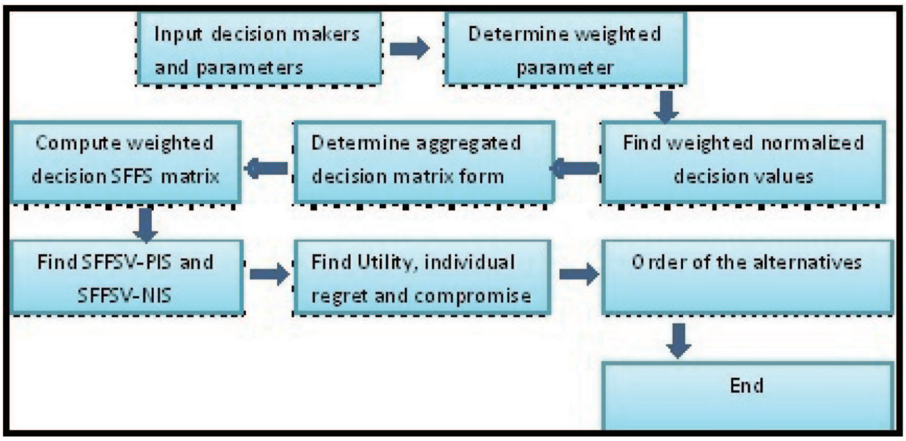

FIGURE 3. Flowchart representation using MCGDM based on VIKOR

Step-3: Determine weighted normalized decision matrix $\widehat{\mathcal{N}} = (\widehat{n}_{ij})_{n\times m}$, where $\widehat{n}_{ij} = \frac{p_{ij}}{\sqrt[3]{\sum_{i=1}^{n} p_{ij}^3}}$ is called the normalized parameter and weighted vector $\mathcal{W} = (m_1, m_2, \ldots, m_m)$, where $m_i = \frac{p_i}{\sqrt[3]{\sum_{l=1}^{n} p_{li}}}$ is the weight of the jth parameter and $p_j = \frac{\sum_{i=1}^{n} \widehat{n}_{ij}}{n}$.

Step-4: Form the SFFS decision matrix $\mathscr{D}_i = \left(z_{jk}^i\right)_{l\times m}$, where z_{jk}^i is an SFFS element for the ith decision maker $\mathscr{D}_i$ for each i. Determine the aggregating matrix $\mathcal{Y} = \frac{\mathscr{D}_1+\mathscr{D}_2+\cdots+\mathscr{D}_n}{n} = (\dot{z}_{jk})_{l\times m}$.

Step-5: Determine the decision weighted SFFS matrix $\mathcal{Z} = (\ddot{z}_{jk})_{l\times m}$, where $\ddot{z}_{jk} = m_k \times \dot{z}_{jk}$.

Step-6: Calculate SFFS-PIS and SFFS-NIS. Now,
SFFS-PIS $= \left(\ddot{z}_1^+, \ddot{z}_2^+, \ldots, \ddot{z}_l^+\right) = \left\{\left(\vee_k \ddot{z}_{jk}, \wedge_k \ddot{z}_{jk}, \wedge_k \ddot{z}_{jk}\right) : k = 1, 2, \ldots, m\right\}$ and
SFFS-NIS $= \left(\ddot{z}_1^-, \ddot{z}_2^-, \ldots, \ddot{z}_l^-\right) = \left\{\left(\wedge_k \ddot{z}_{jk}, \vee_k \ddot{z}_{jk}, \vee_k \ddot{z}_{jk}\right) : k = 1, 2, \ldots, m\right\}$, where $\vee$ represents SFFS union and $\wedge$ represents SFFS intersection.

Step-7: Determine the values for utility $\mathcal{S}_i$, individual regret $\mathcal{R}_i$ and compromise $\mathcal{Q}_i$, where $\mathcal{S}_i = \sum_{j=1}^{m} m_j \cdot \left|\frac{\ddot{z}_{ij}^3 - \ddot{z}_j^{3+}}{\ddot{z}_j^{3+} - \ddot{z}_j^{3-}}\right|$ and $\mathcal{R}_i = \max_{j=1}^{m} m_j \cdot \left|\frac{\ddot{z}_{ij}^3 - \ddot{z}_j^{3+}}{\ddot{z}_j^{3+} - \ddot{z}_j^{3-}}\right|$ and $\mathcal{Q}_i = \kappa\left(\frac{\mathcal{S}_i - \mathcal{S}^-}{\mathcal{S}^+ - \mathcal{S}^-}\right)$ $+ (1-\kappa)\left(\frac{\mathcal{R}_i - \mathcal{R}^-}{\mathcal{R}^+ - \mathcal{R}^-}\right)$, where $\mathcal{S}^+ = \max_i \mathcal{S}_i$, $\mathcal{S}^- = \min_i \mathcal{S}_i$, $\mathcal{R}^+ = \max_i \mathcal{R}_i$ and $\mathcal{R}^- = \min_i \mathcal{R}_i$. The real number κ is called a coefficient of decision mechanism. The role of κ is that if majority compromise solution when $\kappa > 0.5$; and consensus compromise solution when $\kappa = 0.5$; and veto compromise solution when $\kappa < 0.5$. Let m_j represent the weight of the jth parameter/criteria.

Step-8: Determine the rank of choices and derive compromise solution. Arrange $\mathcal{Q}_i$ in increasing order to make ranking list. The alternative $\ddot{z}_\alpha$ will be declared compromise solution if it ranks the best (having least value) in $\mathcal{Q}_i$ and the following two conditions satisfy simultaneously:
$C1$ secure: If $\ddot{z}_\alpha$ and $\ddot{z}_\beta$ represent top alternatives in $\mathcal{Q}$, then $\mathcal{Q}(\ddot{z}_\beta) - \mathcal{Q}(\ddot{z}_\alpha) \geq \frac{1}{n-1}$, where n is the number of parameters.
$C2$ secure: The alternative $\ddot{z}_\alpha$ should be best ranked by $\mathcal{S}_i$ and/or $\mathcal{R}_i$. If $C1$ and $C2$ are not satisfying simultaneously, then there exist multiple compromise solutions.
(i) If $C1$ is true, then the alternatives $\ddot{z}_\alpha$ and $\ddot{z}_\beta$ are called compromise solutions.
(ii) If $C1$ is false, then the alternatives $\ddot{z}_\alpha, \ddot{z}_\beta, \ldots, \ddot{z}_\xi$ are called the multiple compromise solutions, where $\ddot{z}_\xi$ is determined by $\mathcal{Q}(\ddot{z}_\xi) - \mathcal{Q}(\ddot{z}_\alpha) \geq \frac{1}{n-1}$.

Example 4.1. *As Example 3.1 based on VIKOR approach, let us start with step-6.*

Step-6: Find the values for SFFSV-PIS and SFFSV-NIS are listed below.

$\ddot{z}^+$	SFFSV-PIS
$\ddot{z}_1^+$	$(0.1215, 0.0608, 0.076)$
$\ddot{z}_2^+$	$(0.1327, 0.0811, 0.0737)$
$\ddot{z}_3^+$	$(0.12, 0.0565, 0.0706)$
$\ddot{z}_4^+$	$(0.1179, 0.0663, 0.0663)$
$\ddot{z}_5^+$	$(0.1288, 0.0725, 0.0644)$

$\ddot{z}^-$	SFFSV-NIS
$\ddot{z}_1^-$	$(0.0608, 0.1215, 0.1215)$
$\ddot{z}_2^-$	$(0.0663, 0.1179, 0.1253)$
$\ddot{z}_3^-$	$(0.0706, 0.1129, 0.1129)$
$\ddot{z}_4^-$	$(0.0737, 0.1179, 0.1106)$
$\ddot{z}_5^-$	$(0.0805, 0.1288, 0.1288)$

Step-7: Taking $\kappa = 0.5$, we can find the values for utility $\mathcal{S}_i$, individual regret $\mathcal{R}_i$ and compromise $\mathcal{Q}_i$ for each alternative.

$\ddot{z}_i$	$\ddot{z}_1$	$\ddot{z}_2$	$\ddot{z}_3$	$\ddot{z}_4$	$\ddot{z}_5$	$\ddot{z}_6$	$\ddot{z}_7$	$\ddot{z}_8$	$\ddot{z}_9$	$\ddot{z}_{10}$
$\mathcal{S}_i$	0.1726	0.6118	0.4397	0.3201	0.199	0.268	0.5152	0.1969	0.5999	0.4133
$\mathcal{R}_i$	0.0913	0.3151	0.2378	0.1188	0.0879	0.2152	0.214	0.0802	0.3366	0.153
$\mathcal{Q}_i$	0.0217	0.9581	0.6113	0.2432	0.0452	0.3719	0.6511	0.0277	0.9864	0.4161

From the aforementioned information, a graph shows how much each medical firm invests according to MCGDM and VIKOR approaches.

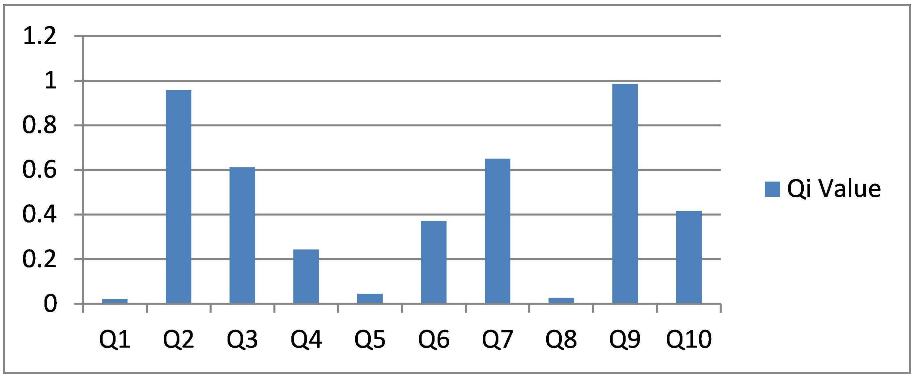

FIGURE 4. Graphical representation using MCGDM based on VIKOR

Step-8: The ranking of medical companies for $\mathcal{Q}_i$ is $\ddot{z}_1 \leq \ddot{z}_8 \leq \ddot{z}_5 \leq \ddot{z}_4 \leq \ddot{z}_6 \leq \ddot{z}_{10} \leq \ddot{z}_3 \leq \ddot{z}_7 \leq \ddot{z}_2 \leq \ddot{z}_9$. Now, $\mathcal{Q}(\ddot{z}_8) - \mathcal{Q}(\ddot{z}_1) = 0.006 \ngeq \frac{1}{4}$. Thus, C1 fails, furthermore $\mathcal{Q}(\ddot{z}_6) - \mathcal{Q}(\ddot{z}_1) = 0.3502 \geq \frac{1}{4}$. Therefore, we establish that $\ddot{z}_1$, $\ddot{z}_8$, $\ddot{z}_5$, $\ddot{z}_4$, $\ddot{z}_6$ are multiple compromise solutions. Hence, the medical company should invest the medicine 30% on $\ddot{z}_1$, 25% on $\ddot{z}_8$, 20% on $\ddot{z}_5$, 15% on $\ddot{z}_4$ and 10% on $\ddot{z}_6$.

5. **Comparison and Discussion.** These two approaches assume a scalar component for each criterion and these two approaches are different from normalization approach. In the case of TOPSIS use vector normalization approach and VIKOR use linear normalization approach. The major difference between two approaches looks in the aggregation function. We can find ranking of values using an aggregating function. The best ranked alternative under VIKOR approach is nearest to the ideal solution. However, the best ranked alternative under TOPSIS approach is the best using ranking index, but is not nearest to the ideal solution. Hence, advantage of VIKOR approach gives to be compromise solution.

6. **Conclusion.** In this present communication, spherical fermatean fuzzy soft set models based on TOPSIS aggregating operator and VIKOR aggregating operator followed by multi criteria group decision making are explained. The main focus of this study is the awareness of spherical fermatean fuzzy soft set models, among a medical company plans to invest some medicine in stock exchange by purchasing some shares of best medical companies and apply some decision making methods in practical applications. Also, we

have inserted a few sorts of statistical charts to image the rankings of alternatives under consideration. Our research can be further extended along the following lines: 1) to consider cubic fermatean fuzzy soft set model; 2) to consider neutrosophic fermatean fuzzy soft set model.

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