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ON INTERVAL VALUED FUZZY ALMOST (m, n)-QUASI-IDEAL IN SEMIGROUPS

Thiti Gaketem

Department of Mathematics
School of Science
University of Phayao
Mae Ka, Mueang, Phayao 56000, Thailand
thti.ga@up.ac.th

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Abstract. *Almost ideal in semigroup was studied by Grošek and Satko in 1980. In this paper, we study the concept of an interval valued fuzzy almost (m, n)-quasi-ideals. We investigate properties of an interval valued fuzzy almost (m, n)-quasi-ideal in semigroups.*
Keywords: Almost ideal, Almost (m, n)-ideal, Fuzzy almost ideal, Interval valued fuzzy almost (m, n)-quasi-ideal

1. **Introduction.** The theory of fuzzy set was presented in 1965 by Zadeh [15]. The theory of fuzzy semigroups was contained by Kuroki in 1979 [10]. Later the theory of interval valued fuzzy sets was a generalization of the notion of fuzzy sets introduced by Zadeh in 1975 [16]. Interval valued fuzzy sets have various applications in several areas like medical science [4], image processing [3], and decision making [17]. In 2006, Narayanan and Manikantan [9] developed the theory of interval valued fuzzy subsemigroup and studied types interval valued fuzzy ideals in semigroups. In 1963, Lajos [7] studied the concepts of (m, n)-ideals in semigroups which was generalized of ideals of semigroups. The research of (m, n)-ideals of semigroups has interested many such as Akram et al. [2], and Yaqoob and Aslam [13]. In 2019 Mahboob et al. [8] extended the ideals of (m, n)-ideals in semigroups to fuzzy sets in semigroup and they characterized the regular semigroup by using fuzzy (m, n)-ideals.

Almost ideal in semigroup was studied by Grošek and Satko in 1980 [5]. Later in 2018, Wattanatripop et al. [14] used knowledge of fuzzy set defined types almost ideal in semigroup and studied properties of those.

In this paper, we give the concept of an interval valued fuzzy almost (m, n)-quasi-ideals which expands the content of fuzzy (m, n)-ideal in semigroups. We prove properties of an interval valued fuzzy almost (m, n)-quasi-ideal in semigroups.

2. **Preliminaries.** In this section, we give some definitions and theories helpful in later sections.

A non-empty subset L of a semigroup G is called

1) a *subsemigroup* of G if $L^2 \subseteq L$,
2) a *left* (*right*) ideal of G if $GL \subseteq L$ ($LG \subseteq G$),
3) an *almost quasi-ideal* of G if $gL \cap Lg \cap L \neq \emptyset$,
4) an *almost* (m, n)*-quasi-ideal* of G if $g^m L \cap Lg^n \cap L \neq \emptyset$ for all $m, n \in \mathbb{Z}$.

For a non-empty subset L of a semigroup G, we denote the

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$$[L](m,n)=\bigcup_{r=1}^{m+n} L^r \cap L^m G L^n \text{ is principal } (m,n)\text{-ideal,}$$

$$[L](m,0)=\bigcup_{r=1}^{m} L^r \cap L^m G \text{ is principal } (m,0)\text{-ideal,}$$

$$[L](0,n)=\bigcup_{r=1}^{n} L^r \cap G L^n \text{ is principal } (0,n)\text{-ideal,}$$

i.e., the smallest (m,n)-ideal, the smallest $(m,0)$-ideal and the smallest $(0,n)$-ideal of G containing L, respectively.

Lemma 2.1. [6] *Let G be a semigroup and m,n positive integers, $[\pi]_{(m,n)}$ the principal (m,n)-ideal generated by the element π. Then*
1) $\left([\pi]_{(m,0)}\right)^m G=\pi^m G$,
2) $G\left([\pi]_{(0,n)}\right)^n=G\pi^n$,
3) $\left([\pi]_{(m,0)}\right)^m G\left([\pi]_{(0,n)}\right)^n=\pi^m G\pi^n$.

For any $p_i\in[0,1]$, where $i\in\mathcal{A}$, define

$$\underset{i\in\mathcal{A}}{\vee} p_i:=\sup_{i\in\mathcal{A}}\{p_i\} \text{ and } \underset{i\in\mathcal{A}}{\wedge} p_i:=\inf_{i\in\mathcal{A}}\{p_i\}.$$

We see that for any $p,q\in[0,1]$, we have

$$p\vee q=\max\{p,q\} \text{ and } p\wedge q=\min\{p,q\}.$$

A *fuzzy set* of a non-empty set T is a function $\omega: L\to[0,1]$.

Definition 2.1. [8] *A fuzzy set ω of a semigroup G is said to be*
1) a fuzzy subsemigroup *of G if $\omega(e_1e_2)\geq\omega(e_1)\wedge\omega(e_2)$ for all $e_1,e_2\in G$,*
2) a fuzzy left (right) ideal *of G if $\omega(e_1e_2)\geq\omega(e_2)$ $(\omega(e_1e_2)\geq\omega(e_1))$ for all $e_1,e_2\in G$,*
3) a fuzzy ideal *of G if it is both a fuzzy left ideal and a fuzzy right ideal of G,*
4) a fuzzy (m,n)-ideal *of G if $\omega(e_1e_2\ldots e_mbd_1d_2\ldots d_n)\geq\omega(e_1)\wedge\omega(e_2)\wedge\cdots\wedge\omega(e_n)\wedge\omega(d_1)\wedge\omega(d_2)\wedge\cdots\wedge\omega(d_n)$ for all $e_1,e_2,\ldots,e_m,d_1,d_2,\ldots,d_n,b\in G$ and m, n are positive integers.*

Definition 2.2. [14] *A fuzzy set ω of a semigroup G such that $\omega\neq 0$ is called* fuzzy almost quasi-ideal *of G if $(\omega\circ\chi_G\cap\chi_G\circ\omega)\cap\omega\neq 0$.*

Let $\Omega[0,1]$ be the set of all closed subintervals of $[0,1]$, i.e.,

$$\Omega[0,1]=\left\{\overline{p}=\left[p^-,p^+\right]\mid 0\leq p^-\leq p^+\leq 1\right\}.$$

We note that $[p,p]=\{p\}$ for all $p\in[0,1]$. For $p=0$ or 1 we shall denote $[0,0]$ by $\overline{0}$ and $[1,1]$ by $\overline{1}$.

Let $\overline{p}=[p^-,p^+]$ and $\overline{q}=[q^-,q^+]\in\Omega[0,1]$. Define the operations $\preceq$, $=$, $\curlywedge$ and $\curlyvee$ as follows:
1) $\overline{p}\preceq\overline{q}$ if and only if $p^-\leq q^-$ and $p^+\leq q^+$,
2) $\overline{p}=\overline{q}$ if and only if $p^-=q^-$ and $p^+=q^+$,
3) $\overline{p}\curlywedge\overline{q}=[(p^-\wedge q^-),(p^+\wedge q^+)]$,
4) $\overline{p}\curlyvee\overline{q}=[(p^-\vee q^-),(p^+\vee q^+)]$.
If $\overline{p}\succeq\overline{q}$, we mean $\overline{q}\preceq\overline{p}$.

For each interval $\overline{p}_i=\left[p_i^-,p_i^+\right]\in\Omega[0,1]$, $i\in\mathcal{A}$ where $\mathcal{A}$ is an index set, we define

$$\underset{i\in\mathcal{A}}{\curlywedge}\overline{p}_i=\left[\underset{i\in\mathcal{A}}{\wedge}p_i^-,\underset{i\in\mathcal{A}}{\wedge}p_i^+\right] \text{ and } \underset{i\in\mathcal{A}}{\curlyvee}\overline{p}_i=\left[\underset{i\in\mathcal{A}}{\vee}p_i^-,\underset{i\in\mathcal{A}}{\vee}p_i^+\right].$$

Definition 2.3. [11] *Let L be a non-empty set. Then the function $\overline{f} : L \to \Omega[0,1]$ is called* interval valued fuzzy set *(shortly, IVF set) of T.*

Definition 2.4. [11] *Let L be a subset of a non-empty set G. An* interval valued characteristic function *of L is defined to be a function $\overline{\chi}_L : T \to \Omega[0,1]$ by*

$$\overline{\chi}_L(e) = \begin{cases} \overline{1} & \text{if } e \in L, \\ \overline{0} & \text{if } e \notin L \end{cases}$$

for all $e \in G$.

For two IVF sets $\overline{\omega}$ and $\overline{\varpi}$ of a non-empty set G, define

1) $\overline{\omega} \sqsubseteq \overline{\varpi} \Leftrightarrow \overline{\omega}(e) \preceq \overline{\varpi}(e)$ for all $e \in G$,
2) $\overline{\omega} = \overline{\varpi} \Leftrightarrow \overline{\omega} \sqsubseteq \overline{\varpi}$ and $\overline{\varpi} \sqsubseteq \overline{\omega}$,
3) $(\overline{\omega} \sqcap \overline{\varpi})(e) = \overline{\omega}(e) \curlywedge \overline{\varpi}(e)$ for all $e \in G$,
4) $(\overline{\omega} \sqcup \overline{\varpi})(e) = \overline{\omega}(e) \curlyvee \overline{\varpi}(e)$ for all $e \in G$.

For two IVF sets $\overline{\omega}$ and $\overline{\varpi}$ in a semigroup G, define the product $\overline{\omega} \circ \overline{\varpi}$ as follows: for all $e \in G$,

$$(\overline{\omega} \circ \overline{\varpi})(e) = \begin{cases} \underset{(t,h)\in F_e}{\curlyvee} \left\{\overline{f}(t) \curlywedge \overline{\varpi}(h)\right\} & \text{if } F_e \neq \emptyset, \\ \overline{0} & \text{if } F_e = \emptyset, \end{cases}$$

where $F_e := \{(t,h) \in G \times G \mid e = th\}$.

Next, we shall give definitions of various types of IVF subsemigroups.

Definition 2.5. [9] *An IVF set $\overline{\omega}$ of a semigroup G is said to be an IVF subsemigroup of G if $\overline{\omega}(e_1e_2) \succeq \overline{\omega}(e_1) \curlywedge \overline{\omega}(e_2)$ for all $e_1, e_2 \in G$.*

Definition 2.6. [9] *An IVF set $\overline{\omega}$ of a semigroup G is said to be an IVF left (right) ideal of G if $\overline{\omega}(e_1e_2) \succeq \overline{\omega}(e_2)\,(\overline{\omega}(e_1e_2) \succeq \overline{\omega}(e_1))$ for all $e_1, e_2 \in G$. An IVF subset $\overline{\omega}$ of G is called an IVF ideal of G if it is both an IVF left ideal and an IVF right ideal of G.*

Let $\overline{\omega}$ be an IVF set of a semigroup G and $m \in \mathbb{Z}$. Then

1) $\omega^0 := \overline{G}$ and $\overline{\omega}^0 \circ \overline{\chi}_G \circ \overline{\omega}^0 := \overline{\chi}_G$,
2) $\overline{\omega}^m := \underbrace{\overline{\omega} \circ \overline{\omega} \circ \cdots \circ \overline{\omega}}_{m-times}$,
3) $\overline{\omega}^m \circ \overline{\chi}_G \circ \overline{\omega}^0 := \overline{\omega}^m \circ \overline{\chi}_G$,
4) $\overline{\omega}^0 \circ \overline{\chi}_G \circ \overline{\omega}^m \circ \overline{\omega}^0 := \overline{\chi}_G \circ \overline{\omega}^m$.

The following theorem can easy to prove.

Theorem 2.1. *Let $\overline{\omega}$, $\overline{\varpi}$ and $\overline{\kappa}$ be IVF set of a semigroup G. Then the following statements hold.*

1) If $\overline{\omega} \sqsubseteq \overline{\varpi}$ then $\overline{\omega}^m \sqsubseteq \overline{\varpi}^m$ for all $m \in \mathbb{Z}$.
2) If $\overline{\omega} \sqsubseteq \overline{\varpi}$ then $\overline{\omega} \circ \overline{\kappa} \sqsubseteq \overline{\varpi} \circ \overline{\kappa}$.
3) If $\overline{\omega} \sqsubseteq \overline{\varpi}$ then $\overline{\omega} \circ \overline{\kappa} \sqcap \overline{\varpi} \circ \overline{\kappa}$.

Definition 2.7. *An IVF set $\overline{\omega}$ of a semigroup G is called an* IVF almost (m,n)-ideal *of G if $\left(\overline{\omega}^m \circ \overline{G} \circ \overline{\omega}^n\right) \sqcap \overline{\omega} \neq \overline{0}$ for all $m, n \in \mathbb{Z}$.*

3. **On Interval Valued Fuzzy Almost (m,n)-Quasi-Ideal in Semigroups.** In this section, we give the concept of an interval valued fuzzy almost (m,n)-quasi-ideals and investigate properties of an interval valued fuzzy almost (m,n)-quasi-ideal in semigroups.

Definition 3.1. *An IVF subsemigroup $\overline{\omega}$ of a semigroup G is called an* IVF almost (m,n)-quasi-ideal *of G if $\left(\overline{\omega}^m \circ \overline{G} \sqcap \overline{G} \circ \overline{\omega}^n\right) \sqcap \overline{\omega} \neq \overline{0}$ for all $m, n \in \mathbb{Z}$.*

Theorem 3.1. *Suppose that $\overline{\omega}$ is an IVF almost (m,n)-quasi-ideal and $\overline{\varpi}$ is an IVF set of a semigroup G and $m, n \in \mathbb{Z}$. Then the following statements hold.*

1) If $\overline{\omega} \sqsubseteq \overline{\varpi}$, then $\overline{\varpi}$ is an IVF almost (m,n)-qausi-ideal of G.
2) $\overline{\omega} \sqcup \overline{\varpi}$ is an IVF almost (m,n)-quasi-ideal of G.

Proof:

1) Suppose that $\overline{\omega} \sqsubseteq \overline{\varpi}$. Then $\overline{0} \neq \left(\overline{\omega}^m \circ \overline{G} \sqcap \overline{G} \circ \overline{\omega}^n\right) \sqcap \overline{\omega} \sqsubseteq \left(\overline{\varpi}^m \circ \overline{G} \sqcap \overline{G} \circ \overline{\varpi}^n\right) \sqcap \overline{\varpi} \neq \overline{0}$. Thus, $\overline{\varpi}$ is an IVF almost (m,n)-quasi-ideal of G.
2) Clearly $\overline{\omega} \sqsubseteq \overline{\omega} \sqcup \overline{\varpi}$. By 1) we have $\overline{\omega} \sqcup \overline{\varpi}$ is an IVF almost (m,n)-quasi-ideal of G. □

Note that for a subset L of G, define $L^0 := G$.

Lemma 3.1. *Let L be a non-empty subset of a semigroup G and $m \in \mathbb{Z}$. Then $(\overline{\chi}_L)^m = \overline{\chi}_L^m$.*

Theorem 3.2. *Let L be a non-empty subset of a semigroup G. Then L is an almost (m,n)-quasi-ideal of G if and only if the characteristic function $\overline{\chi}_L$ is an IVF almost (m,n)-quasi-ideal of G for all $m,n \in \mathbb{Z}$.*

Proof: Suppose that L is an almost (m,n)-bi-ideal of G and let $d \in G$. Then by assumption, there exists $e \in g^m L \cap L g^n \cap L \neq \emptyset$ such that $\left(\overline{G}^m \circ \overline{\chi}_L \sqcap \overline{\chi}_L \circ \overline{G}^n\right) \sqcap \overline{\chi}_L(e) \neq \overline{0}$. Thus, $\left(\overline{G}^m \circ \overline{\chi}_L \sqcap \overline{\chi}_L \circ \overline{G}^n\right)(e) \neq \overline{0}$ and $\overline{\chi}_L(e) = \overline{1}$. Hence

$$\left(\overline{\chi}_L^m \circ \overline{G} \circ \overline{\chi}_L^n \sqcap \overline{\chi}_L\right)(e) \neq \overline{0}.$$

This implies that $\overline{\chi}_L$ is an IVF almost (m,n)-quasi-ideal of G.

Conversely, suppose that $\overline{\chi}_L$ is an IVF almost (m,n)-quasi-ideal of G and let $d \in G$. Then $\left(\overline{\chi}_L \circ \overline{G}^m \sqcap \overline{\chi}_L \circ \overline{G}^n\right) \sqcap \overline{\chi}_L \neq \overline{0}$. Thus, there exists $e \in G$ such that $\left(\overline{\chi}_L \circ \overline{G}^m \sqcap \overline{\chi}_L \circ \overline{G}^n \sqcap \overline{\chi}_L\right)(e) \neq \overline{0}$. Thus, $e \in g^m L \cap L g^n \cap L$. Hence $g^m L \cap L g^n \cap L \neq \emptyset$. Consequently L is an almost (m,n)-quasi-ideal of G. □

For IVF set $\overline{\omega}$ of a semigroup G, define $\text{supp}(\overline{\omega}) := \left\{e \in G \mid \overline{\omega}(e) \neq \overline{0}\right\}$.

Theorem 3.3. *Let $\overline{\omega}$ be an IVF set of a semigroup G. Then $\overline{\omega}$ is an almost (m,n)-quasi-ideal of G if and only if* $\text{supp}(\overline{\omega})$ *is an IVF almost (m,n)-quasi-ideal of G for all $m,n \in \mathbb{Z}$.*

Proof: Suppose that $\overline{\omega}$ is an almost (m,n)-quasi-ideal of G and let $d \in G$. Then there exists $e \in G$ such that $\left(\overline{\chi}_L \circ \overline{G}^m \sqcap \overline{\chi}_L \circ \overline{G}^n \sqcap \overline{\chi}_L\right)(e) \neq \overline{0}$. Thus, $e = c_1^n k = k c_2$ for some $c_1, c_2 \in G$ such that $\overline{\omega}(e) \neq \overline{0}$, $\overline{\omega}(c_1) \neq \overline{0}$, $\overline{\omega}(c_2) \neq \overline{0}$. So $c_1, c_2, d \in \text{supp}(\overline{\omega})$. It implies that

$$\left(\overline{\chi}_{\text{supp}(\omega)} \circ \overline{G}^m \sqcap \overline{G}^n \circ \overline{\chi}_{\text{supp}(\omega)}\right)(e) \neq \overline{0} \text{ and } \overline{\chi}_{\text{supp}(\omega)}(e) \neq \overline{0}.$$

Hence $\left(\overline{G}^m \circ \overline{\chi}_{\text{supp}(\omega)} \sqcap \overline{G}^n \circ \overline{\chi}_{\text{supp}(\overline{\omega})} \sqcap \overline{\chi}_{\text{supp}(\overline{\omega})}\right)(e) \neq \overline{0}$. Thus, $\overline{\chi}_{\text{supp}(\overline{\omega})}$ is an IVF almost (m,n)-quasi-ideal of G. By Theorem 3.2, $\text{supp}(\overline{\omega})$ is an almost (m,n)-quasi-ideal of G.

Conversely, suppose that $\text{supp}(\overline{\omega})$ is an IVF almost (m,n)-quasi-ideal of G and let $r \in G$. Then by Theorem 3.2, $\overline{\chi}_{\text{supp}(\overline{\omega})}$ is an IVF almost (m,n)-quasi-ideal of G. Thus,

$$\left(\overline{\chi}_{\text{supp}(\overline{\omega})} \circ \overline{G}^m \sqcap \overline{G}^n \circ \overline{\chi}_{\text{supp}(\overline{\omega})} \sqcap \overline{\chi}_{\text{supp}(\overline{\omega})}\right) \neq \overline{0}.$$

So there exists $e \in S$ such that $\left(\overline{\chi}_{\text{supp}(\overline{\omega})} \circ \overline{G}^m \sqcap \overline{G}^n \circ \overline{\chi}_{\text{supp}(\overline{\omega})} \sqcap \overline{\chi}_{\text{supp}(\overline{\omega})}\right)(e) \neq \overline{0}$. Hence $\left(\overline{\chi}_{\text{supp}(\overline{\omega})} \circ \overline{G}^m \sqcap \overline{G}^n \circ \overline{\chi}_{\text{supp}(\overline{\omega})} \sqcap \overline{\chi}_{\text{supp}(\overline{\omega})}\right)(e) \neq \overline{0}$ and $\overline{\chi}_{\text{supp}(\overline{\omega})}(e) \neq \overline{0}$. Thus, there exist $c_1, c_2, d \in G$ and $e = c_1^n k = k c_2$ for some $c_1, c_2 \in G$. So $\overline{\omega}(e) \neq \overline{0}$, $\overline{\omega}(c_1) \neq \overline{0}$, $\overline{\omega}(c_2) \neq \overline{0}$. Hence $\left(\overline{\omega} \circ \overline{G}^m \sqcap \overline{G}^n \circ \overline{\omega} \sqcap \overline{\omega}\right) \neq \overline{0}$. Therefore, $\overline{\omega}$ is an almost (m,n)-quasi-ideal of G. □

Definition 3.2. *An IVF almost quasi-ideal $\overline{\omega}$ is called* minimal *if for all nonzero IVF almost quasi-ideals $\overline{\varpi}$ of a semigroup G such that $\overline{\varpi} \sqsubseteq \overline{\omega}$ implies* $\text{supp}(\overline{\omega}) = \text{supp}(\overline{\varpi})$.

Definition 3.3. *An IVF almost (m,n)-quasi-ideal $\overline{\omega}$ is called* minimal *if for all nonzero IVF almost (m,n)-quasi-ideals $\overline{\varpi}$ of a semigroup G such that $\overline{\varpi} \sqsubseteq \overline{\omega}$ implies* $\text{supp}(\overline{\omega}) = \text{supp}(\overline{\varpi})$.

Theorem 3.4. *Let L be a non-empty subset of a semigroup G. Then L is a minimal almost (m,n)-quasi-ideal of G if and only if $\overline{\chi}_L$ is a minimal IVF almost (m,n)-quasi-ideal of G.*

Proof: Suppose that L is a minimal almost (m,n)-quasi-ideal of G and let $\overline{\omega}$ be an IVF almost (m,n)-quasi-ideal of S such that $\overline{\omega} \sqsubseteq \overline{\chi}_L$. Then $\text{supp}(\overline{\omega}) \sqsubseteq \text{supp}(\overline{\chi}_L) = L$. By Theorem 3.3, $\text{supp}(\overline{\omega})$ is an almost (m,n)-bi-ideal of G. By supposition, $\text{supp}(\overline{\omega}) = L = \text{supp}(\overline{\chi}_L)$. Hence $\overline{\chi}_L$ is a minimal IVF almost (m,n)-quasi-ideal of G.

Conversely, suppose that $\overline{\chi}_L$ is a minimal IVF almost (m,n)-quasi-ideal of G and let D be an almost (m,n)-quasi-ideal of G such that $D \subseteq L$. Then $\overline{\chi}_D$ is an IVF almost (m,n)-quasi-ideal of G such that $\overline{\chi}_D \sqsubseteq \overline{\chi}_L$. Thus, $D = \text{supp}(\overline{\chi}_B) = \text{supp}(\overline{\chi}_L) = L$. Therefore, L is a minimal almost (m,n)-quasi-ideal of G. □

Corollary 3.1. *Let G have no proper almost (m,n)-quasi-ideal if and only if for all IVF almost (m,n)-quasi-ideal $\overline{\omega}$ of G, $\text{supp}(\overline{\omega}) = G$.*

Proof: It follows from Theorem 3.4. □

4. **Conclusion.** In Section 3, we define interval valued fuzzy almost (m,n)-quasi-ideals in semigroup with extended study fuzzy alomst quasi-ideals in semigroup and we study properties of those.

In the future work, we can study interval valued fuzzy almost (m,n)-quasi-ideals in generalizations of semigroups. For example, we can define interval valued fuzzy almost (m,n)-quasi-ideals in semihypergroups.

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