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ADAPTIVE FEED-BACK STABILIZATION PROBLEM FOR HIGH-ORDER SYSTEM WITH CONTROL FUNCTION AND UNKNOWN POWER

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Abstract. *This paper investigates the stability problem for a class of nonlinear high-order system with control function. Two novel adaptive state-feedback controllers are successfully designed to achieve global stabilization for two cases of the high-order system. In contrast with the past investigations on nonlinear high-order system, not only the unknown control direction but the unknown input power in system are assumed in this paper. The feature of the controllers lies in the design scheme with upper and lower bounds and novel Nussbaum type gain method in strategy that can respectively realize the stability for high-order system in both two cases. The stability and effectiveness of the system are conducted based on Lyapunov theory. Finally, two illustrative examples are used to validate effectiveness for the system (1) respectively for two cases.*
Keywords: Unknown power, High-order nonlinear system, Control coefficient, Unknown control direction

1. **Introduction.** Adaptive control theory is an active area in control theory design, and some results were reported recently [1,2]. Control law design for high-order systems has become a hot topic in last two decades and much effort also has been devoted to the control design for high-order nonlinear systems, see [3-5] and the references therein. Compared with existing papers that the input powers are one, the higher powers do exist in practical system and can be regarded as a more general case. The control problems of uncertain high-order nonlinear systems has become a hot topic, see [6,7-11]. Compared with the classic nonlinear systems, the high-order one owns more nonlinearities due to the presence of higher powers that make it difficult to design a controller, especially for the stabilization problem, see [5,6,8]. And many significant progresses have been achieved by adding a power integrator, see [5,6]. In [6], by introducing sign function and necessarily modifying the method of adding a power integrator, an adaptive controller design method with the aid of integrator was proposed, which can not only compensate the serious system uncertainties, but overcome the major obstruction from unknown powers. However, when the powers remain unknown, the technique by adding a power integrator is no longer capable of designing a controller. And for recent works, significant results for high-order system with unknown powers have been proposed [7]. In [10], the developed controller is independent of the powers and can be capable of ensuring global stability for high-order system.

However, the unknowns including unknown control direction and unknown powers that are caused by model errors, disturbances and other unknown issues challenge the control design of nonlinear system largely. These aforementioned investigations cannot meet the existence of the unknowns we have mentioned in high-order nonlinear system. However,

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most of the existing results only focused on known control direction in high-order system, see [14,15]. And we have noticed that unknown control direction problem has not been fully studied in high-order system. So, it is worthy of intensively studying in the control theory. With the introduction of powerful Nussbaum-gain technique tool, since the 1980s, abundant achievements on unknown direction control coefficients have been obtained, see [13,14]. Then, when the input powers are known odd constants, stabilization or tracking problem system for high-order system with unknown control direction has also been solved by employing the Nussbaum gain based regulation design technique, see [15,16], etc.

According to the adaptive control theory, the existing results of a class of high-order nonlinear system could only be as follows: either the control direction in the systems remains known with unknown power [11], or the systems remain unknown control direction with known odd input power [14-20]. All supposed the sign of the control direction or the input power to be known. Then, a natural problem here is that how to achieve the desired control objective under the situation that both two assumptions do not exist. Is it possible to further relax both of these two assumptions in high-order nonlinear system? So, these reports cannot straightforwardly extend to solve this problem. That gives the birth to this paper, which motivates us to handle the problem for high-order system with unknown odd power and unknown control direction.

Inspired by the above observation, the problem of control design is thoroughly investigated for high-order system with unknowns including unknown control direction and unknown odd power, which can effectively help to design a controller to realize the stabilization for system. Contributions of this paper can be considered as the following.

1) In contrast with the results, the unknown control direction is concerned for the first time in the literature for a class of high-order nonlinear system with unknown odd power.

2) Compared with the existing literature for high-order system, which applied Nussbaum technic solving the problem of unknown control direction and known input power see [11], this paper assumes that the input power is unknown odd constant.

The rest of the paper is organized as follows. In Section 2, the problem formulation is introduced. In Section 3 and Section 4, design procedure of the control law and adaptive rate are proposed respectively for two cases and stability analysis is demonstrated. In Section 5, some simulation results are given to show the effectiveness of the control design that is proposed in this paper. In Section 6, we draw the conclusion.

2. **Problem Statement.**

Consider the following high-order nonlinear system

$$\begin{cases} \dot{x} = g(x)u^p + d(x) \\ y = x \end{cases} \tag{1}$$

where $x \in R$, $u \in R$, $y \in R$ are respectively the system state, input and output; $d(x)$ is called bounded disturbance of the system, which is unknown locally Lipschitz continuous functions. $p \in {Q^{\geq 1}}_{odd}$ is called the power of the system. $g(x)$ denotes the control function.

The control objective is to seek a smooth state feedback control law so that it can stabilize the system state to the origin, while ensuring global stability of the resulting closed-loop system.

3. **Adaptive Stabilization Controller Design for System with Known Direction.**

This section is devoted to designing a state feedback controller for the system with known control direction and unknown power. For this seek, the following lemma and assumptions are imposed for system (1).

Lemma 3.1. [12] *If $p \in {Q^{\geq 1}}_{odd}$ and for $\forall x_i \in R$, $i = 1, \ldots, m$, the following inequality holds that*

$$\sum_{i=1}^{n} |x_i|^{p_i} \leq \left(\sum_{i=1}^{n} |x_i|\right)^{p_i} \tag{2}$$

Lemma 3.2. [12] *If $p \in Q^{\geq 1}{}_{odd}$, and satisfies $0 < \underline{p} \leq p \leq \bar{p}$, then for any $x \in R$, the following inequality holds that*

$$|x|^p \leq |x|^{\underline{p}} + |x|^{\bar{p}} \tag{3}$$

Assumption 3.1. *Unknown power p is bounded, which satisfied $\bar{p} \geq p \geq \underline{p} \geq 1$, where $\underline{p}$ and $\bar{p}$ are known constants.*

Assumption 3.2. *There exists an unknown constant θ, for $\forall x \in R$, there holds that*

$$|d(x)| \leq \theta h(x)|x|^{\overline{p}} \tag{4}$$

where $h(x)$ is known positive smooth function.

Assumption 3.3. *There exist known constants $\underline{g} > 0 \in R$ and $\bar{g} > 0 \in R$, such that*

$$\underline{g} \leq g(x) \leq \bar{g} \tag{5}$$

where $\underline{g}$ and $\bar{g}$ are respectively the lower and upper bounds of $g(x)$.

3.1. **Design of the controller and stability analysis.** Under Assumption 3.1, we have

$$1 + p \leq 1 + \bar{p} \leq 1 + \bar{p}p \tag{6}$$

Under Lemma 3.1 and (6), one has that

$$g(x)\left(\frac{1}{\underline{g}} + \frac{1}{\underline{g}^{\frac{1}{\bar{p}}}}\right)^p \geq g(x)^{1-p} + g(x)^{1-\frac{p}{\bar{p}}} \geq 1 \tag{7}$$

From (6), it can be obtained that

$$\left(x + x^{\overline{p}}\right)^p x = \left(|x|^p + |x|^{\overline{p}p}\right)|x| \geq |x|^{1+\bar{p}} \tag{8}$$

Similar to the analysis of (8), we have

$$\left(\left(\hat{\theta}h(x)\right)^{\frac{1}{\bar{p}}} + \left(\hat{\theta}h(x)\right)^{\frac{1}{\underline{p}}}\right)^p \geq \hat{\theta}h(x) \tag{9}$$

Then, from the analysis (6)-(9) and Assumption 3.3, we define the control u as follows:

$$u = -\left(\frac{1}{\underline{g}} + \frac{1}{\underline{g}^{\frac{1}{p}}}\right)(\Delta + 1)\left(x + x^{\bar{p}}\right) \quad \Delta = \left(\hat{\theta}h(x)\right)^{\frac{1}{\bar{p}}} + \left(\hat{\theta}h(x)\right)^{\frac{1}{\underline{p}}} \tag{10}$$

Define

$$V = \frac{1}{2}x^2 + \frac{1}{2}\tilde{\theta}^2 \tag{11}$$

where $\tilde{\theta} = \theta - \hat{\theta}$, $\tilde{\theta}$ denotes the error of estimate.

The time derivative of along the trajectories of the system (1) is

$$\dot{V} = x\dot{x} - \dot{\hat{\theta}}\tilde{\theta} \tag{12}$$

Considering system (1) and (12), and under Assumption 3.2, we have that

$$\dot{V} \leq g(x)u^p x + \theta h(x)|x|^{\bar{p}}x - \dot{\hat{\theta}}\tilde{\theta} \tag{13}$$

Substituting (10) into (13), we have

$$\begin{aligned}
\dot{V} &\leq g(x)\left(-\left(\frac{1}{\underline{g}} + \frac{1}{\underline{g}^{\frac{1}{\bar{p}}}}\right)(\Delta + 1)\left(x + x^{\bar{p}}\right)\right)^p x + \theta h(x)|x|^{1+\bar{p}} - \tilde{\theta}\dot{\hat{\theta}} \\
&\leq -g(x)\left(\frac{1}{\underline{g}} + \frac{1}{\underline{g}^{\frac{1}{\bar{p}}}}\right)^p (\Delta + 1)^p \left(x + x^{\bar{p}}\right)^p x + \theta h(x)|x|^{1+\bar{p}} - \tilde{\theta}\dot{\hat{\theta}} \\
&\leq -\left(\left(\hat{\theta}h(x)\right)^{\frac{p}{\bar{p}}} + \left(\hat{\theta}h(x)\right)^{\frac{p}{\underline{p}}} + 1\right)\left(x + x^{\bar{p}}\right)^p x + \theta h(x)|x|^{1+\bar{p}} - |x|^{1+\bar{p}} - \tilde{\theta}\dot{\hat{\theta}}
\end{aligned}$$

$$\leq \theta h(x)|x|^{1+\bar{p}} - \hat{\theta} h(x)|x|^{1+\bar{p}} - |x|^{1+\bar{p}} - \tilde{\theta}\dot{\hat{\theta}}$$
$$\leq \tilde{\theta}\left(h(x)|x|^{1+\bar{p}} - \dot{\hat{\theta}}\right) - |x|^{1+\bar{p}} \tag{14}$$

Define the adaptive law as

$$\dot{\hat{\theta}} = h(x)|x|^{1+\bar{p}} \tag{15}$$

Substituting (15) into (14), it can be obtained that

$$\dot{V} \leq -|x|^{1+\bar{p}} \tag{16}$$

Theorem 3.1. *For high-order system (1) with Assumptions 3.1-3.3, the state-feedback controller (11) guarantees that the system state is globally asymptotically stable and the closed-loop signals converge to the origin.*

Proof: From (16), it is easily found that, if $x \neq 0$, $\dot{V} < 0$. From Lyapunov stability theory, we can obtain that $V \to 0$. From (11), we further obtain that x converges to the origin and $\tilde{\theta} \to 0$, which means that the control objective is achieved.

4. **Adaptive Stabilization Controller Design for System with Unknown Direction.** Although we have shown the feasibility of controller for system (1) under Assumption 3.3, it is not our task since the sign of control gain remains known. Therefore, in this section, we extend to investigate more general case which is considered as high-order system with unknown control direction and unknown power. And the corresponding function expression and properties are formulated in the following lemma.

Lemma 4.1. *Let $V(\cdot)$ and $\varsigma(\cdot)$ be smooth functions defined on $[0, t_f)$, $V(t) \geq 0$, $\forall t \in [0, t_f)$ and $N(\cdot)$ be an even smooth Nussbaum-type function. The following inequality holds:*

$$V(t) \leq c_0 + e^{-c_1 t}\int_0^t g(\tau)N(\varsigma)\dot{\varsigma}e^{c_1\tau}d\tau + e^{-c_1 t}\int_0^t \dot{\varsigma}e^{c_1\tau}d\tau$$

where constant $c_1 > 0$, $g(\cdot)$ is a time-varying parameter which takes values in the unknown closed intervals $I \in [l^{-1}, l^{+1}]$ with $0 \notin I$ and c_0 represents some suitable constant, then $V(t)$, $\varsigma(t)$ and $\int_0^t gN(\varsigma)\dot{\varsigma}d\tau$ must be bounded on $[0, t_f)$.

Lemma 4.2. *For $\forall x \in R$, $|x| - \tanh(x/\eta)x \leq 0.2785\eta$, where $\eta > 0 \in R$.*

Assumption 4.1. *The sign of $g(x)$ remains unknown, with the upper and lower bounds, such that*

$$g_0 \leq |g(x)| \leq g_1 \tag{17}$$

where g_0 and g_1 are respectively the lower and upper bounds of $g(x)$.

Remark 4.1. *The final expression of $N^p(\tau)$ is different from those in existing results where the one is chosen as $N(\tau) = e^{\varsigma^2}\cos((\pi/2)\varsigma)$. Since, $N(\tau)$ is the dynamic regulation parameter, based on the above function properties of $N^p(\tau)$, and chosen by nonnegative, it dose not even necessarily need the $N(\tau)$ in the lemma.*

4.1. **Design of the controller and stability analysis.** Now, consider the following function:

$$V_z = \frac{1}{2}x^2 \tag{18}$$

Considering the system (1) under Assumption 4.1 and differentiating (18), one has that

$$V_z = x\dot{x} = g(x)u^p x + d(x)x \tag{19}$$

From Lemma 3.2 and Assumption 4.1, the control strategy is defined as follows:

$$u = N(\cdot)\cdot\left(|\nabla|^{\frac{1}{\bar{p}}} + |\nabla|^{\frac{1}{\underline{p}}}\right)\nabla = cx + \hat{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right) \tag{20}$$

$$\dot{\tau} = cx^2 + \hat{a}h(x)\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)|x|^{\bar{p}+1} \tag{21}$$

where c and γ are deigned parameters, and $\hat{a}$ is the estimate of a^*. Then, by substituting (20) into (19) and adding and subtracting the term of $\dot{\tau}$, one has that

$$\dot{V}_z = g(x)\left[N(\cdot)\cdot\left(|\nabla|^{\frac{1}{\bar{p}}} + |\nabla|^{\frac{1}{\underline{p}}}\right)\right]^p x + d(x)x + \dot{\tau} - cx^2 - \hat{a}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)|x|^{\bar{p}+1} \tag{22}$$

From Lemma 3.2 and Lemma 4.2 and under Assumption 4.1, it follows that

$$\begin{aligned}
\dot{V}_z \leq{}& |g(x)||N(\cdot)|^p\left(|\nabla|^{\frac{1}{\bar{p}}} + |\nabla|^{\frac{1}{\underline{p}}}\right)^p|x| + \theta h(x)|x|^{\bar{p}+1} + \dot{\tau} - cx^2 \\
&+ \hat{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)|x|^{\bar{p}+1} \\
\leq{}& |g(x)||N^p(\tau)|\left(cx^2 + \hat{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)\right) + \dot{\tau} - cx^2 \\
&- a^*\left(h(x)|x|^{\bar{p}} - \tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)\right) - h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right) \\
\leq{}& \tanh(\alpha)g(x)N^p(\tau)\left(cx + \hat{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)\right)x + 0.2785\eta_1 + \dot{\tau} - cx^2 \\
&- a^*\left(h(x)|x|^{\bar{p}} - \tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)\right) - h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)
\end{aligned} \tag{23}$$

where $\tanh(\alpha) = \tanh\left(\frac{g(x)N^p(\tau)\left(cx+\hat{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)\right)}{\eta_1}\right)$.

Consider the following Lyapunov function:

$$V = \frac{1}{2}x^2 + \frac{1}{2\rho}\tilde{a}^2 \tag{24}$$

Differentiating the Lyapunov Function, one has

$$\dot{V} = x\dot{x} + \frac{1}{\rho}\tilde{a}\dot{\hat{a}} \tag{25}$$

Define the adaptive laws as follows

$$\dot{\hat{a}} = \rho h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right) - \sigma\hat{a} \tag{26}$$

where $\sigma > 0$, $\gamma > 0$, $\rho > 0$ are all designed parameters.

Substituting adaptive law (26) and (23) into (25), one has that

$$\begin{aligned}
\dot{V} \leq{}& \tanh(\alpha)g(x)N(\tau)^p\dot{\tau}x + 0.2785\eta_1 + \theta h(x)|x|^{\bar{p}+1} + \dot{\tau} - cx^2 + a^*0.2785\eta_2 \\
&+ \tilde{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right) - \tilde{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right) + \frac{\sigma}{\rho}\tilde{a}\hat{a}
\end{aligned} \tag{27}$$

By Young's inequality, we have

$$\frac{\sigma}{\rho}\tilde{a}\hat{a} \leq \frac{\sigma}{2\rho} \parallel a^* \parallel^2 - \frac{\sigma_2}{2\rho}\tilde{a}^2 \tag{28}$$

By substituting (28) into (27), we have

$$\begin{aligned}
\dot{V} \leq{}& -cx^2 + \tanh(\alpha)g(x)N(\tau)^p\dot{\tau} + \dot{\tau} + \mu + \tilde{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right) \\
&+ \frac{\sigma}{2\rho} \parallel a^*\parallel^2 - \frac{\tilde{a}^2}{2\rho} - \tilde{a}h(x)|x|^{\bar{p}}\tanh\left(\frac{h(x)|x|^{\bar{p}}}{\gamma}\right)
\end{aligned} \tag{29}$$

$$\dot{V} \leq -DV + (\tanh(\alpha)g(x)N^p(\tau) + 1)\dot{\tau} + \mu$$

where $\mu = \left(0.2785\eta_1 + a^*0.2785\eta_2 + \frac{\sigma}{2\rho} \parallel a^*\|^2\right)$, $D = \min(2c, \sigma)$

$$\dot{V}e^{Dt} \leq -DVe^{Dt} + (\tanh(\alpha)g(x)N^p(\tau) + 1)\dot{\tau}e^{Dt} + \mu e^{Dt}$$

$$\frac{d}{dt}\left(Ve^{Dt}\right) \leq (\tanh(\alpha)g(x)N^p(\tau) + 1)\dot{\tau} + \mu)e^{Dt} \tag{30}$$

Integrating both side of Inequality (30), we have that

$$V \leq \int_0^t ((\tanh(\alpha)g(x)N^p(\tau) + 1)\dot{\tau} + \mu)e^{-Dt}d\tau + \frac{\mu}{D} + V(0) \tag{31}$$

From Lemma 4.1 and Lemma 4.2, it can be easily known that $V(t)$, $\hat{a}$, $\dot{\tau}$ are all bounded in $[0, t_f)$. From [21], the same results can be obtained in $[0, +\infty)$. Then we obtain that x is bounded.

Theorem 4.1. *Consider system (1) and if Assumptions 3.1, 3.2, 4.1 hold, control law and adaptive rate are defined by (26). Then, the high-order system with unknown control direction is asymptotically bounded with the tracking error converging to a neighborhood of the origin.*

Proof: It is not difficult to prove from the above analysis. So, the detailed proof is omitted.

5. **Numerical Example.** In this example 1, consider a class of high-order system with known direction as follows:

$$\begin{cases} \dot{x} = 0.05x^2u^3 + \theta\cos x \cdot x^5 \\ y = x \end{cases} \tag{32}$$

where $h(x) = \cos x$, $p = 3$, $\theta = 0.8$, $\underline{g} = 0.05$, $\bar{p} = 5$, $\underline{p} = 1$. Obviously, the system (32) satisfies Assumptions 3.1-3.3. The initial conditions are set as $x(0) = 1$, $\hat{\theta}(0) = 1$. According to the controller (10) and adaptive law (15). It can be respectively obtained as follows:

$$\begin{cases} u = -\left(\dfrac{1}{\underline{g}} + \dfrac{1}{\underline{g}^{\frac{1}{5}}}\right)\left(\left(\hat{\theta}\cos x\right)^{\frac{1}{5}} + \hat{\theta}\cos x + 1\right)(x + x^5) \\ \dot{\hat{\theta}} = \cos x|x|^6 \end{cases} \tag{33}$$

Simulation results are given in Figure 1, which shows that x asymptotically converges to the origin.

In this example 2, consider a class of high-order system with unknown direction as follows:

$$\begin{cases} y = x \\ \dot{x} = -0.05x^2u^3 + \theta\cos x|x|^5 \end{cases} \tag{34}$$

where $h(x) = \cos x$, $g(x) = -0.05x^2$, $\bar{p} = 5$, $p = 3$. Clearly, system (34) satisfies Assumptions 4.1. The initial conditions are set as $x(0) = 1$, $\hat{a}(0) = 1$. According to the controller (20) and adaptive law (26), it can be respectively obtained as follows:

$$\begin{cases} u = N(\tau)\left(|\nabla|^{\frac{1}{5}} + |\nabla|\right) \\ \dot{\tau} = 2x^2 + \hat{a}\cos x\tanh\left(\dfrac{\cos x|x|^{\bar{p}}}{2}\right)|x|^6 \\ \dot{\hat{a}} = 0.5\hat{a}\cos x|x|^5\tanh\left(\dfrac{\cos x|x|^5}{2}\right) - 2\hat{a} \end{cases} \tag{35}$$

Simulation results are given in Figures 2 and 3, which show that x asymptotically converges to the origin in Figure 2 and the u is bounded in Figure 3.

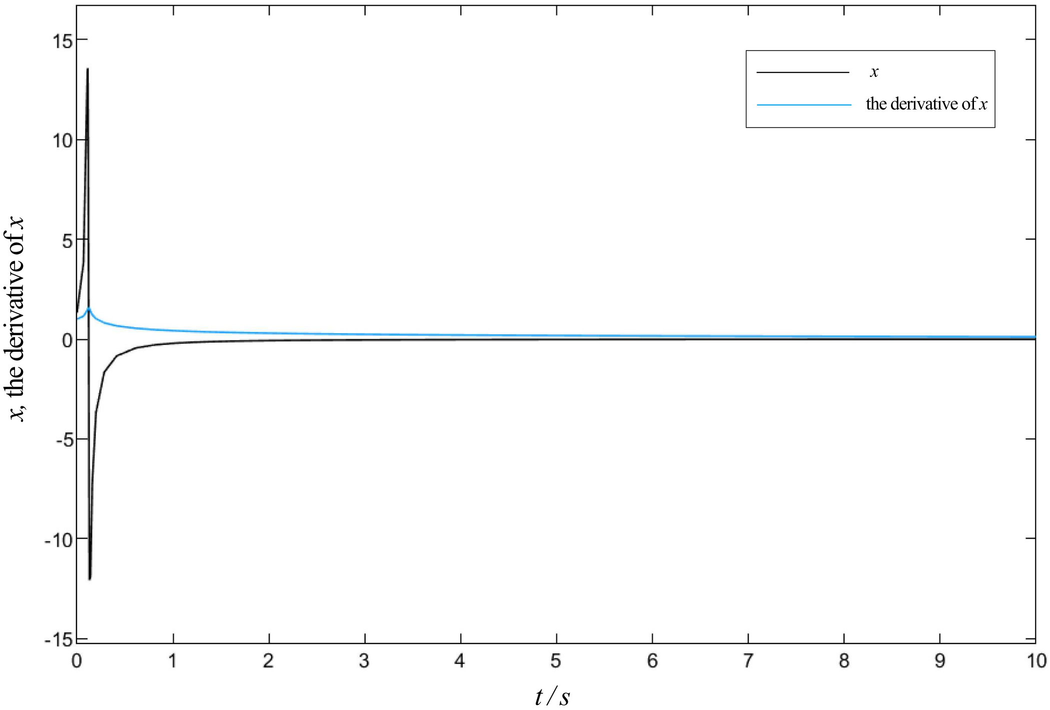

FIGURE 1. The trajectory state of x and its derivative

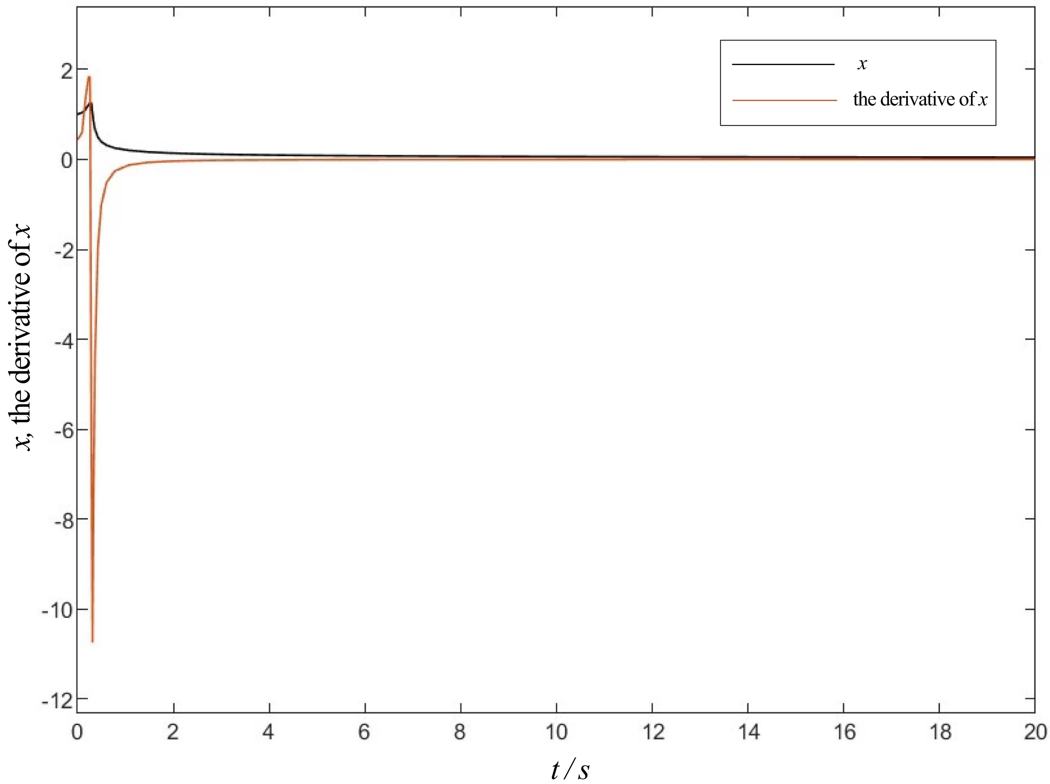

FIGURE 2. The trajectory state of x and its derivative

6. **Conclusions.** This article has studied the problem of stabilization for high-order nonlinear systems with unknown power and control function. By combining the Nussbaum type gain approach and the upper and lower bounds of p in themes respectively for two cases for the high-order system, we have presented two control strategies. Two examples are given to prove the effectiveness of the results. However, there is problem that needs

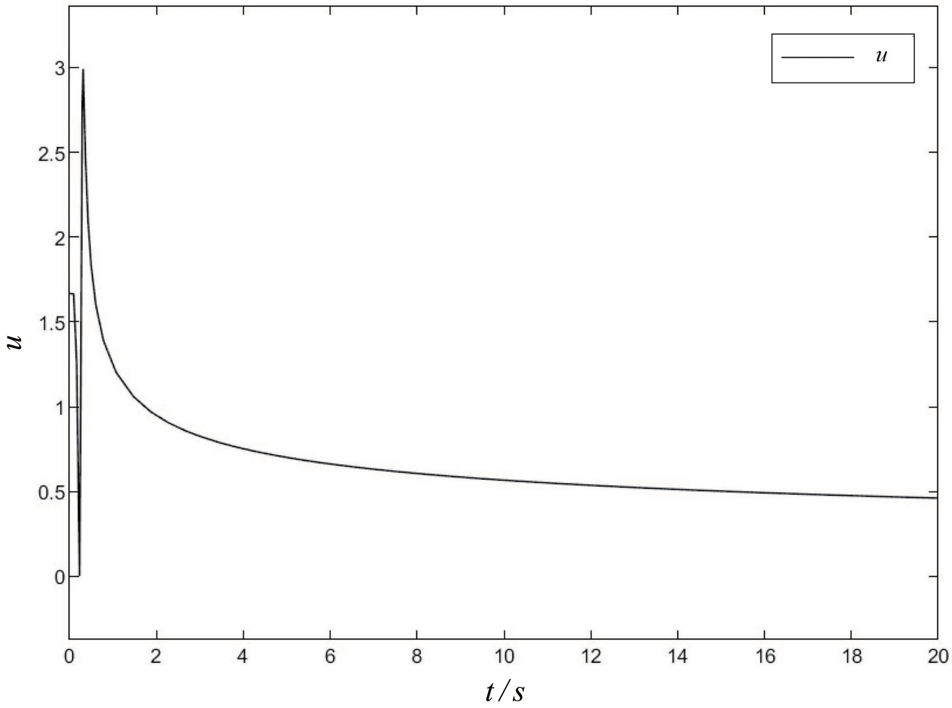

FIGURE 3. The trajectory of control law u

to be considered: control design in this paper needs Assumption 4.1 that the $\bar{p}$, $\underline{p}$ must be known. If we relax this assumption, how to design a controller that can stabilize the system (1) will be our future work.

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