ICIC Express Letters ICIC International ©2022 ISSN 1881-803X
Volume **16**, Number **6**, June **2022** pp. **595–604**

FIXED-TIME EVENT-TRIGGERED DISTRIBUTED CONTROLLER FOR SECONDARY VOLTAGE RESTORATION IN MICROGRID

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Received October 2021; accepted December 2021

Abstract. *This paper proposes a fixed-time event-triggered distributed secondary voltage controller for microgrid. A simplified microgrid model is established. An event-triggering function using the latest information transmitted from its neighbors, itself states both at the latest triggering moment and at current time and a constant threshold function is developed. Based on the developed triggering function, a fixed-time event-triggered controller is presented to achieve fixed-time restoration of voltage to the reference value. Lyapunov stability analysis and non-zeno behavior analysis show the developed control strategy achieves fixed-time convergence and excludes zeno behavior. Numerical simulation is given to verify the correctness of the obtained results.*
Keywords: Fixed-time control, Event-triggered control, Distributed control, Secondary voltage restoration, Microgrid

1. **Introduction.** With massive integration of renewable energy resources and energy storage, microgrid has been widely used in practical power system. Microgrid can work in islanded and grid-connected modes. The microgrid normally works in grid-connected mode and in the event of unexpected disturbance or planned operation, microgrid may be separated from the main grid and work in islanded mode. During islanding process, the reactive and active power of the microgrid will lose balance and voltage as well as frequency stability will be lost. Once it detects frequency and voltage deviation, primary control is taken into action immediately to restore the frequency and voltage to a region around desired value. Nevertheless, due to the droop property, primary control cannot recover the voltage to the exact reference value assigned by the leader. This affects power supply quality and cannot meet the requirement of some voltage/frequency-sensitive loads. Secondary voltage control is required to recover the voltage to the exact reference value. Distributed model predictive control [1], adaptive resilient control [2] and optimal output feedback control [3] were proposed to design distributed secondary voltage controller. Nevertheless, only asymptotical secondary voltage stabilization can be achieved in these results.

Fixed-time control can ensure secondary voltage restoration within bounded time. Compared with the above-mentioned asymptotical secondary voltage controllers, fixed-time controller has faster convergence speed, stronger robustness and higher convergence accuracy. Besides, the settling time of fixed-time controller has nothing to do with initial state and one can select appropriate controller parameters to satisfy the requirement of

DOI: 10.24507/icicel.16.06.595

settling time. Fixed-time consensus problems for multi-agent system (MAS) with disturbance [4], wheeled mobile robots [5], constrained MAS [6] and MAS under digraph [7] have been studied. However, these results fail to take resource constraint into account. In fact, many networked systems have limited energy resource and continuous communication consumes vast amounts of energy. As a result, continuous communication will shorten working time. Furthermore, capability constraints of microprocessor together with communication and actuation modules in communicating, calculation and actuation disable continuous communication and control update. Besides, frequent information exchange in communication environment with limited bandwidth will result in delay, data dropout and other undesirable phenomena. This deteriorates controller performance and gives rise to instability. In fact, there will be no need to conduct frequent communication and control update in steady state and without disturbances.

Event-triggered communication and control can save limited resources. Only when the errors exceed the threshold function, will the event be triggered, information be exchanged and control input be updated, thereby reducing limited resources. Event-triggered consensus problems for MAS under quantized communication [8], DOS attack [9] and event-triggered formation [10] and containment [11] problems have been investigated. However, these results cannot achieve fixed-time system stabilization.

This paper will give a fixed-time event-triggered secondary voltage control for microgrid system under digraph. A simplified microgrid consisting of distributed generator, power network and load is established. An event-triggering function using the latest information transmitted from its neighbors, itself states both at the latest triggering moment and at current time and a constant threshold is developed. Then, a fixed-time event-triggered controller is developed to achieve fixed-time restoration of voltage to the reference value assigned by the leader. Lyapunov stability analysis has shown the developed control strategy achieves fixed-time convergence and non-zeno analysis has been conducted to show there is no zeno behavior in the proposed event-triggered control. Numerical simulation is given to verify the correctness of the obtained results. As far as we know, this is the first time to report fixed-time event-triggered secondary voltage control.

The rest of this paper is organized as follows. Section 2 formulates the problem and gives some preliminaries. Section 3 presents the main results and numerical simulation is provided in Section 4. Finally, the conclusion is drawn in Section 5.

2. **Problem Statement and Preliminaries.**

2.1. **Microgrid model.**

Distributed generators, power networks and loads comprise a simple microgrid. Neglecting fast dynamics and according to [12], the simplified microgrid model can be established as

$$
\begin{aligned}
&\dot{\delta}_i = \omega_{ni} - m_{P_i} P_i - \omega_{com} \\
&k_{V_i} \dot{V}_{odi} = V_{ni} - V_{odi} - n_{Q_i} Q_i + u_i \\
&\dot{P}_i = -\omega_{ci} P_i + \omega_{ci} (V_{odi} i_{odi} + V_{oqi} i_{oqi}) \\
&\dot{Q}_i = -\omega_{ci} Q_i + \omega_{ci} (V_{oqi} i_{odi} - V_{odi} i_{oqi}) \\
&\dot{i}_{odi} = -\frac{R_{ci}}{L_{ci}} i_{odi} + \omega_i i_{oqi} + \frac{1}{L_{ci}} (V_{odi} - V_{bdi}) \\
&\dot{i}_{oqi} = -\frac{R_{ci}}{L_{ci}} i_{oqi} - \omega_i i_{odi} + \frac{1}{L_{ci}} (V_{oqi} - V_{bqi}) \\
&\dot{i}_{ldi} = -\frac{r_{li}}{L_{li}} i_{ldi} + \omega_{com} i_{lqi} + \frac{1}{L_{li}} (V_{bdi} - V_{bdj}) \\
&\dot{i}_{lqi} = -\frac{r_{li}}{L_{li}} i_{lqi} - \omega_{com} i_{ldi} + \frac{1}{L_{li}} (V_{bqi} - V_{bqj})
\end{aligned}
$$

$$\dot{i}_{Ldi} = -\frac{r_{Li}}{L_{Li}} i_{Ldi} + \omega_{com} i_{Lqi} + \frac{1}{L_{Li}} V_{bdi}$$
$$\dot{i}_{Lqi} = -\frac{r_{Li}}{L_{Li}} i_{Lqi} - \omega_{com} i_{Ldi} + \frac{1}{L_{Li}} V_{bqi} \tag{1}$$

where δ_i is angle deviation of the ith distributed generator (DG) from the reference axis, ω_{ni} and V_{ni} represent reference angular speed and voltage, m_{P_i} and n_{Q_i} denote droop coefficients of active power and reactive power, u_i is secondary voltage control input, ω_{com} is angular speed of common reference axis, k_{V_i} is voltage control coefficient, P_i and Q_i represent the produced active and reactive power. ω_{ci} denotes cut-off frequency of the filter. V_{odi}, V_{oqi}, i_{odi}, i_{oqi}, V_{bdi} and V_{bqi} are direct axis and quadrature axis components of output voltage and current, and bus voltage. R_{ci} and L_{ci} are resistor and inductor of output connector, r_{li} and L_{li} are resistance and inductance of the line connecting between node i and node j, i_{ldi} and i_{lqi} are d-axis and q-axis line current, r_{Li}, L_{Li} are resistance and inductance of the load, and i_{Ldi}, i_{Lqi} represent d-axis and q-axis load current.

The studied microgrid is an MAS. One of the distributed generators is assigned as leader which provides reference voltage for the microgrid. The other distributed generators are assigned as followers. It is assumed that the communication link between distributed generators is directed graph.

2.2. **Problem formulation.** The secondary voltage control issue studied in this paper is to design a leader-following consensus protocol such that all the DGs' voltage synchronize to the desired value given by the leader within a fixed time and reduce communication overhead as much as possible. To attain this goal, the voltage controller is designed as $u_i = v_i - V_{ni} + V_{odi} + n_{Q_i} Q_i$. With this controller, the voltage dynamics becomes:

$$\dot{V}_{odi} = v_i \tag{2}$$

Then, the studied issue is to design an event-triggered control v_i to force V_{odi} to practically converge to the reference value V_{ref} within a fixed time, i.e., there exists a small positive constant ϵ and function $T(V_{odi}(0))$ bounded by a constant $T_{\max}$ such that $\lim_{t \to T(V_{odi}(0))} |V_{odi}(t) - V_{odj}(t)| \le \epsilon$, $\lim_{t \to T(V_{odi}(0))} |V_{odi}(t) - V_{ref}(t)| \le \epsilon$.

2.3. **Preliminaries.** Some useful lemmas helpful for the proof of main results are given as follows.

Lemma 2.1. [13] *For* $\gamma_i \in R^{\ge 0}$ $(i = 1, \ldots, N)$ *and* $\theta > 1$, *one has* $N^{1-\theta} \left(\sum_{i=1}^N \gamma_i\right)^\theta \le \sum_{i=1}^N \gamma_i^\theta$.

Lemma 2.2. [13] *For* $\gamma_i \in R^{\ge 0}$ $(i = 1, \ldots, N)$ *and* $0 < \theta < 1$, *one has* $\left(\sum_{i=1}^N \gamma_i\right)^\theta \le \sum_{i=1}^N \gamma_i^\theta$.

Lemma 2.3. [13] *For* $\gamma \in R^N$ *and* $s > l > 0$, *we have* $\|\gamma\|_s \le \|\gamma\|_l \le N^{\frac{1}{l} - \frac{1}{s}} \|\gamma\|_s$.

3. **Main Results.** The following fixed-time event-triggered consensus tracking controller is designed to reduce communication overhead and force all the DGs' voltage to track the reference trajectory assigned by the leader:

$$v_i = - a_i \mathrm{sig} \left(\sum_{j=0}^N a_{ij} \left(V_{odi}\left(t_k^i\right) - V_{odj}\left(t_{k_j'}^j\right) \right) \right)^{\frac{c}{2d-c}}$$
$$- b_i' \mathrm{sig} \left(\sum_{j=0}^N a_{ij} \left(V_{odi}\left(t_k^i\right) - V_{odj}\left(t_{k_j'}^j\right) \right) \right)^{\frac{2d-c}{c}}$$

$$-\lambda\text{sign}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}\left(t_k^i\right)-V_{odj}\left(t^j_{k_j'}\right)\right)\right) \tag{3}$$

where $b_i' = \frac{1}{2}\left[2^{\frac{2m-2n}{n}}b_i - 2^{-\frac{2m-2n}{n}}b_i\right]\text{sign}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)+\frac{1}{2}\left[2^{\frac{2m-2n}{n}}b_i + 2^{-\frac{2m-2n}{n}}b_i\right]$, $a_i, b_i > 0$, the positive odd numbers c and d satisfying $d > c$, the event time sequence satisfying $k_j' = \arg\min_{s\in N^+:t_k^i\geq t_s^j}(t-t_s^j)$, λ is the upper bound of the leader's control u_0.

Define errors as $e_{i1} = -a_i\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}\left(t_k^i\right)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)^{\frac{c}{2d-c}} - b_i'\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}\left(t_k^i\right)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)^{\frac{2d-c}{c}} - \lambda\text{sign}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}\left(t_k^i\right)-V_{odj}\left(t^j_{k_j'}\right)\right)\right) - \left(-a_i\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)^{\frac{c}{2d-c}} - b_i'\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)^{\frac{2d-c}{c}} - \lambda\text{sign}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)\right)$, $e_{i2} = V_{odi}\left(t_{k_i}^i\right) - V_{odi}(t)$ and the triggering function is designed as

$$\psi_{i1} = |e_{i1}(t)| - \zeta_{i1}, \quad \psi_{i2} = |e_{i2}(t)| - \zeta_{i2} \tag{4}$$

The next triggering time can be determined by

$$t_{k+1}^i = \inf\left\{t > t_k^i, \psi_{i1}\geq 0 \text{ or } \psi_{i2}\geq 0\right\} \tag{5}$$

Theorem 3.1. *For the secondary voltage dynamics (2), the controller (3) with the trigger condition (4) can force the voltage V_{odi} to practically converge to the reference value V_{ref} after a fixed time $t_0 \leq \frac{d}{d-c}\frac{1}{1-\sigma}\frac{2\kappa_1}{\lambda_{\min}(Q)\kappa_2}$ and zeno behavior can be avoided.*

Proof: Consider the following Lyapunov function:

$$V_1 = \sum_{i=1}^{N} r_i\left(\frac{a_i}{\frac{2d}{2d-c}}|\tilde{z}_i|^{\frac{2d}{2d-c}} + \frac{b_i}{\frac{2d}{c}}|\tilde{z}_i|^{\frac{2d}{c}}\right) \tag{6}$$

where $\tilde{z}_i = \sum_{j=0}^{N}a_{ij}(V_{odi}-V_{odj})$.

Define $ee_i = -a_i\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)^{\frac{c}{2d-c}} - b_i'\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right)^{\frac{2d-c}{c}} - \lambda\text{sign}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t^j_{k_j'}\right)\right)\right) + a_i\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}(t)\right)\right)^{\frac{c}{2d-c}} + b_i\text{sig}\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}(t)\right)\right)^{\frac{2d-c}{c}} + \lambda\text{sign}\left(\sum_{j=0}^{N}a_{ij}(V_{odi}(t)-V_{odj}(t))\right)$. Then, $v_i = v_{i1}+v_{i2}+v_{i3}+v_{i4} = e_i + ee_i - a_i\text{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}} - b_i\text{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}} - \lambda\text{sign}\left(\tilde{z}_i\right)$, where $v_{i1} = -a_i\text{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}} - b_i\text{sig}(\tilde{z}_i)^{\frac{2d-c}{c}}$, $v_{i2} = -\lambda\text{sign}\left(\tilde{z}_i\right)$, $v_{i3} = e_i$, $v_{i4} = ee_i$. Taking time derivative of V_1 produces

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{N} r_i\left(a_i\text{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}} + b_i\text{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=0}^{N}a_{ij}(v_i-v_j)\right)\\ &\leq \sum_{i=1}^{N} r_i\left(a_i\text{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}} + b_i\text{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=1}^{N}a_{ij}(v_{i1}-v_{j1}) + a_{i0}v_{i1}\right.\end{aligned}$$

$$+\sum_{j=1}^{N} a_{ij}(v_{i2}-v_{j2})+a_{i0}(v_{i2}-v_{i0})+\sum_{j=1}^{N} a_{ij}(v_{i3}-v_{j3})+a_{i0}v_{i3}$$
$$\left.+\sum_{j=1}^{N} a_{ij}(v_{i4}-v_{j4})+a_{i0}v_{i4}\right) \tag{7}$$

Note that $v_{i2}-v_{j2} \leq -\lambda\mathrm{sign}\left(\tilde{z}_i\right)+\lambda$, $v_{i2}-u_0 \leq -\lambda\mathrm{sign}\left(\tilde{z}_i\right)+\lambda$, follow the same analysis and one has

$$\sum_{i=1}^{N} r_i\left(a_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}}+b_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=0}^{N} a_{ij}(v_{i2}-v_{j2})\right)\leq 0 \tag{8}$$

According to triggering condition (4), one has $v_{i3}-v_{j3}=e_i-e_j\leq\zeta_{i1}+\zeta_{j1}$. Then, we have

$$\sum_{i=1}^{N} r_i\left(a_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}}+b_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=1}^{N} a_{ij}(v_{i3}-v_{j3})+a_{i0}v_{i3}\right)$$
$$\leq\sum_{i=1}^{N} r_i\left(a_i\left|\tilde{z}_i\right|^{\frac{c}{2d-c}}+b_i\left|\tilde{z}_i\right|^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=1}^{N} a_{ij}\left(\zeta_{i1}+\zeta_{j1}\right)+a_{i0}\zeta_{i1}\right) \tag{9}$$

According to [14,15], we obtain $|v_{i4}| \leq 3a_i\left|\sum_{j=0}^{N} a_{ij}\zeta_{j2}(t)\right|^{\frac{c}{2d-c}}+\left(2^{\frac{4d-3c}{c}}+1\right)b_i\left|\sum_{j=0}^{N} a_{ij}\zeta_{j2}(t)\right|^{\frac{2d-c}{c}}+2\lambda=\Xi_i$. Further, one has

$$\sum_{i=1}^{N} r_i\left(a_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}}+b_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=1}^{N} a_{ij}(v_{i4}-v_{j4})+a_{i0}v_{i4}\right)$$
$$\leq\sum_{i=1}^{N} r_i\left(a_i|\tilde{z}_i|^{\frac{c}{2d-c}}+b_i|\tilde{z}_i|^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=1}^{N} a_{ij}(\Xi_i+\Xi_j)+a_{i0}\Xi_i\right) \tag{10}$$

Define $g_1(\tilde{z})=\sum_{i=1}^{N} a_i^2|\tilde{z}_i|^{\frac{2c}{2d-c}}+2\sum_{i=1}^{N} a_ib_i|\tilde{z}_i|^{\frac{c}{2d-c}+\frac{2d-c}{c}}+\sum_{i=1}^{N} b_i^2|\tilde{z}_i|^{\frac{2(2d-c)}{c}}$, $g_2(\tilde{z})=\sum_{i=1}^{N} a_i|\tilde{z}_i|^{\frac{c}{2d-c}}+\sum_{i=1}^{N} b_i|\tilde{z}_i|^{\frac{2d-c}{c}}$, $a=2d/c$, $b=1$. Using Lemma 2.3, it can be obtained that

$$\sum_{i=1}^{N}|\tilde{z}_i|^{\frac{2d-c}{c}}\leq\frac{2^{\frac{c}{2d}}V_1^{\frac{2d-c}{2d}}}{\left(\underline{r}_{\frac{b}{\frac{2d}{c}}}\right)^{\frac{2d-c}{2d}}N^{\frac{-c}{2d}}},\quad \sum_{i=1}^{N}|\tilde{z}_i|^{\frac{c}{2d-c}}\leq\frac{2^{1-\frac{c}{2d}}V_1^{\frac{c}{2d}}}{\left(\underline{r}_{\frac{a}{\frac{2d}{2d-c}}}\right)^{\frac{c}{2d}}N^{\frac{c-2d}{2d}}} \tag{11}$$

According to [14,15], one has

$$\sum_{i=1}^{N} r_i\left(a_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{c}{2d-c}}+b_i\mathrm{sig}\left(\tilde{z}_i\right)^{\frac{2d-c}{c}}\right)\times\left(\sum_{j=1}^{N} a_{ij}\left(v_{i1}-v_{j1}\right)+a_{i0}v_{i1}\right)$$
$$\leq-\frac{1}{2}\left(a\mathrm{sig}\left(\tilde{z}\right)^{\frac{c}{2d-c}}+b\mathrm{sig}\left(\tilde{z}\right)^{\frac{2d-c}{c}}\right)^T\left(RH+H^TR\right)\left(a\mathrm{sig}\left(\tilde{z}\right)^{\frac{c}{2d-c}}+b\mathrm{sig}\left(\tilde{z}\right)^{\frac{2d-c}{c}}\right)$$
$$\leq-\frac{1}{2}\lambda_{\min}(Q)\left(\sum_{i=1}^{N} a_i^2|\tilde{z}_i|^{\frac{2c}{2d-c}}+2\sum_{i=1}^{N} a_ib_i|\tilde{z}_i|^{\frac{c}{2d-c}+\frac{2d-c}{c}}+\sum_{i=1}^{N} b_i^2|\tilde{z}_i|^{\frac{2(2d-c)}{c}}\right)$$
$$\leq-\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\left(V_1^{\frac{c}{d}}+V_1^{\frac{2d-c}{d}}\right) \tag{12}$$

where $\kappa_1 = 2^{\frac{d-c}{d}} \left(\bar{r}\frac{\bar{a}}{\frac{2d}{2d-c}}\right)^{\frac{2d-c}{d}} + 2^{\frac{d-c}{d}} \left(\bar{r}\frac{\bar{b}}{\frac{2d}{c}}\right)^{\frac{2d-c}{d}} + \left(\bar{r}\frac{\bar{a}}{\frac{2d}{2d-c}}\right)^{\frac{c}{d}} + \left(\bar{r}\frac{\bar{b}}{\frac{2d}{c}}\right)^{\frac{c}{d}}$, with $\bar{a} = \max\{a_i\}$, $\bar{b} = \max\{b_i\}$. κ_2 has the following form:

$$\kappa_2 = \begin{cases} \min\left\{\underline{b}^2 N^{3-\frac{4d}{c}}, \underline{a}^2\right\}, & \text{if } 0 < c/d < 2/3 \\ \min\left\{\underline{b}^2 N^{3-\frac{4d}{c}}, \underline{a}^2 N^{1-\frac{2c}{2d-c}}\right\}, & \text{if } 2/3 \le c/d < 1 \end{cases} \tag{13}$$

Substituting (8)-(13) into (7), we obtain

$$\begin{aligned} \dot{V}_1 \le & \left(\frac{2^{\frac{c}{2d}}\bar{b}}{\left(\underline{r}\frac{b}{\frac{2d}{c}}\right)^{\frac{2d-c}{2d}} N^{-\frac{c}{2d}}} V_1^{\frac{2d-c}{2d}} + \frac{\bar{a}2^{1-\frac{c}{2d}}}{\left(\underline{r}\frac{a}{\frac{2d}{2d-c}}\right)^{\frac{c}{2d}} N^{\frac{c-2d}{2d}}} V_1^{\frac{c}{2d}} \right) \times \underline{\Gamma} \\ & - \frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\left(V_1^{\frac{c}{d}} + V_1^{\frac{2d-c}{d}}\right) \end{aligned} \tag{14}$$

where $\underline{\Gamma}$ is the minimum of Γ and $\Gamma = [r_1(\Psi_1+\Lambda_1), \dots, r_N(\Psi_N+\Lambda_N)]^T$, $\Psi_i = \sum_{j=1}^N a_{ij}(\zeta_{i1}+\zeta_{j1}) + a_{i0}\zeta_{i1}$, $\Lambda_i = \sum_{j=1}^N a_{ij}(\Xi_i + \Xi_j) + a_{i0}\Xi_i$.

When $\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma V_1^{\frac{c}{d}} \ge \frac{\underline{\Gamma}\bar{a}2^{1-\frac{c}{2d}} V_1^{\frac{c}{2d}}}{\left(\underline{r}\frac{a}{\frac{2d}{2d-c}}\right)^{\frac{c}{2d}} N^{\frac{c-2d}{2d}}}$ and $\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma V_1^{\frac{2d-c}{d}} \ge \frac{\underline{\Gamma}2^{\frac{c}{2d}} V_1^{\frac{2d-c}{2d}}\bar{b}}{\left(\underline{r}\frac{b}{\frac{2d}{c}}\right)^{\frac{2d-c}{2d}} N^{-\frac{c}{2d}}}$, then $\dot{V}_1 \le -\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}(1-\sigma)\left(V_1^{\frac{c}{d}} + V_1^{\frac{2d-c}{d}}\right)$. The system will converge into the region $\Omega_{V_1} = \left\{ V_1 : V_1 \le \max\left\{ \left(\frac{\frac{\underline{\Gamma}\bar{a}2^{1-\frac{c}{2d}}}{\left(\underline{r}\frac{a}{\frac{2d}{2d-c}}\right)^{\frac{c}{2d}} N^{\frac{c-2d}{2d}}}}{\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma} \right)^{\frac{2d}{c}}, \left(\frac{\frac{\underline{\Gamma}2^{\frac{c}{2d}}\bar{b}}{\left(\underline{r}\frac{b}{\frac{2d}{c}}\right)^{\frac{2d-c}{2d}} N^{-\frac{c}{2d}}}}{\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma} \right)^{\frac{2d}{2d-c}} \right\} \right\}$. Note that $V_1 = \sum_{i=1}^N r_i \left(\frac{a_i}{\frac{2d}{2d-c}}|\tilde{z}_i|^{\frac{2d}{2d-c}} + \frac{b_i}{\frac{2d}{c}}|\tilde{z}_i|^{\frac{2d}{c}} \right) \ge \underline{r}\sum_{i=1}^N \left(\frac{a}{\frac{2d}{2d-c}}|\tilde{z}_i|^{\frac{2d}{2d-c}} + \frac{b}{\frac{2d}{c}}|\tilde{z}_i|^{\frac{2d}{c}} \right)$. Since $2d-c > c$, $\frac{2d}{2d-c} < \frac{2d}{c}$. When $|\tilde{z}_i| \le 1$, one has $|\tilde{z}_i|^{\frac{2d}{2d-c}} \ge |\tilde{z}_i|^{\frac{2d}{c}}$. When $|\tilde{z}_i| > 1$, one has $|\tilde{z}_i|^{\frac{2d}{2d-c}} < |\tilde{z}_i|^{\frac{2d}{c}}$. Denote $\tilde{z}_m = \max_{1\le i\le N}\{|\tilde{z}_i|\}$. One has $|\tilde{z}_m|^{\frac{2d}{2d-c}} \le \sum_{i=1}^N |\tilde{z}_i|^{\frac{2d}{2d-c}}$, $|\tilde{z}_m|^{\frac{2d}{c}} \le \sum_{i=1}^N |\tilde{z}_i|^{\frac{2d}{c}}$. Considering these, one has $V_1 \ge \underline{r}\left(\frac{a}{\frac{2d}{2d-c}} + \frac{b}{\frac{2d}{c}}\right)\min\left\{|\tilde{z}_m|^{\frac{2d}{2d-c}}, |\tilde{z}_m|^{\frac{2d}{c}}\right\}$. Further, one obtains

$$|\tilde{z}_i| \le |\tilde{z}_m| \le \max\left\{ \left(\frac{\left(\frac{\frac{\underline{\Gamma}\bar{a}2^{1-\frac{c}{2d}}}{\left(\underline{r}\frac{a}{\frac{2d}{2d-c}}\right)^{\frac{c}{2d}} N^{\frac{c-2d}{2d}}}}{\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma} \right)^{\frac{2d}{c}}}{\underline{r}\left(\frac{a}{\frac{2d}{2d-c}} + \frac{b}{\frac{2d}{c}}\right)} \right)^{\frac{2d-c}{2d}}, \left(\frac{\left(\frac{\frac{\underline{\Gamma}\bar{a}2^{1-\frac{c}{2d}}}{\left(\underline{r}\frac{a}{\frac{2d}{2d-c}}\right)^{\frac{c}{2d}} N^{\frac{c-2d}{2d}}}}{\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma} \right)^{\frac{2d}{c}}}{\underline{r}\left(\frac{a}{\frac{2d}{2d-c}} + \frac{b}{\frac{2d}{c}}\right)} \right)^{\frac{c}{2d}},$$

$$\left.\left(\frac{\left(\frac{\frac{\underline{\Gamma}2^{\frac{c}{2d}}\bar{b}}{\left(\underline{r}\frac{b}{\frac{2d}{c}}\right)^{\frac{2d-c}{2d}}N^{-\frac{c}{2d}}}}{\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma}\right)^{\frac{2d}{2d-c}}}{\underline{r}\left(\frac{a}{\frac{2d}{2d-c}}+\frac{b}{\frac{2d}{c}}\right)}\right)^{\frac{2d-c}{2d}}, \left(\frac{\left(\frac{\frac{\underline{\Gamma}2^{\frac{c}{2d}}\bar{b}}{\left(\underline{r}\frac{b}{\frac{2d}{c}}\right)^{\frac{2d-c}{2d}}N^{-\frac{c}{2d}}}}{\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}\sigma}\right)^{\frac{2d}{2d-c}}}{\underline{r}\left(\frac{a}{\frac{2d}{2d-c}}+\frac{b}{\frac{2d}{c}}\right)}\right)^{\frac{c}{2d}}\right\} = \mu_i \quad (15)$$

Now, the time derivative of Lyapunov function is $\dot{V}_1 \leq -\frac{1}{2}\lambda_{\min}(Q)\frac{\kappa_2}{\kappa_1}(1-\sigma)\left(V_1^{\frac{c}{d}} + V_1^{\frac{2d-c}{d}}\right)$. As a result, the convergence time bound is estimated as $t_0 \leq \frac{d}{d-c}\frac{1}{1-\sigma}\frac{\pi\kappa_1}{\lambda_{\min}(Q)\kappa_2}$.

Next, we will prove there is no zeno behavior. The time derivative of measurement error e_{i1} is

$$\begin{aligned}&\left|\frac{\mathrm{d}e_{i1}}{\mathrm{d}t}\right|\\ &\leq \left(a_i\frac{c}{2d-c}\left|\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t_{k_j'}^{j}\right)\right)\right|^{\frac{2c-2d}{2d-c}} + b_i'\frac{2d-c}{c}\left|\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)\right.\right.\right.\\ &\left.\left.\left.\quad - V_{odj}\left(t_{k_j'}^{j}\right)\right)\right|^{\frac{2d-2c}{c}} + 2\lambda\delta\left(\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t)-V_{odj}\left(t_{k_j'}^{j}\right)\right)\right)\right)\times\sum_{j=0}^{N}a_{ij}\left|\dot{V}_{odi}\right| \quad (16)\end{aligned}$$

where $\delta(\cdot)$ is dirac function.

Since $\left|\sum_{j=0}^{N}a_{ij}\left(V_{odi}\left(t_k^i\right)-V_{odj}\left(t_{k_j'}^{j}\right)\right)\right| = \left|\sum_{j=0}^{N}a_{ij}(V_{odi}(t)+e_{i2}-V_{odj}(t)-e_{j2})\right| \leq \left|\sum_{j=0}^{N}a_{ij}(V_{odi}(t)-V_{odj}(t)+\zeta_{i2}+\zeta_{j2})\right| \leq \left|\sum_{j=0}^{N}a_{ij}(\zeta_{i2}+\zeta_{j2})+\mu_i\right|$, $\left|\sum_{j=0}^{N}a_{ij}\left(V_{odi}(t) - V_{odj}\left(t_{k_j'}^{j}\right)\right)\right| = \left|\sum_{j=0}^{N}a_{ij}(V_{odi}(t)-V_{odj}(t)-e_{j2})\right| \leq \left|\sum_{j=0}^{N}a_{ij}(V_{odi}(t)-V_{odj}(t)+\zeta_{j2})\right| \leq \left|\sum_{j=0}^{N}a_{ij}\zeta_{j2}+\mu_i\right|$, (16) becomes

$$\left|\frac{\mathrm{d}e_{i1}}{\mathrm{d}t}\right| \leq \vartheta_{V_{odi2}}\sum_{j=0}^{N}a_{ij}\vartheta_{V_{odi1}} \quad (17)$$

where $\vartheta_{V_{odi1}} = a_i\left|\sum_{j=0}^{N}a_{ij}(\zeta_{i2}+\zeta_{j2})+\mu_i\right|^{\frac{c}{2d-c}} + b_i'\left|\sum_{j=0}^{N}a_{ij}(\zeta_{i2}+\zeta_{j2})+\mu_i\right|^{\frac{2d-c}{c}} + \lambda$, $\vartheta_{V_{odi2}} = a_i\frac{c}{2d-c}\left|\mu_i+\sum_{j=0}^{N}a_{ij}\zeta_{j2}\right|^{\frac{2c-2d}{2d-c}} + b_i'\frac{2d-c}{c}\left|\mu_i+\sum_{j=0}^{N}a_{ij}\zeta_{j2}\right|^{\frac{2d-2c}{c}} + 2\lambda$.

Integrating both sides of (17) over $\left(t_{k1}^i, t_{(k+1)1}^i\right)$ and using triggering condition produce $\left|e_{i1}\left(t_{(k+1)1}^i\right)\right| = \zeta_{i1} \leq \int_{t_{k1}^i}^{t_{(k+1)1}^i}\vartheta_{V_{odi2}}\sum_{j=0}^{N}a_{ij}\vartheta_{V_{odi1}} = \vartheta_{V_{odi2}}\sum_{j=0}^{N}a_{ij}\vartheta_{V_{odi1}}\left(t_{(k+1)1}^i - t_{k1}^i\right)$. The inter-event period satisfies $t_{(k+1)1}^i - t_{k1}^i \geq \frac{\zeta_{i1}}{\vartheta_{V_{odi2}}\sum_{j=0}^{N}a_{ij}\vartheta_{V_{odi1}}}$. Similarly, from triggering condition (4), one has $\left|e_{i2}\left(t_{(k+1)2}^i\right)\right| = \zeta_{i2} \leq \int_{t_{k2}^i}^{t_{(k+1)2}^i}\vartheta_{V_{odi2}} = \vartheta_{V_{odi2}}\left(t_{(k+1)2}^i - t_{k2}^i\right)$. The inter-event period satisfies $t_{(k+1)2}^i - t_{k2}^i \geq \frac{\zeta_{i2}}{\vartheta_{V_{odi2}}}$. This follows that it will be triggered at most 2 times during the time interval $\left(t, t + \min\left\{\frac{\zeta_{i1}}{\vartheta_{V_{odi2}} \sum_{j=0}^{N} a_{ij} \vartheta_{V_{odi1}}}, \frac{\zeta_{i2}}{\vartheta_{V_{odi2}}}\right\}\right)$. Therefore, there is no zeno behavior in the proposed distributed observer. The proof is completed.

4. **Numerical Simulations.** The validity of the presented secondary voltage control is demonstrated by simulating a typical microgrid system given in Figure 1. The studied microgrid includes 3 DGs and 2 loads in connection with DG1 and DG3 with the parameters chosen the same as [16]. The dashed line in Figure 1 gives the communication connection relationship between the DGs where we can see that it is a digraph. The consensus control scheme parameters are selected as $a_i = 10$, $b_i = 10$, $c = 11$, $d = 13$, $\lambda = 0$, $\zeta_{i1} = 2.4$, $\zeta_{i2} = 1.5$ which satisfy the conditions presented in Section 3. The response of

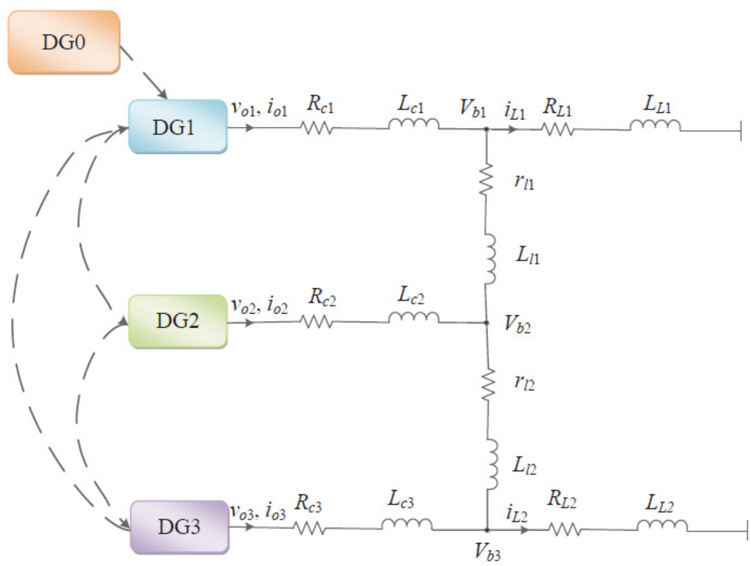

FIGURE 1. The structure of microgrid

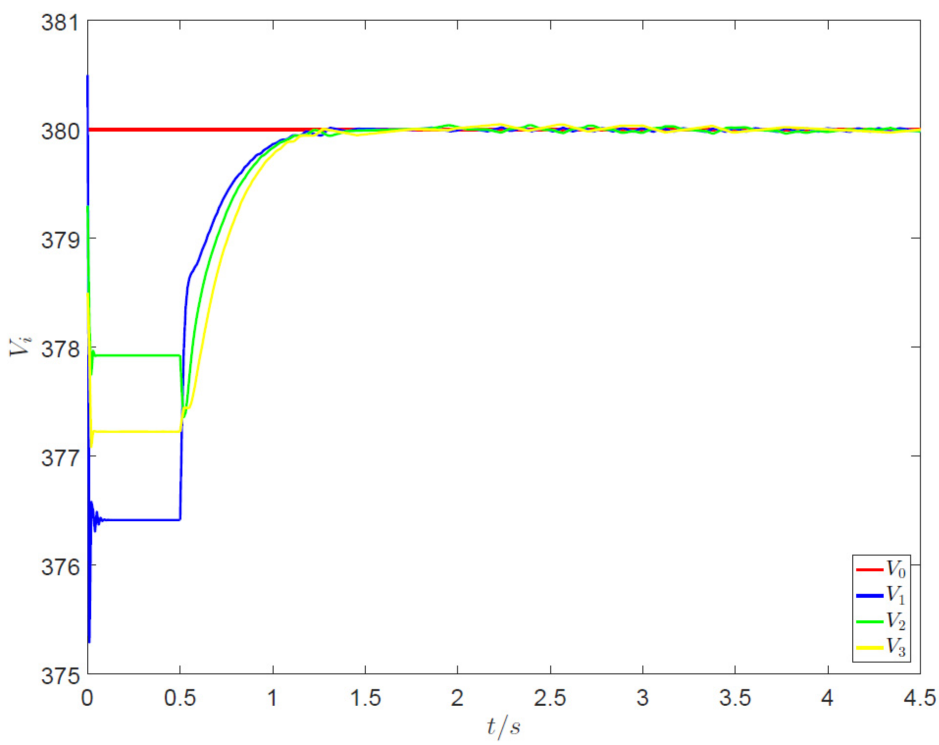

FIGURE 2. The voltage responses of the DGs under the proposed controller

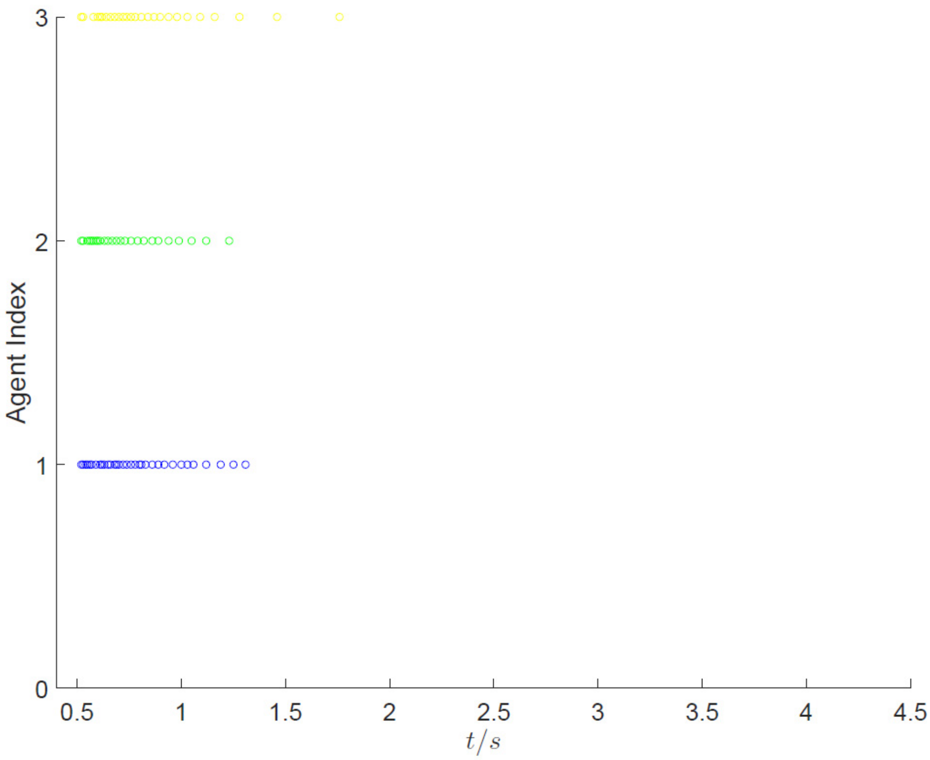

FIGURE 3. Triggering moment of each agent

microgrid frequency is neglected since the focus of this paper is secondary voltage control. Suppose that the microgrid is separated from the main grid at $t = 0$. Immediately after that, the primary control takes into action and the voltage will reduce to a certain value due to droop effect, as shown in Figure 2. Then, at $t = 0.5$s, the secondary control takes into action and forces the voltage to reach a small neighborhood around the desired value within finite time $t = 1.19$s. Figure 3 gives the triggering moment of each agent, where it is obvious that the presented fixed-time event-triggered control strategy effectively reduces the communication frequency, thereby reducing communication overhead. All the results show the correctness of the obtained results.

5. **Conclusions.** This paper addresses fixed-time event-triggered secondary voltage control for microgrid with directed interaction topology to restore the voltage to the desirable value pre-assigned by the leader. A simplified microgrid model is built. An event-triggering function using the latest information transmitted from its neighbors, itself states both at the latest triggering moment and at current time and a constant threshold function is developed. Next, an event-triggered controller is proposed to eliminate the deviation of droop control and restore the voltage to the reference value assigned by the leader within a fixed time. Then, fixed-time convergence of the developed control strategy and non-zeno behavior analysis are conducted. Finally, numerical simulation is given to verify the correctness of the obtained results. Our future work will extend the obtained results to formation control [17].

Acknowledgment. This work is partially supported by the National Natural Science Foundation of China under Grant 61903302.

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