PREDICTION OF PORT AIR POLLUTANT EMISSION BASED ON GRAY COMBINATION MODEL – TAKING DALIAN PORT AS AN EXAMPLE

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ABSTRACT. Due to the implementation of "Double-carbon Goal" and "ship emission control area", the development of green port is becoming more and more important. Accurate pollution discharge prediction is the key to the realization of port pollution discharge management and the basis of realizing port green development. However, at present, the prediction of port pollutant emission relying on a single prediction model cannot meet the accuracy requirements. In this paper, a combined prediction model of port pollutant emission composed of grey prediction model (GM) and BP artificial neural network model (BPNN) is proposed. Two single models: GM(1,1), FGM(1,1) and two combined models: GM(1,1)-BPNN, FGM(1,1)-BPNN are compared and analyzed. The results show that the proposed FGM(1,1)-BPNN model has a higher prediction accuracy than the GM(1,1)-BPNN combined model and any single model of GM(1,1), FGM(1,1), which is more suitable for air pollutant emissions prediction in port cities. Then this paper predicts the air pollutant emission of Dalian Port from 2021 to 2025, which can provide a scientific basis for the port air pollutant emission management and the implementation of green ports.

Keywords: Combination model, Port management, Air pollutant emissions, Prediction

1. Introduction. Globally speaking, ship emissions are the third largest source of air pollutant, with the top two being industrial gas emissions and motor vehicle exhaust. It is estimated that a medium-sized container ship using ordinary fuel (with 3.5% sulfur content) for one day emits pollutant equivalent to the emissions of 500,000 auto mobiles. The international navigation area has been classified based on MARPOL (ANNEX VI) with the control amount of sulfur oxides (SOx) and particulate matter (PM) [1,2]. China Emissions Control Area is one of the global SECA areas.

On December 10th 2018, the Ministry of Transport of China announced the Implementation Plan of the Ship Air Pollutant Emission Control Zone [3]. Since the plan took effect on January 1st 2019, the discharge of sulfide and other pollutants from three major ports in China (Bohai Rim, Yangtze River Delta and Pearl River Delta) has shown a steady downward trend. The Plan mainly aims at the emission of pollutants such as sulfide and other pollutants in ship fuel-heavy oil, in which the sulfur content of heavy oil must be no greater than 0.5%. Starting from January 1st 2022, ships shall use Marine fuel of no more than 0.1% in Hainan waters of the coastal control area, and gradually decrease in the future until the sulfur content is 0. With the adjustment of China's development strategy, the concept of "mountains and rivers green are mountains of silver and gold" has been deeply rooted in people's hearts. Since 2020, the Chinese government has gradually formed the "Peak Carbon Dioxide Emissions" by 2030 and the "Double-carbon Goal" of

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"Carbon Neutrality" by 2060. Therefore, it is critical to strengthen the emission management of port pollutants and realize the green development of the port. The accurate prediction of port pollutant discharge is the basis for the realization of port pollutant discharge management. Therefore, this paper will study and establish a prediction model for port city air pollutant emission and carry out a prediction of the air pollutant emission in Dalian Port.

2. Literature Review. As presently China has clearly put forward an important task of continuously improving environmental quality, optimizing the management of ports air pollutant discharge is an effective way to alleviate air pollution, for which the accurate prediction of port air pollutant emissions is the basis [4]. At present, the methods of air pollutant emission prediction mainly include two categories: statistical methods and artificial intelligence methods. The former is mainly time series methods, these methods take a series of sequence values in accordance with temporal order, and finds its characteristic mode to establish mathematical models, including exponential smoothing method, autoregressive moving average (ARMA), gray model (GM), etc. [4-7]. The latter includes artificial neural network (ANN), support vector machine (SVM), fuzzy logic (FL), hidden Markov model (HMM), etc., for example, Lin et al., and Martínez-España et al. [8,9]. The advantage of statistical model is that it needs a small number of samples and has good prediction accuracy within the sample. However, due to the strict assumption of data distribution, those models have weak generalization ability and low prediction accuracy outside the sample. The advantage of artificial intelligence models is that it does not make strict assumptions about data distribution, has strong generalization ability, but needs a large number of samples. The so-called combined prediction model refers to the combination of different prediction models in an appropriate way, making comprehensive use of the advantages of various prediction models to achieve more accurate prediction. The prediction model that combines two or more prediction models to give full play to their respective advantages has been applied in many fields, for example, wind speed prediction [10], and prognostic risk prediction [11]. However, as far as we know, there is no application in the field of air pollution emission, especially in the prediction of port pollutant emission. In this paper, the grey prediction model is selected as the representative of time series model, the artificial neural network model is selected as the representative of artificial intelligence model, and a combined model is constructed. Taking Dalian Port as an example, the prediction of port pollutant emission is realized.

1) Grey prediction model: the gray model is suitable for the prediction of small samples and incomplete information, which through the mining system of raw data, establishes the corresponding differential equation, so as to predict the future development situation. Port air pollutant emissions datasets are consistent with such characteristics. In this paper we tend to use fractional accumulation which optimizes the traditional gray model to obtain better prediction results [12-14].

2) Neural network prediction model: artificial neural network (ANN) is an effective tool to describe nonlinear phenomena. The air pollutant discharge in the port is affected by the physical and chemical processes between air pollutants emissions at different times; hence it has strong non-linear characteristics. One of the ANNs is BP neural network which is widely used in air pollutant prediction [15-18].

Considering the poor prediction effect used alone, in this paper, a combination model composed of linear model and nonlinear model is constructed and used to predict the air pollutant emission of Dalian Port. The reasons are as follows. 1) As port air pollutant emissions are affected by multiple factors, a single model cannot make full advantage of the full information of the system, while the prediction accuracy can be improved by using combined models. 2) The emission of port air pollutants is affected by the economic activities with strong time-series characteristics, so the time-series model was used as one of the prediction models. 3) Emission prediction of port pollutants with annual data is an uninformative gray system which uses small sample data, so the gray models are used. Last but not least, fractional order GM(1,1), time-series models and BP neural network models are widely and maturely used by other scholars. Therefore, this paper establishes a gray combination model of FGM(1,1) and BP neural network, with the historical data, takes the Dalian Port as an example to carry out a prediction of the next five years, so as to provide a scientific basis for the port air pollutant emission management and the implementation of green port.

3. Models & Methodologies.

3.1. The design of model structure. In order to select the model with higher prediction accuracy, this paper constructs four competing models, namely GM(1,1), FGM(1,1), GM-BPNN, FGM-BPNN. The modeling logic of these four models is shown in Table 1.



TABLE 1. The modeling logic of these four models

Later in this section, we will introduce the modeling process of M1 and M2 and build M3 and M4 based on ANN.

3.2. Modeling of the gray fractional FGM(1,1) model. In 1982, Professor Deng [19] proposed and introduced the concept of the gray system [20]. The advantage of the gray GM(1,1) model is that it is able to handle gray information and poor data, but the model also has shortcomings and has large errors on individual problem. Wu et al. [21,22] first placed and improved the fractional accumulation on the gray system model, and greatly improved the prediction accuracy of the gray model. The fractional gray model FGM(1,1), aiming for the deficiency of the GM(1,1) model, reduces the error by selecting the appropriate additive order and obtains better predictions. The basic process of the FGM(1,1) model is given below.

1) From the raw non-negative data, the raw sequence is obtained as follows:

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\right) \tag{1}$$

2) Based on the original non-negative sequence, the order r accumulation sequence is

$$X^{(r)} = \left(x^{(r)}(1), x^{(r)}(2), \dots, x^{(r)}(n)\right)$$
(2)

$$x^{(r)} = \sum_{i=1}^{k} C_{k-i+r-1}^{k-i} x^{(0)}(i)$$
(3)

$$C_{k-i+r-1}^{k-i} = \frac{(k-i+r-1)(k-i+r-2)\cdots(r+1)r}{(k-i)!}$$
(4)

where

$$C_{r-1}^0 = 1, \quad C_k^{k+1} = 0$$
 (5)

3) The whitening differential equation was established as $\frac{dx^{(r)}(t)}{dt} + ax^{(r)}(t) = b$. The form of the solution is an exponential function:

$$x^{(r)}(t+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-at} + \frac{b}{a}$$
(6)

solved by the least square method, for \hat{a} and \hat{b} :

$$\left(\frac{\hat{a}}{\hat{b}}\right) = \left(B^T B\right)^{-1} B^T Y \tag{7}$$

where

$$B = \begin{pmatrix} -0.5 \left(x^{(r)}(1) + x^{(r)}(2) \right) & 1 \\ -0.5 \left(x^{(r)}(2) + x^{(r)}(3) \right) & 1 \\ \vdots & \vdots \\ -0.5 \left(x^{(r)}(n-1) + x^{(r)}(n) \right) & 1 \end{pmatrix}, \quad Y = \begin{pmatrix} x^{(r)}(2) - x^{(r)}(1) \\ x^{(r)}(3) - x^{(r)}(2) \\ \vdots \\ x^{(r)}(n) - x^{(r)}(n-1) \end{pmatrix}$$
(8)

4) Time response function solved is $\hat{x}^{(r)}(k+1) = \left| x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right| e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}}$. While $\hat{x}^{(r)}(k+1)$

is the value of the time k + 1. 5) For the sequence $\hat{X}^{(r)} = \left\{ \hat{x}^{(r)}(1), \hat{x}^{(r)}(2), \dots, a^{(1)}\hat{x}^{(r)}(n) \right\}$, the reduction of the sequence is

$$a^{(r)}\hat{X}^{(r)} = \left\{ a^{(1)}\hat{x}^{(r)(1-r)}(1), a^{(1)}\hat{x}^{(r)(1-r)}(2), \dots, a^{(1)}\hat{x}^{(r)(1-r)}(n) \right\}$$
(9)

where $a^{(1)}\hat{x}^{(r)(1-r)}(k) = \hat{x}^{(r)(1-r)}(k) - \hat{x}^{(r)(1-r)}(k-1)$. Through the subtraction operation, the prediction sequence is $\left\{\hat{x}^{(0)}(1), \hat{x}^{(0)}(2), \dots, \hat{x}^{(0)}(n)\right\}$.

6) The model was evaluated using the mean absolute percentage error (MAPE), where

$$MAPE = \frac{1}{n}x\sum_{k=1}^{n} \left|\frac{x^{(0)}(k) - \hat{x}^{(0)}(k-1)}{x^{(0)}(k)}\right| \times 100\%$$
(10)

At that time when r = 1, the gray fractional FGM(1,1) model is the gray GM(1,1) model.

3.3. The process of establishing the BP neural network. Artificial neural network is a simplified model based on the cognition of human brain neural network and the abstract of human brain neural network, among which, BP neural network is a multi-layer feedforward neural network (MLFNN) using the error back propagation method, which is according to the set prediction error value. If with the output prediction value compared with the actual value, the error exceeds the prediction range, then back propagation initiates and constantly adjusts the weights and thresholds, so that the prediction value of the network model constantly approaches the actual value [23]. The main feature is the signal forward transmission and error back propagation. The error signal of each layer unit is obtained as a basis for modifying the weights of each unit. The BP algorithm only uses the mean-square error function for first derivative (gradient) of weight and threshold, so the convergence rate of the algorithm is slow and easy to fall into local minimum and other defects. In order to solve this problem, Hinton and Salakhutdinov proposed an unsupervised greedy layer wise training algorithm, a machine learning method of deep neural network based on human brain learning thought, which brought some new thinking to solving the optimization problem related to deep structure [24].

The target function of BP neural network is the average relative error (MRE):

$$MRE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{Y_i - \hat{Y}_i}{Y_i} \right|$$
(11)

In this article we use MRE to train the BPNN model.

3.4. Establishment process of the gray combination model. The multi-forecaster systems can be divided into two classes, depending on how their components interact to deliver the consensual decision: 1) chained (hierarchical) systems, where the output of a forecasting sub-system serves as input for some upper forecasting level; 2) unchained (non-hierarchical) systems, where independent forecasting sub-systems have their outputs combined. This paper focuses on the former class of systems, which is frequently cited by Bates and Granger as a seminal reference [25,26].

This paper seeks to establish a gray combination model based on the fractional GM(1,1) model and BP neural network. Fractional gray model makes linear prediction of the data, and the neural network performs nonlinear processing of the data, thus better capturing the linear and implied non-linear change characteristics of the data. First, we use GM(1,1) and FGM(1,1) to process the raw data and obtain the predicted data, which will then be processed by the BP neural network, and after test and qualitative analysis, obtain the final results.

4. Data Processing & Results.

4.1. Verifying accuracy of the combined model. To verify the accuracy of the gray combination model, eight kinds of air pollutants CO_2 , SO_x , NO_x , PM10, PM2.5, CH_4 , CO, N_2O from the year 2015 to 2020 were selected and predicted as a sample (here, we assume that CO_2 is an air pollutant for prediction). We first used the data from 2015-2018 into the GM(1,1) model, the FGM(1,1) model, and the gray combined model, respectively, to predict the results of 2019-2020 and compare the results with the actual data of 2019-2020, thus to verify the prediction accuracy of the gray combination model.

Before processing the output of the gray model in the BP neural network (BPNN), the data needs to be normalized, and then execute an inverse process after training to obtain the results. Given that the different characteristics of data: the gray models are more suitable for small sample data, while BPNN is more suitable for large sample data. Therefore, this paper uses bootstrap to repeatedly sample the gray model prediction results to obtain large sample data and then be trained by BPNN.

In this paper, the simulated data results predicted by the four models are as shown in Tables 2-5.

Vear	COa	SO	NO	PM10	PM2 5	CH	CO	$N_{2}O$	Total	Total's
1041		ыох	ΝO _X	1 1110	1 1/12.0	0114	00	1120	1000	error
2015	83105.45	36.00	1357.29	25.25	23.23	1.16	55.80	5.73	84609.91	0.00%
2016	89344.00	39.00	1480.00	27.00	25.00	1.00	61.00	6.00	90983.00	9.70%
2017	85940.00	37.00	1423.00	26.00	24.00	1.00	59.00	6.00	87516.00	-12.44%
2018	82666.00	36.00	1369.00	25.00	23.00	1.00	56.00	6.00	84182.00	-0.35%
2019	79517.00	34.00	1317.00	24.00	22.00	1.00	54.00	5.00	80974.00	6.13%
2020	76488.00	33.00	1266.00	23.00	21.00	1.00	52.00	5.00	77889.00	-0.53%
MAPE	7.15%	7.12%	7.19%	7.02%	7.03%	15.32%	7.25%	6.54%		

TABLE 2. GM(1,1) (unit: ton)

Vear	COa	SO	NO	PM10	PM2 5	CH	CO	NaO	Total	Total's
rear		ыох	NO _X	1 1110	1 1012.0	\mathbf{OII}_4	00	1.20	rotar	error
2015	83105.45	36.00	1357.29	25.25	23.23	1.16	55.80	5.73	84609.91	0.00%
2016	87106.50	37.73	1441.19	26.55	24.43	1.23	59.61	5.98	88703.22	6.95%
2017	86778.38	37.59	1437.99	26.46	24.34	1.23	59.27	5.95	88371.21	-11.58%
2018	83141.76	36.01	1377.41	25.35	23.32	1.18	56.57	5.70	84667.30	0.23%
2019	78171.03	33.86	1294.14	23.83	21.92	1.10	52.93	5.37	79604.18	4.33%
2020	72876.14	31.56	1205.52	22.21	20.43	1.03	49.04	5.00	74210.93	-5.23%
MAPE	5.77%	5.78%	5.88%	5.81%	5.81%	5.77%	6.25%	5.76%		

TABLE 3. Fractional GM(1,1) r = 0.5 (unit: ton)

TABLE 4. GM-BPNN combination model (unit: ton)

	CO	SO	NO	DM10	DM9 5	СН	CO	NaO	Total	Total	Trained	Totalla
Year	nom.	nom.	nom.	nom.	$r_{\rm nom.}$	nom.	nom.	nom.	nom.	nom.	data	error
										trained	anti-norm.	
2015	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.46	84609.91	0.00%
2016	0.71	0.67	0.71	0.68	0.69	0.26	0.68	0.61	0.45	0.43	83754.91	0.98%
2017	0.59	0.56	0.61	0.58	0.58	0.26	0.61	0.61	1.00	0.89	96864.98	-3.08%
2018	0.49	0.50	0.52	0.47	0.48	0.26	0.51	0.61	0.50	0.57	87744.93	3.87%
2019	0.38	0.39	0.43	0.37	0.37	0.26	0.44	0.21	0.23	0.24	78339.87	2.67%
2020	0.28	0.33	0.34	0.26	0.27	0.26	0.37	0.21	0.30	0.32	80619.89	2.95%

TABLE 5. FGM-BPNN combination model (unit: ton)

Vear	CO_2	SO_{x}	NO _x	PM10	PM2.5	CH_4	СО	N_2O	Total	Total	Trained data	Total's
lear	nom.	nom.	nom.	nom.	nom.	nom.	nom.	nom.	nom.	trained	anti-norm.	error
2015	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.49	84609.91	0.00%
2016	0.63	0.60	0.64	0.64	0.63	0.60	0.63	0.60	0.45	0.40	81894.01	-1.26%
2017	0.62	0.59	0.64	0.63	0.62	0.60	0.62	0.59	1.00	0.92	97585.87	-2.36%
2018	0.50	0.50	0.53	0.51	0.51	0.53	0.53	0.49	0.50	0.43	82799.31	-1.98%
2019	0.34	0.38	0.39	0.35	0.36	0.41	0.40	0.36	0.23	0.26	77669.28	1.80%
2020	0.16	0.25	0.24	0.18	0.21	0.31	0.26	0.21	0.30	0.23	76763.98	-1.97%

As can be seen from the above four tables: when using gray model alone: average MAPE is 8.08%, maximum is 15.32%, average error absolute value is 5.83% and maximum error absolute value is 12.44%. When using the fractional gray model FGM(1,1) alone: the average MAPE is 5.85%, the maximum is 6.25%, the average absolute value is 5.66% and the maximum absolute value is 11.85%. When combining GM-BPNN predictions: it produces small error with mean error absolute value of 2.71% and maximum error absolute value of 3.87%. When combining FGM-BPNN predictions: small error was generated with mean absolute value of 1.87% and maximum absolute value of 2.36%. Mean average error comparison is shown in Figure 1.

Thus, we can see that the gray combination model FGM-BPNN has better prediction accuracy and stability than other separate models or ordinary gray combination models, and the resulting errors satisfy the prediction demand of air pollutant emissions in Dalian Port.

4.2. Predictions of 2021-2025 for Dalian Port. We use the gray combination model, with the data from 2015-2020, to predict Dalian Port air pollutant emissions for 2021-2025. The predictions were first made using FGM(1,1), and the predicted data was substituted into the BP neural network model for training. During training, when the epoch



FIGURE 1. Mean error comparison of the four types of models



FIGURE 2. Prediction results of air pollutant discharge in Dalian Port

approached 2400, the error decreased rapidly to 0.01. Then we executed an inverse process of normalization of the training results of the BP neural network and obtained the prediction results. Thus, the results are shown in Figure 2.

5. Conclusions. This paper establishes a gray combination model to predict port air pollutant emissions. The results show that the accuracy and stability of the combined model are better than the traditional single prediction model, the data are well fitted, and the accuracy and stability meet the prediction needs of the port pollutant emissions. This paper predicts the air pollutant emissions of Dalian Port from 2021-2025. The results show that the emissions will decrease year by year, which can provide a scientific basis for future port management authorities to formulate development strategies and improve in port resources development, "Double-carbon Goal" and related infrastructure construction.

In this paper we let the r of FGM(1,1) be 0.5, but it may not be the best, so in the future research we will use optimization methods to select the best r of FGM(1,1). Furthermore, in this paper we use chained (hierarchical) systems, and in the future we will compare the chained (hierarchical) systems with unchained (non-hierarchical) systems.

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