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ALMOST BI-INTERIOR IDEALS AND THEIR FUZZIFICATION OF SEMIGROUPS

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Abstract. *The concept of bi-interior ideals, as a generalization of quasi-ideals, bi-ideals and interior ideals of semigroups, was introduced by Rao in 2018. In this paper, we define the notion of almost bi-interior ideals of semigroups and study some generalization of bi-interior ideals by using the concept of almost bi-interior ideals of semigroups. Then, we investigate some properties of almost bi-interior ideals in semigroups. Moreover, we introduce the concept of fuzzy almost bi-interior ideals of semigroups and give the connections between almost bi-interior ideals and fuzzy almost bi-interior ideals of semigroups.*
Keywords: Almost bi-interior ideals, Fuzzy almost bi-interior ideals

1. **Introduction and Preliminaries.** Zadeh [1] introduced the concept of fuzzy subsets as a function from a nonempty set X to the unit interval $[0, 1]$. The fuzzy subsets are an extension of classical sets in mathematics. Rosenfeld [2] developed the fuzzy subsets to define the notion of fuzzy subgroups of groups which provided the first inspiration for various algebraic structures. In many algebraic structures, the fuzzy subsets are now being studied (see, e.g., [3, 4, 5, 6]). Baupradist et al. [7] studied the connections between essential (resp., 0-essential) ideals and fuzzy essential (resp., 0-essential) ideals of semigroups. Next, Bashir et al. [8] used the concepts of m-polar fuzzy ideals and m-polar fuzzy bi-ideals for characterizing regular and intra-regular semigroups. Furthermore, the classes regular and intra-regular semigroups were characterized in terms of their picture fuzzy bi-ideals by Nakkhasen [9]. The concepts of left, right and two-sided almost ideals of semigroups were introduced by Grosek and Satko [10] in 1980. They investigated the characterizations of these ideals when a semigroup contains no proper left, right, two-sided ideals. Afterward, Bogdanović [11] introduced the notion of almost bi-ideals of semigroups as a generalization of bi-ideals. In 2018, Wattanatripop et al. [12] introduced the concept of fuzzy almost bi-ideals by using the concepts of almost bi-ideals and fuzzy ideals of semigroups, and they studied some relationships between almost bi-ideals and fuzzy almost bi-ideals of semigroups. Later, the notions of almost interior ideals and weakly almost interior ideals of semigroups were introduced in 2020 by Kaopusek et al. [13]. Recently, Krailoet et al. [14] extended the fuzzy subsets to introduce the concepts of fuzzy almost interior ideals and weakly fuzzy almost interior ideals by using the concepts of almost interior ideals and weakly almost interior ideals of semigroups, and they discussed

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relations between almost interior ideals and fuzzy almost interior ideals in semigroups. Moreover, other types of almost ideals and their fuzzifications were studied in other algebraic structures (for example, in Γ-semigroups [15, 16, 17], in ternary semigroups [18], in semihypergroups [19, 20, 21], in ordered Γ-semihypergroups [22], and in left almost semihypergroups [23]).

Firstly, we recall some of the basic concepts and properties, which are necessary for this paper. Let f and g be any two fuzzy subsets of a nonempty set X. Then,

(*i*) $f \subseteq q$ if and only if $f(x) \leq g(x)$ for all $x \in X$;
(*ii*) the *intersection* of f and g, denoted by $f \cap g$, defined by $(f \cap g)(x) = \min\{f(x), g(x)\}$ for all $x \in X$;
(*iii*) the *union* of f and g, denoted by $f \cup g$, defined by $(f \cup g)(x) = \max\{f(x), g(x)\}$ for all $x \in X$.

For any fuzzy subset f of a nonempty set X, the *support* of f is a subset of X defined by $supp(f) := \{x \in X \mid f(x) \neq 0\}$. Let A be any subset of a nonempty set X. Then the *characteristic mapping* C_A of A is a fuzzy subset of X defined by for every $x \in X$,

$$C_A(x) = \begin{cases} 1 & \text{if } x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

For every element s of a nonempty set X and $\alpha \in (0, 1]$, a *fuzzy point* s_α [24] of X is a fuzzy subset of X defined by for any $x \in X$,

$$s_\alpha(x) = \begin{cases} \alpha & \text{if } x = s, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 1.1. [19] *Let A and B be nonempty subsets of a nonempty set X and let f and g be fuzzy subsets of X. Then the following statements hold:*

(*i*) $C_{A\cap B} = C_A \cap C_B$;
(*ii*) $A \subseteq B$ *if and only if* $C_A \subseteq C_B$;
(*iii*) $supp(C_A) = A$;
(*iv*) *if* $f \subseteq g$, *then* $supp(f) \subseteq supp(g)$.

Let f and g be fuzzy subsets of a semigroup S. A *product* $f \circ g$ is a fuzzy subset of S defined by for each $x \in S$,

$$(f \circ g)(x) = \begin{cases} \sup\limits_{x=yz} [\min\{f(y), g(z)\}] & \text{if } \exists\ y, z \in S \text{ such that } x = yz, \\ 0 & \text{otherwise.} \end{cases}$$

Lemma 1.2. [19] *If A and B are subsets of a semigroup S, then $C_A \circ C_B = C_{AB}$.*

Next, we recall the definitions of almost bi-ideals, almost interior ideals and weakly almost interior ideals of semigroups which will be used in the next section.

Definition 1.1. [11] *A nonempty subset B of a semigroup S is called an almost bi-ideal of S if $BsB \cap B \neq \emptyset$ for all $s \in S$.*

Definition 1.2. [13] *A nonempty subset I of a semigroup S is called*

(*i*) *an almost interior ideal of S if $aIb \cap I \neq \emptyset$ for all $a, b \in S$,*
(*ii*) *a weakly almost interior ideal of S if $aIa \cap I \neq \emptyset$ for all $a \in S$.*

It is known that the notion of quasi-ideals of semigroups is a generalization of one-sided ideals, while the bi-ideals are a generalization of quasi-ideals in semigroups. However, the concept of interior ideals is a generalization of ideals of semigroups. In 2018, Rao [25] defined the notion of bi-interior ideals, which is a generalization of quasi-ideals, bi-ideals and interior ideals of semigroups as the following definition.

Definition 1.3. [25] *A nonempty subset B of a semigroup S is said to be a bi-interior ideal of S if $BSB \cap SBS \subseteq B$.*

The purpose of this paper is to define the concept, as some generalization of bi-interior ideals in semigroups, namely almost bi-interior ideals. Then, we investigate some of its properties. Furthermore, we introduce the notion of fuzzy almost bi-interior ideals of semigroups and study the relationships between almost bi-interior ideals and fuzzy almost bi-interior ideals of semigroups.

2. **Almost Bi-interior Ideals of Semigroups.** In this section, we introduce the new concept, namely almost bi-interior ideals, as some generalization of bi-interior ideals of semigroups and study some properties of almost bi-interior ideals of semigroups.

Definition 2.1. *A nonempty subset B of a semigroup S is called an almost bi-interior ideal of S if $(sBs \cap BsB) \cap B \neq \emptyset$ for all $s \in S$.*

Proposition 2.1. *Every bi-interior ideal B of a semigroup S is either $aBa \cap BaB = \emptyset$ for some $a \in S$ or an almost bi-interior ideal of S.*

Proof: Let B be a bi-interior ideal of a semigroup S. Assume that $aBa \cap BaB \neq \emptyset$ for all $a \in S$. Let $s \in S$. Then, $\emptyset \neq sBs \cap BsB \subseteq SBS \cap BSB \subseteq B$. This implies that $(sBs \cap BsB) \cap B \neq \emptyset$. Hence, B is an almost bi-interior ideal of S. □

Example 2.1. *Consider the semigroup $(\mathbb{Z}_3, \cdot)$ under the multiplication $\cdot$ on $\mathbb{Z}_3$. We can see that a subset $B = \{\overline{0}, \overline{2}\}$ of $\mathbb{Z}_3$ is an almost bi-interior ideal of $\mathbb{Z}_3$. However, $B\mathbb{Z}_3B \cap \mathbb{Z}_3B\mathbb{Z}_3 = \mathbb{Z}_3 \nsubseteq B$. Hence, B is not a bi-interior ideal of $\mathbb{Z}_3$.*

From Example 2.1, we obtain that an almost bi-interior ideal of a semigroup S needs not to be a bi-interior ideal of S.

Proposition 2.2. *Every almost bi-interior ideal of a semigroup S is also an almost bi-ideal of S.*

Proof: Let B be an almost bi-interior ideal of a semigroup S. Then, $\emptyset \neq (sBs \cap BsB) \cap B \subseteq BsB \cap B$ for all $s \in S$. It follows that $BsB \cap B \neq \emptyset$ for all $s \in S$. Hence, B is an almost bi-ideal of S. □

The following proposition can be proven similar to Proposition 2.2.

Proposition 2.3. *Every almost bi-interior ideal of a semigroup S is also a weakly almost interior ideal of S.*

Example 2.2. *Consider $S = \{a, b, c\}$ with the following multiplication table:*

$\cdot$	a	b	c
a	a	a	a
b	b	b	b
c	c	c	c

Then, $(S, \cdot)$ is a semigroup. Let $B = \{b\}$. By routine calculations, we have that B is an almost bi-ideal of S, but B is not an almost bi-interior ideal of S because $(aBa \cap BaB) \cap B = \emptyset$.

Example 2.3. *Let $S = \{a, b, c, d, e\}$. Define a binary operation $\cdot$ on S by the following table:*

$\cdot$	a	b	c	d	e
a	a	a	a	d	d
b	a	b	c	d	e
c	a	c	b	d	e
d	d	d	d	a	a
e	d	d	d	a	a

Then, $(S, \cdot)$ is a semigroup [17]. *Let $B = \{a, b\}$. We can show that B is a weakly almost interior ideal of S, but B is not an almost bi-interior ideal of S since $(dBd \cap BdB) \cap B = \emptyset$.*

By Example 2.2, we can conclude that an almost bi-ideal of a semigroup S needs not to be an almost bi-interior ideal of S. Similarly, a weakly almost interior ideal of a semigroup S needs not to be an almost bi-interior ideal of S as shown in Example 2.3.

Lemma 2.1. *Let B be an almost bi-interior ideal of a semigroup S. If A is a subset of S containing B, then A is also an almost bi-interior ideal of S.*

Proof: Assume that A is a subset of S containing B. Let $s \in S$. Then, $\emptyset \neq (sBs \cap BsB) \cap B \subseteq (sAs \cap AsA) \cap A$. Hence, A is an almost bi-interior ideal of S. □

Corollary 2.1. *If A and B are almost bi-interior ideals of a semigroup S, then $A \cup B$ is also an almost bi-interior ideal of S.*

It is observed that the set of all almost bi-interior ideals of a semigroup S forms a semigroup under the binary operation of union. On the other hand, it cannot form a semigroup under the binary operation of intersection, as shown by the following example.

Example 2.4. *Consider $S = \{a, b, c, d, e\}$ together with the binary operation $\cdot$ on S defined in Example 2.3. Let $B_1 = \{a, d\}$ and $B_2 = \{c, d\}$. By routine computations, we obtain that B_1 and B_2 are almost bi-interior ideals of S. However, $B_1 \cap B_2 = \{d\}$ is not an almost bi-interior ideal of S because $(a\{d\}a \cap \{d\}a\{d\}) \cap \{d\} = \emptyset$.*

Theorem 2.1. *Let S be a semigroup and $|S| > 1$. Then S has no proper almost bi-interior ideals if and only if for every $a \in S$, there exists $s_a \in S$ such that*

$$s_a(S \setminus \{a\})s_a \cap (S \setminus \{a\})s_a(S \setminus \{a\}) \subseteq \{a\}.$$

Proof: Assume that S has no proper almost bi-interior ideals and let $a \in S$. Then, $S \setminus \{a\}$ is not an almost bi-interior ideal of S. Thus, there exists $s_a \in S$ such that $[s_a(S \setminus \{a\})s_a \cap (S \setminus \{a\})s_a(S \setminus \{a\})] \cap (S \setminus \{a\}) = \emptyset$. This implies that $s_a(S \setminus \{a\})s_a \cap (S \setminus \{a\})s_a(S \setminus \{a\}) \subseteq \{a\}$.

Conversely, suppose that S contains a proper almost bi-interior ideal. Let A be a proper almost bi-interior ideal of S. Thus, there exists $a \in S$ such that $a \notin A$. By assumption, there exists $s_a \in S$ such that $s_a(S \setminus \{a\})s_a \cap (S \setminus \{a\})s_a(S \setminus \{a\}) \subseteq \{a\}$. It follows that $[s_a(S \setminus \{a\})s_a \cap (S \setminus \{a\})s_a(S \setminus \{a\})] \cap (S \setminus \{a\}) = \emptyset$. This means that $S \setminus \{a\}$ is not an almost bi-interior ideal of S. Since $A \subseteq S \setminus \{a\}$ and by Lemma 2.1, we have that $S \setminus \{a\}$ is an almost bi-interior ideal of S which is a contradiction. Therefore, S has no proper almost bi-interior ideals. □

3. **Fuzzy Almost Bi-interior Ideals of Semigroups.** In this section, we introduce the concept of fuzzy almost bi-interior ideals of semigroups and investigate some connection between almost bi-interior ideals and fuzzy almost bi-interior ideals of semigroups.

Definition 3.1. *Let f be a fuzzy subset of a semigroup S such that $f \neq 0$. Then f is said to be a fuzzy almost bi-interior ideal of S if $(s_\alpha \circ f \circ s_\alpha \cap f \circ s_\alpha \circ f) \cap f \neq 0$ for all fuzzy point s_α of S.*

Theorem 3.1. *Let f be a fuzzy almost bi-interior ideal of a semigroup S. If g is a fuzzy subset of S such that $f \subseteq g$, then g is also a fuzzy almost bi-interior ideal of S.*

Proof: Assume that g is a fuzzy subset of S such that $f \subseteq g$. Clearly, $g \neq 0$. Let s_α be any fuzzy point of S. Then, $0 \neq (s_\alpha \circ f \circ s_\alpha \cap f \circ s_\alpha \circ f) \cap f \subseteq (s_\alpha \circ g \circ s_\alpha \cap g \circ s_\alpha \circ g) \cap g$. We obtain that g is a fuzzy almost bi-interior ideal of S. □

Corollary 3.1. *The union of any two fuzzy almost bi-interior ideals of a semigroup S is also a fuzzy almost bi-interior ideal of S.*

Example 3.1. *Consider $S = \{a, b, c, d, e\}$ together with the binary operation $\cdot$ on S defined in Example 2.3. Define two fuzzy subsets f and g of S by for every $x \in S$,*

$$f(x) = \begin{cases} 0.5 & \text{if } x \in \{a, e\}, \\ 0 & \text{otherwise,} \end{cases} \quad \text{and } g(x) = \begin{cases} 0.9 & \text{if } x \in \{c, d\}, \\ 0 & \text{otherwise.} \end{cases}$$

For every fuzzy points x_α and y_β of S, we have that

$$[(x_\alpha \circ f \circ x_\alpha) \cap (f \circ x_\alpha \circ f) \cap f](a) \neq 0 \text{ and } [(y_\beta \circ g \circ y_\beta) \cap (g \circ y_\beta \circ g) \cap g](d) \neq 0.$$

This shows that f and g are fuzzy almost bi-interior ideals of S. However, $f \cap g$ is not a fuzzy almost bi-interior ideal of S.

From Example 3.1, we conclude that the intersection of any two fuzzy almost bi-interior ideals of a semigroup S needs not to be a fuzzy almost bi-interior ideal of S. Next, we investigate the characterizations of almost bi-interior ideals and fuzzy almost bi-interior ideals of semigroups.

Theorem 3.2. *Let B be a nonempty subset of a semigroup S. Then B is an almost bi-interior ideal of S if and only if C_B is a fuzzy almost bi-interior ideal of S.*

Proof: Assume that B is an almost bi-interior ideal of S. Let $x \in S$ and $\alpha \in (0, 1]$. Then, $(xBx \cap BxB) \cap B \neq \emptyset$. Thus, there exists $a \in S$ such that $a \in xBx \cap BxB$ and $a \in B$. This implies that $[(x_\alpha \circ C_B \circ x_\alpha) \cap (C_B \circ x_\alpha \circ C_B)](a) \neq 0$ and $C_B(a) = 1$. Hence, $[(x_\alpha \circ C_B \circ x_\alpha) \cap (C_B \circ x_\alpha \circ C_B)] \cap C_B \neq 0$. Therefore, C_B is a fuzzy almost bi-interior ideal of S.

Conversely, assume that C_B is a fuzzy almost bi-interior ideal of S. Let $s \in S$. Then, $[(s_1 \circ C_B \circ s_1) \cap (C_B \circ s_1 \circ C_B)] \cap C_B \neq 0$. So, there exists $x \in S$ such that $[(s_1 \circ C_B \circ s_1) \cap (C_B \circ s_1 \circ C_B) \cap C_B](x) \neq 0$. We obtain that $x \in (sBs \cap BsB) \cap B$, that is, $(sBs \cap BsB) \cap B \neq \emptyset$. Consequently, B is an almost bi-interior ideal of S. □

Theorem 3.3. *Let f be a fuzzy subset of a semigroup S. Then f is a fuzzy almost bi-interior ideal of S if and only if $supp(f)$ is an almost bi-interior ideal of S.*

Proof: Assume that f is a fuzzy almost bi-interior ideal of S. Let $s \in S$ and $\alpha \in (0, 1]$. Then, $[(s_\alpha \circ f \circ s_\alpha) \cap (f \circ s_\alpha \circ f)] \cap f \neq 0$. Thus, there exists $x \in S$ such that $[(s_\alpha \circ f \circ s_\alpha) \cap (f \circ s_\alpha \circ f) \cap f](x) \neq 0$. Hence, there exist $a_1, a_2, a_3 \in S$ such that $x = sa_1s = a_2sa_3$, $f(a_1) \neq 0$, $f(a_2) \neq 0$, $f(a_3) \neq 0$ and $f(x) \neq 0$. It follows that $x, a_1, a_2, a_3 \in supp(f)$. We obtain that

$$\left[\left(s_\alpha \circ C_{supp(f)} \circ s_\alpha\right) \cap \left(C_{supp(f)} \circ s_\alpha \circ C_{supp(f)}\right)\right](x) \neq 0 \text{ and } C_{supp(f)}(x) = 1.$$

That is, $\left[\left(s_\alpha \circ C_{supp(f)} \circ s_\alpha\right) \cap \left(C_{supp(f)} \circ s_\alpha \circ C_{supp(f)}\right)\right] \cap C_{supp(f)} \neq 0$. It turns out that $C_{supp(f)}$ is a fuzzy almost bi-interior ideal of S. By Theorem 3.2, we have that $supp(f)$ is an almost bi-interior ideal of S.

Conversely, assume that $supp(f)$ is an almost bi-interior ideal of S. By Theorem 3.2, $C_{supp(f)}$ is a fuzzy almost bi-interior ideal of S. Let s_α be any fuzzy point of S. Then, $\left[\left(s_\alpha \circ C_{supp(f)} \circ s_\alpha\right) \cap \left(C_{supp(f)} \circ s_\alpha \circ C_{supp(f)}\right)\right] \cap C_{supp(f)} \neq 0$. So, there exists $x \in S$ such that

$$\left[\left(s_\alpha \circ C_{supp(f)} \circ s_\alpha\right) \cap \left(C_{supp(f)} \circ s_\alpha \circ C_{supp(f)}\right)\right](x) \neq 0 \text{ and } C_{supp(f)}(x) \neq 0.$$

This implies that there exist $y_1, y_2, y_3 \in S$ such that $x = sy_1s = y_2sy_3$, $f(y_1) \neq 0$, $f(y_2) \neq 0$, $f(y_3) \neq 0$ and $f(x) \neq 0$. Hence, $[(s_\alpha \circ f \circ s_\alpha) \cap (f \circ s_\alpha \circ f)] \cap f \neq 0$. Therefore, f is a fuzzy almost bi-interior ideal of S. □

Now, we provide some connection between minimal almost bi-interior ideals and minimal fuzzy almost bi-interior ideals of semigroups.

Definition 3.2. *A fuzzy almost bi-interior ideal f of a semigroup S is called minimal if $supp(g) = supp(f)$ for every fuzzy almost bi-interior ideal g of S such that $g \subseteq f$.*

Theorem 3.4. *Let B be a nonempty subset of a semigroup S. Then B is a minimal almost bi-interior ideal of S if and only if C_B is a minimal fuzzy almost bi-interior ideal of S.*

Proof: Assume that B is a minimal almost bi-interior ideal of S. By Theorem 3.2, C_B is a fuzzy almost bi-interior ideal of S. Let g be a fuzzy almost bi-interior ideal of S such that $g \subseteq C_B$. By Lemma 1.1, $supp(g) \subseteq supp(C_B) = B$. In addition, $supp(g)$ is an almost bi-interior ideal of S by Theorem 3.3. By the given assumption, we obtain that $supp(g) = B = supp(C_B)$. Therefore, C_B is a minimal fuzzy almost bi-interior ideal of S.

Conversely, assume that C_B is a minimal fuzzy almost bi-interior ideal of S. Also, B is an almost bi-interior ideal of S by Theorem 3.2. Let A be any almost bi-interior ideal of S such that $A \subseteq B$. By Lemma 1.1 and Theorem 3.2, we have that C_A is a fuzzy almost bi-interior ideal of S such that $C_A \subseteq C_B$. Since C_B is minimal, it implies that $A = supp(C_A) = supp(C_B) = B$. Consequently, B is a minimal almost bi-interior ideal of S. □

The following result can be achieved by Theorem 3.2 and Theorem 3.3.

Corollary 3.2. *Let S be a semigroup. Then S has no proper almost bi-interior ideals if and only if for every fuzzy almost bi-interior ideal f of S, $supp(f) = S$.*

Let S be a semigroup. An almost bi-interior ideal P of S is called *prime* if for any almost bi-interior ideals A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost bi-interior ideal P of S is called *semiprime* if for any almost bi-interior ideal A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost bi-interior ideal P of S is called *strongly prime* if for any almost bi-interior ideals A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 3.3. *A fuzzy almost bi-interior ideal h of a semigroup S is called a fuzzy prime almost bi-interior ideal of S if for any two fuzzy almost bi-interior ideals f and g of S, $f \circ g \subseteq h$ implies that $f \subseteq h$ or $g \subseteq h$.*

Definition 3.4. *A fuzzy almost bi-interior ideal h of a semigroup S is said to be a fuzzy semiprime almost bi-interior ideal of S if for any fuzzy almost bi-interior ideal f of S, $f \circ f \subseteq h$ implies that $f \subseteq h$.*

Definition 3.5. *A fuzzy almost bi-interior ideal h of a semigroup S is called a fuzzy strongly prime almost bi-interior ideal of S if for any two fuzzy almost bi-interior ideals f and g of S, $(f \circ g) \cap (g \circ f) \subseteq h$ implies that $f \subseteq h$ or $g \subseteq h$.*

We note that every fuzzy strongly prime almost bi-interior ideal of a semigroup S is a fuzzy prime almost bi-interior ideal of S, and every fuzzy prime almost bi-interior ideal of a semigroup S is a fuzzy semiprime almost bi-interior ideal of S. Finally, we present the relations between prime (resp., semiprime, strongly prime) almost bi-interior ideals and their fuzzifications of semigroups.

Theorem 3.5. *Let P be a nonempty subset of a semigroup S. Then P is a prime almost bi-interior ideal of S if and only if C_P is a fuzzy prime almost bi-interior ideal of S.*

Proof: Assume that P is a prime almost bi-interior ideal of S. By Theorem 3.2, C_P is a fuzzy almost bi-interior ideal of S. Let f and g be fuzzy almost bi-interior ideals of S such that $f \circ g \subseteq C_P$. Suppose that $f \nsubseteq C_P$ and $g \nsubseteq C_P$. Then, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$, while $C_P(x) = 0$ and $C_P(y) = 0$. Thus, $x \in supp(f)$ and $y \in supp(g)$ such that $x, y \notin P$. This means that $supp(f) \nsubseteq P$ and $supp(g) \nsubseteq P$. By Theorem 3.3, we get that $supp(f)$ and $supp(g)$ are almost bi-interior ideals of S. Since P is prime, $supp(f)supp(g) \nsubseteq P$. So, there exists $t = ab$ for some $a \in supp(f)$ and for some $b \in supp(g)$ such that $t \notin P$. It turns out that $C_P(t) = 0$ and then $(f \circ g)(t) = 0$ because $f \circ g \subseteq C_P$. Since $f(a) \neq 0$ and $g(b) \neq 0$, it follows that $(f \circ g)(t) = \sup_{t=ab}[\min\{f(a), g(b)\}] \neq 0$, which is a contradiction. Hence, $f \subseteq C_P$ or $g \subseteq C_P$. Therefore, C_P is a fuzzy prime almost bi-interior ideal of S.

Conversely, assume that C_P is a fuzzy prime almost bi-interior ideal of S. Then, P is an almost bi-interior ideal of S by Theorem 3.2. Let A and B be two almost bi-interior ideals of S such that $AB \subseteq P$. Again by Theorem 3.2, C_A and C_B are fuzzy almost bi-interior ideals of S. By Lemma 1.1 and Lemma 1.2, we have that $C_A \circ C_B = C_{AB} \subseteq C_P$. By the given assumption, $C_A \subseteq C_P$ or $C_B \subseteq C_P$. This implies that $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost bi-interior ideal of S. □

The following theorem can be proven similar to Theorem 3.5.

Theorem 3.6. *Let P be a nonempty subset of a semigroup S. Then P is a semiprime almost bi-interior ideal of S if and only if C_P is a fuzzy semiprime almost bi-interior ideal of S.*

Theorem 3.7. *Let P be a nonempty subset of a semigroup S. Then P is a strongly prime almost bi-interior ideal of S if and only if C_P is a fuzzy strongly prime almost bi-interior ideal of S.*

Proof: Assume that P is a strongly prime almost bi-interior ideal of S. Then, C_P is a fuzzy almost bi-interior ideal of S by Theorem 3.2. Let f and g be any two fuzzy almost bi-interior ideals of S such that $(f \circ g) \cap (g \circ f) \subseteq C_P$. Suppose that $f \nsubseteq C_P$ and $g \nsubseteq C_P$. Then, there exist $x, y \in S$ such that $f(x) \neq 0$ and $g(y) \neq 0$ where $C_P(x) = 0$ and $C_P(y) = 0$. Thus, $x \in supp(f)$ and $y \in supp(g)$ such that $x \notin P$ and $y \notin P$. This implies that $supp(f) \nsubseteq P$ and $supp(g) \nsubseteq P$. By Theorem 3.3 and the given assumption, we have that $[supp(f)supp(g)] \cap [supp(g)supp(f)] \nsubseteq P$. It follows that there exists $t \in [supp(f)supp(g)] \cap [supp(g)supp(f)]$ such that $t \notin P$. So, $C_P(t) = 0$ and then $[(f \circ g) \cap (g \circ f)](t) = 0$, because $(f \circ g) \cap (g \circ f) \subseteq C_P$. On the other hand, since $t \in [supp(f)supp(g)] \cap [supp(g)supp(f)]$, we get that $t = a_1b_1$ and $t = b_2a_2$ for some $a_1, a_2 \in supp(f)$ and for some $b_1, b_2 \in supp(g)$. It turns out that

$$(f \circ g)(t) = \sup_{t=a_1b_1}[\min\{f(a_1), g(b_1)\}] \neq 0 \text{ and } (g \circ f)(t) = \sup_{t=b_2a_2}[\min\{g(b_2), f(a_2)\}] \neq 0.$$

This implies that $\min\{(f \circ g)(t), (g \circ f)(t)\} \neq 0$, that is, $[(f \circ g) \cap (g \circ f)](t) \neq 0$. This is a contradiction with the fact that $[(f \circ g) \cap (g \circ f)](t) = 0$. Hence, $f \subseteq C_P$ or $g \subseteq C_P$. Therefore, C_P is a fuzzy strongly prime almost bi-interior ideal of S.

Conversely, assume that C_P is a fuzzy strongly prime almost bi-interior ideal of S. By Theorem 3.2, P is an almost bi-interior ideal of S. Let A and B be any two almost bi-interior ideals of S such that $AB \cap BA \subseteq P$. It follows that C_A and C_B are fuzzy almost bi-interior ideals of S by Theorem 3.2. By Lemma 1.1 and Lemma 1.2, we have that

$$(C_A \circ C_B) \cap (C_B \circ C_A) = C_{AB} \cap C_{BA} = C_{AB \cap BA} \subseteq C_P.$$

By the hypothesis, it follows that $C_A \subseteq C_P$ or $C_B \subseteq C_P$. This implies that $A \subseteq P$ or $B \subseteq P$. Consequently, P is a strongly prime almost bi-interior ideal of S. □

4. **Conclusions.** In 2018, Rao [25] introduced the notion of bi-interior ideals, which is a generalization of quasi-ideals, bi-ideals and interior ideals of semigroups. In this paper, we define the concept of almost bi-interior ideals of semigroups. Then, we show that the almost bi-interior ideal of semigroups is some generalization of bi-interior ideals in Proposition 2.1. In Section 3, we introduce the notion of fuzzy almost bi-interior ideals of semigroups. Then, we give some connections between almost bi-interior ideals and fuzzy almost bi-interior ideals of semigroups which were shown in Theorem 3.4, Theorem 3.5, Theorem 3.6 and Theorem 3.7. In our future work, we can study many kinds of almost ideals and their fuzzifications in other algebraic structures.

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