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DISTRIBUTED TARGET-ENCIRCLEMENT CONTROL OF MULTIPLE QUADROTORS WITH A LEADER-FOLLOWER FRAMEWORK

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Abstract. *This paper addresses a distributed target-encirclement control problem of multiple quadrotors using a leader-follower framework. In the task of encirclement control, the leader and followers have different functions and roles. Only the leader can obtain the information of the target, while followers cannot obtain directly. Two distributed fixed-time observers are proposed for each follower to estimate the information of the target and leader. Based on observed states, sliding mode controllers are presented for the leader and followers such that the target can be tracked and encircled by multi-quadrotors. Simulation results show the effectiveness of the proposed encirclement control strategy.*
Keywords: Distributed control, Leader-follower framework, Fixed-time observers, Target-encirclement

1. **Introduction.** In recent years, distributed cooperative control of multi-agent systems has been a hot research topic due to its wide applications from multi-robots missions [1], spacecraft formation [2], to an active traffic management [3]. Consensus control implies that all agents reach an agreement with a specific state using neighbors' information. Consensus control consists of leaderless consensus [4] and leader-follower consensus tracking [5]. So far abundant results for leader-follower tracking problems have been reported for first-order [6], second-order [7, 8] and high-order multi-agent systems [9].

Over the past decades, encirclement control problems have received widely concern owing to various applications in military and civilian fields including cooperative hunting for adversary, protection of important members such as unmanned ground vehicles. Model Predictive Control (MPC) method was applied to research a single UAV encircling a stationary target, a single Unmanned Aerial Vehicle (UAV) encircling a moving target, and a group of UAVs encircling a stationary target, respectively [10]. The work in [11] proposed an improved MPC algorithm based on reinforcement learning to solve a target encirclement problem in obstacle rich environment. A cooperative encirclement hunting-like guidance strategy was presented in [12] where the initial states or the interception formation of multiple missiles were optimized by using a finite covering theorem. The

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work in [13] considered a moving target in 3D environment and presented an encirclement control scheme with guaranteed collision avoidance for a multi-robot system.

However, for above-mentioned encirclement strategies, the centralized control approaches were applied to developing controllers. Centralized control is simple and easy to realize in practical applications. However, it also increases the computation and communication load of control center, especially for large-scale multi-agent systems. For a distributed control strategy, each agent has its own control unit where only local neighbors' information is used. Hence, distributed control methods are more flexible and applicable for large-scale multi-agent systems. Two distributed control schemes were developed in [14] to realize encirclement of point target and disk target, where only local bearing measurements are required. The work in [15] solved a collective multi-target rotating encirclement formation problem of second-order multi-agent systems using fixed-time estimators and Lyapunov theory.

In most of the existing encirclement control schemes, a group of agents have the same role and function. Nevertheless, for many special tasks, for example, military confrontation not only needs fighters but a reconnaissance plane. Hence, agents with different functions are required in practical applications. A leader-follower framework [16] was applied in the target-encirclement control, where only a leader has detection ability and can access to the information of the target, while followers cannot obtain directly. Observers were used to estimate the states of the target and leader. The work in [17] further considered target-encirclement control problem of fractional-order multi-agent systems applying leader-follower construction. However, the above research works did not consider any practical model, where the control objects are only with first-order or fractional-order systems. In practical applications, many systems are characterized by higher order dynamics, such as quadrotors and mobile robots. Moreover, compared with finite-time observers in [16] and [17], fixed-time observers not only guarantee that the real values are estimated with a finite time, but also the estimated time is global bounded for any initial state [18].

Hence, inspired by above discussions, this paper addresses a distributed target-encirclement control problem of multiple quadrotors using a leader-follower framework. In this work, the information of the target is only accessible to a leader which has detection ability, while it is unnecessary for followers to detect the target. Fixed-time observers are presented to estimate the information of the target and leader, and estimated time is global bounded independent of initial conditions. Then, based on the observed states, the distributed sliding mode encirclement control algorithm is designed for leader and followers such that they can cooperate to enclose the target. The simulation results are presented to show the effectiveness of the proposed encirclement control strategy.

2. Problem Statement and Preliminaries.

In this section, we will introduce the system model of a quadrotor and define the control objective of target-encirclement.

2.1. System model.

The model of the quadrotor is adopted in [11]. The dynamics are written as

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x = \{x, y, \dot{x}, \dot{y}, \ddot{x}, \ddot{y}\}^T$, $u = \{u_x, u_y\}^T$,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -5.6956 & 0 \\ 0 & 0 & 0 & 0 & 0 & -5.6956 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 5.6956 & 0 \\ 0 & 5.6956 \end{bmatrix}$$

Then, the dynamics of the target can be described as

$$\dot{x}_T(t) = Ax_T(t) \tag{2}$$

The dynamics of a leader is

$$\dot{x}_0(t) = Ax_0(t) + Bu_0(t) \tag{3}$$

The dynamics of followers are

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t) \tag{4}$$

To facilitate the expression, define $p = (x, y)$, $v = (\dot{x}, \dot{y}) = (v_x, v_y)$, $a = (\ddot{x}, \ddot{y}) = (a_x, a_y)$ as position, velocity and acceleration, respectively.

2.2. **Control objective.** In order to achieve the target-encirclement control of multiple quadrotors, we define the following control objective.

Definition 2.1. *The target-encirclement control problem of systems (3) and (4) can be solved, if design controllers u_0 and u_i for the leader and followers such that*

$$\begin{aligned}
&\lim_{t\to\infty} \|p_i - p_T\| = l \\
&\lim_{t\to\infty} (v_i - v_T) = 0 \\
&\lim_{t\to\infty} (a_i - a_T) = 0 \\
&\lim_{t\to\infty} \left(\theta_i - \theta_j - \frac{2\pi(i-j)}{N}\right) = 0
\end{aligned} \tag{5}$$

where l is the desired distance between the target and agents, θ_i represents the desired angle between the target and agent i, and N is the total number of agents.

Lemma 2.1. [19] *If the communication topology G is undirected and connected, then matrix $H = L + B$ is symmetric and positive definite.*

Lemma 2.2. [5] *For a finite dimensional linear space, the following inequality holds:*

$$\|y\|_m \le \|y\|_n \le N^{\frac{1}{n}-\frac{1}{m}} \|y\|_m \tag{6}$$

where $m > n > 0$, and the norms $\|y\|_c = \left(\sum_{i=1}^N |y_i|^c\right)^{\frac{1}{c}}$, $y = [y_1, y_2, \ldots, y_N]^T \in \mathbb{R}^n$, $c > 0$.

Lemma 2.3. [20] *Assume that there exists a continuously differentiable positive definite and radially unbounded function $V(z) : \mathbb{R}^N \to \mathbb{R}_+$ such that*

$$\dot{V}(z) \le -aV^p(z) - bV^q(z), \quad z \ne 0 \tag{7}$$

where $a, b > 0$, $p = 1 - 1/\mu$, $q = 1 + 1/\mu$, $\mu \ge 1$. Then the origin of the system is globally fixed-time stable with convergence time estimated by

$$T(z_0) \le T_{\max} = \frac{\pi\mu}{2\sqrt{ab}} \tag{8}$$

Assumption 2.1. *The communication graph G is connected and at least one follower can access to the information of leader.*

Assumption 2.2. *The control input of leader u_0 is bounded and satisfies $|u_0| \le \bar{u}_0$, where $\bar{u}_0$ is a known positive constant.*

3. **Main Results.** In this section, a fixed-time observer method is applied for followers to estimating the information of the target and leader. Then, based on observers, a distributed sliding mode control strategy is proposed to solve the target-encirclement problem.

3.1. **Design of fixed-time observers.** In this paper, a leader and followers have different roles and functions, where a leader can detect the target, while followers cannot obtain the information of the target directly. Moreover, the information of a leader is only accessible to a part of followers. Hence, fixed-time observers are designed for followers to estimate the states of the target and leader.

The distributed fixed-time observer is designed as follows for each follower to estimate the states of the target

$$\begin{aligned}
\dot{\varphi}_{pi} &= \varphi_{vi} - \alpha_{pi}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{pi}-\varphi_{pj}\right) + a_{i0}\left(\varphi_{pi}-\varphi_{p0}\right)\right)^2 \\
&\quad - \beta_{pi}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{pi}-\varphi_{pj}\right) + a_{i0}\left(\varphi_{pi}-\varphi_{p0}\right)\right) \\
\dot{\varphi}_{vi} &= \varphi_{ai} - \alpha_{vi}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{vi}-\varphi_{vj}\right) + a_{i0}\left(\varphi_{vi}-\varphi_{v0}\right)\right)^2 \\
&\quad - \beta_{vi}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{vi}-\varphi_{vj}\right) + a_{i0}\left(\varphi_{vi}-\varphi_{v0}\right)\right) \\
\dot{\varphi}_{ai} &= -r\varphi_{ai} - \alpha_{ai}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{ai}-\varphi_{aj}\right) + a_{i0}\left(\varphi_{ai}-\varphi_{a0}\right)\right)^2 \\
&\quad - \beta_{ai}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{ai}-\varphi_{aj}\right) + a_{i0}\left(\varphi_{ai}-\varphi_{a0}\right)\right)
\end{aligned} \tag{9}$$

where $\alpha_{pi} > 0$, $\beta_{pi} > 0$, $\alpha_{vi} > 0$, $\beta_{vi} > 0$, $\alpha_{ai} > 0$, $\beta_{ai} > 0$, $r = 5.6956$. Since the leader could detect the states of the target, then we define

$$\begin{aligned}
\varphi_{p0}(t) &= p_T(t) \\
\varphi_{v0}(t) &= v_T(t) \\
\varphi_{a0}(t) &= a_T(t)
\end{aligned} \tag{10}$$

Theorem 3.1. *Suppose Assumption 2.1 holds, and it uses the distributed observer (9) for any initial states* $\varphi_{pi}(t_0)$, $\varphi_{vi}(t_0)$, $\varphi_{ai}(t_0)$. *Then it has* $\varphi_{pi}(t) = p_T(t)$, $\varphi_{vi}(t) = v_T(t)$, $\varphi_{ai}(t) = a_T(t)$ *when* $t \geq T_1$, *and the convergence time* T_1 *is bounded by*

$$\begin{aligned}
T_1 \leq T_1^{\max} &= \frac{\pi N^{\frac{1}{4}}}{2\lambda_{\min}(H)\sqrt{\alpha_{a\min}\beta_{a\min}}} + \frac{\pi N^{\frac{1}{4}}}{2\lambda_{\min}(H)\sqrt{\alpha_{v\min}\beta_{v\min}}} \\
&\quad + \frac{\pi N^{\frac{1}{4}}}{2\lambda_{\min}(H)\sqrt{\alpha_{p\min}\beta_{p\min}}}
\end{aligned} \tag{11}$$

where $\alpha_{a\min} = \min\{\alpha_{a1}, \alpha_{a2}, \ldots, \alpha_{aN-1}\}$, $\alpha_{v\min} = \min\{\alpha_{v1}, \alpha_{v2}, \ldots, \alpha_{vN-1}\}$, $\alpha_{p\min} = \min\{\alpha_{p1}, \alpha_{p2}, \ldots, \alpha_{pN-1}\}$, $\beta_{a\min} = \min\{\beta_{a1}, \beta_{a2}, \ldots, \beta_{aN-1}\}$, $\beta_{v\min} = \min\{\beta_{v1}, \beta_{v2}, \ldots, \beta_{vN-1}\}$, $\beta_{p\min} = \min\{\beta_{p1}, \beta_{p2}, \ldots, \beta_{pN-1}\}$, $\lambda_{\min}(H)$ *is the minimum eigenvalue of matrix* H.

Proof: Let $\tilde{\varphi}_{pi} = \varphi_{pi} - \varphi_{p0}$, $\tilde{\varphi}_{vi} = \varphi_{vi} - \varphi_{v0}$, $\tilde{\varphi}_{ai} = \varphi_{ai} - \varphi_{a0}$, then it has

$$\begin{aligned}
\dot{\tilde{\varphi}}_{pi} &= \tilde{\varphi}_{vi} - \alpha_{pi}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{pi}-\varphi_{pj}\right) + a_{i0}\left(\varphi_{pi}-\varphi_{p0}\right)\right)^2 \\
&\quad - \beta_{pi}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{pi}-\varphi_{pj}\right) + a_{i0}\left(\varphi_{pi}-\varphi_{p0}\right)\right)
\end{aligned}$$

$$\begin{aligned}\dot{\tilde{\varphi}}_{vi} &= \tilde{\varphi}_{ai} - \alpha_{vi}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{vi} - \varphi_{vj}\right) + a_{i0}\left(\varphi_{vi} - \varphi_{v0}\right)\right)^2 \\ &\quad - \beta_{vi}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{vi} - \varphi_{vj}\right) + a_{i0}\left(\varphi_{vi} - \varphi_{v0}\right)\right) \\ \dot{\tilde{\varphi}}_{ai} &= -r\tilde{\varphi}_{ai} - \alpha_{ai}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{ai} - \varphi_{aj}\right) + a_{i0}\left(\varphi_{ai} - \varphi_{a0}\right)\right)^2 \\ &\quad - \beta_{ai}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\varphi_{ai} - \varphi_{aj}\right) + a_{i0}\left(\varphi_{ai} - \varphi_{a0}\right)\right)\end{aligned} \tag{12}$$

Define $\tilde{\varphi}_p = [\tilde{\varphi}_{p1}, \tilde{\varphi}_{p2}, \ldots, \tilde{\varphi}_{pN-1}]^T$, $\tilde{\varphi}_v = [\tilde{\varphi}_{v1}, \tilde{\varphi}_{v2}, \ldots, \tilde{\varphi}_{vN-1}]^T$, $\tilde{\varphi}_a = [\tilde{\varphi}_{a1}, \tilde{\varphi}_{a2}, \ldots, \tilde{\varphi}_{aN-1}]^T$, $\alpha_p = \mathrm{diag}\{\alpha_{p1}, \alpha_{p2}, \ldots, \alpha_{pN-1}\}$, $\beta_p = \mathrm{diag}\{\beta_{p1}, \beta_{p2}, \ldots, \beta_{pN-1}\}$, $\alpha_v = \mathrm{diag}\{\alpha_{v1}, \alpha_{v2}, \ldots, \alpha_{vN-1}\}$, $\beta_v = \mathrm{diag}\{\beta_{v1}, \beta_{v2}, \ldots, \beta_{vN-1}\}$, $\alpha_a = \mathrm{diag}\{\alpha_{a1}, \alpha_{a2}, \ldots, \alpha_{aN-1}\}$, $\beta_a = \mathrm{diag}\{\beta_{a1}, \beta_{a2}, \ldots, \beta_{aN-1}\}$, then we have

$$\begin{aligned}\dot{\tilde{\varphi}}_p &= \tilde{\varphi}_v - \alpha_p\mathrm{sig}\left(H\tilde{\varphi}_p\right)^2 - \beta_p\mathrm{sign}\left(H\tilde{\varphi}_p\right) \\ \dot{\tilde{\varphi}}_v &= \tilde{\varphi}_a - \alpha_v\mathrm{sig}\left(H\tilde{\varphi}_v\right)^2 - \beta_v\mathrm{sign}\left(H\tilde{\varphi}_v\right) \\ \dot{\tilde{\varphi}}_a &= -r\tilde{\varphi}_a - \alpha_a\mathrm{sig}\left(H\tilde{\varphi}_a\right)^2 - \beta_a\mathrm{sign}\left(H\tilde{\varphi}_a\right)\end{aligned} \tag{13}$$

Choosing the Lyapunov function $V_1 = \frac{1}{2}\tilde{\varphi}_a^T H\tilde{\varphi}_a$, then the derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= \tilde{\varphi}_a^T H\dot{\tilde{\varphi}}_a \\ &= \tilde{\varphi}_a^T H\left(-r\tilde{\varphi}_a - \alpha_a\mathrm{sig}\left(H\tilde{\varphi}_a\right)^2 - \beta_a\mathrm{sign}\left(H\tilde{\varphi}_a\right)\right) \\ &\le -\alpha_{a\min}\left\|H\tilde{\varphi}_a\right\|_3^3 - \beta_{a\min}\left\|H\tilde{\varphi}_a\right\|_1\end{aligned}$$

Applying Lemma 2.2, the following inequality is satisfied

$$\begin{aligned}\dot{V}_1 &\le -\alpha_{a\min}N^{-\frac{1}{2}}\left\|H\tilde{\varphi}_a\right\|_2^3 - \beta_{a\min}\left\|H\tilde{\varphi}_a\right\|_2 \\ &\le -\alpha_{a\min}N^{-\frac{1}{2}}\left(2\lambda_{\min}(H)\right)^{\frac{3}{2}}V_1^{\frac{3}{2}} - \beta_{a\min}\left(2\lambda_{\min}(H)\right)^{\frac{1}{2}}V_1^{\frac{1}{2}}\end{aligned} \tag{14}$$

According to Lemma 2.3, the observer error $\tilde{\varphi}_{ai}$ can converge to zero. After $\tilde{\varphi}_a$ converge to zero, equality (13) is

$$\dot{\tilde{\varphi}}_v = -\alpha_v\mathrm{sig}\left(H\tilde{\varphi}_v\right)^2 - \beta_v\mathrm{sign}\left(H\tilde{\varphi}_v\right) \tag{15}$$

Choose Lyapunov function $V_2 = \frac{1}{2}\tilde{\varphi}_v^T H\tilde{\varphi}_v$ and $V_3 = \frac{1}{2}\tilde{\varphi}_p^T H\tilde{\varphi}_p$. Then following the same process as above, we can obtain that $\tilde{\varphi}_v$ and $\tilde{\varphi}_p$ converge to zero and the convergence time T_1 is bounded by (11). The proof is completed.

Similarly, the distributed fixed-time observer is designed as follows for each follower to estimate the states of the leader

$$\begin{aligned}\dot{\hat{p}}_i &= \hat{v}_i - \gamma_{pi}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\hat{p}_i - \hat{p}_j\right) + a_{i0}\left(\hat{p}_i - \hat{p}_0\right)\right)^2 \\ &\quad - \kappa_{pi}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\hat{p}_i - \hat{p}_j\right) + a_{i0}\left(\hat{p}_i - \hat{p}_0\right)\right) \\ \dot{\hat{v}}_i &= \hat{a}_i - \gamma_{vi}\mathrm{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\hat{v}_i - \hat{v}_j\right) + a_{i0}\left(\hat{v}_i - \hat{v}_0\right)\right)^2 \\ &\quad - \kappa_{vi}\mathrm{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\hat{v}_i - \hat{v}_j\right) + a_{i0}\left(\hat{v}_i - \hat{v}_0\right)\right)\end{aligned}$$

$$\dot{\hat{a}}_i = r\hat{a}_i - \gamma_{ai}\text{sig}\left(\sum_{i=1}^{N-1} a_{ij}\left(\hat{a}_i - \hat{a}_j\right) + a_{i0}\left(\hat{a}_i - \hat{a}_0\right)\right)^2 - \kappa_{ai}\text{sign}\left(\sum_{i=1}^{N-1} a_{ij}\left(\hat{a}_i - \hat{a}_j\right) + a_{i0}\left(\hat{a}_i - \hat{a}_0\right)\right) \tag{16}$$

Theorem 3.2. *Suppose Assumption 2.1 holds, and it uses the distributed observer (16) for any initial states $\hat{p}_i(t_0)$, $\hat{v}_i(t_0)$, $\hat{a}_i(t_0)$. Then it has $\hat{p}_i(t) = p_0(t)$, $\hat{v}_i(t) = v_0(t)$, $\hat{a}_i(t) = a_0(t)$ when $t \geq T_2$, and the convergence time T_2 is bounded by*

$$T_2 \leq T_2^{\max} = \frac{\pi N^{\frac{1}{4}}}{2\lambda_{\min}(H)\sqrt{\gamma_{a\min}\kappa_{a\min}}} + \frac{\pi N^{\frac{1}{4}}}{2\lambda_{\min}(H)\sqrt{\gamma_{v\min}\kappa_{v\min}}} + \frac{\pi N^{\frac{1}{4}}}{2\lambda_{\min}(H)\sqrt{\gamma_{p\min}\kappa_{p\min}}} \tag{17}$$

where $\gamma_{a\min} = \min\{\gamma_{a1}, \gamma_{a2}, \dots, \gamma_{aN-1}\}$, $\gamma_{v\min} = \min\{\gamma_{v1}, \gamma_{v2}, \dots, \gamma_{vN-1}\}$, $\gamma_{p\min} = \min\{\gamma_{p1}, \gamma_{p2}, \dots, \gamma_{pN-1}\}$, $\kappa_{a\min} = \min\{\kappa_{a1}, \kappa_{a2}, \dots, \kappa_{aN-1}\}$, $\kappa_{v\min} = \min\{\kappa_{v1}, \kappa_{v2}, \dots, \kappa_{vN-1}\}$, $\kappa_{p\min} = \min\{\kappa_{p1}, \kappa_{p2}, \dots, \kappa_{pN-1}\}$.

Proof: This proof is similar to one of Theorem 3.1. Hence, here it is omitted.

4. **Design of Sliding Mode Encirclement Controllers.** In this section, we present a distributed sliding mode strategy for the leader and followers to realize the encirclement of target. Firstly, the errors between the target and leader can be defined as

$$\begin{aligned} e_{p0} &= p_0 - p_T - [l\cos\theta_0 \ l\sin\theta_0]^T \\ e_{v0} &= \dot{e}_{p0} = v_0 - v_T \\ e_{a0} &= \ddot{e}_{p0} = a_0 - a_T \end{aligned} \tag{18}$$

where l and θ_0 are desired distance and desired angle between the target and leader, respectively. For followers, the information of the target and leader is not accessible to them. Hence, the estimated values are used to define the errors for followers. The desired position of followers is determined by the location of target and leader. Hence, the errors for followers are defined as

$$\begin{aligned} e_{pi} &= p_i - \varphi_{pi} - \begin{bmatrix} \cos\frac{2\pi i}{N} & -\sin\frac{2\pi i}{N} \\ \sin\frac{2\pi i}{N} & \cos\frac{2\pi i}{N} \end{bmatrix}\left(\hat{p}_i - \varphi_{pi}\right) \\ e_{vi} &= \dot{e}_{pi} = v_i - \varphi_{vi} - \begin{bmatrix} \cos\frac{2\pi i}{N} & -\sin\frac{2\pi i}{N} \\ \sin\frac{2\pi i}{N} & \cos\frac{2\pi i}{N} \end{bmatrix}\left(\hat{v}_i - \varphi_{vi}\right) \\ e_{ai} &= \ddot{e}_{pi} = a_i - \varphi_{ai} - \begin{bmatrix} \cos\frac{2\pi i}{N} & -\sin\frac{2\pi i}{N} \\ \sin\frac{2\pi i}{N} & \cos\frac{2\pi i}{N} \end{bmatrix}\left(\hat{a}_i - \varphi_{ai}\right) \end{aligned} \tag{19}$$

Based on the defined errors, an integral sliding mode surface is designed as

$$s_i = e_{ai} + \int_0^t \left(k_{3i}e_{ai}(\tau) + k_{2i}e_{vi}(\tau) + k_{1i}e_{pi}(\tau)\right)d(\tau), \quad i = 0, 1, \dots, N-1 \tag{20}$$

where k_{1i}, k_{2i}, k_{3i} are designed later.

Based on sliding mode surface (20), the distributed controller for leader and followers is proposed as

$$u_i = e_{ai} - \frac{1}{r}\left(k_{3i}e_{ai} + k_{2i}e_{vi} + k_{1i}e_{pi}\right) - \frac{1}{r}\sigma_i \text{sign}(s_i) \tag{21}$$

where $\sigma_i > 0$.

Theorem 4.1. *Considering quadrotors systems (3)-(4) with the observers (9) and (16), sliding mode surface (20) and control law (21), then the leader and followers can encircle the target (2).*

Proof: The proof consists of the reachability and existence of sliding mode surface. Firstly, we give the reachability of sliding mode surface. The Laypunov function is chosen as $V_4 = \sum_{i=0}^{N-1} \frac{1}{2} s_i^T s_i$, and then the derivative of V_4 is

$$\dot{V}_4 = \sum_{i=0}^{N-1} s_i \dot{s}_i$$

Applying sliding surface (20) and control law (21), it has

$$\begin{aligned}
\dot{V}_4 &= \sum_{i=0}^{N-1} s_i \left(\dot{e}_{ai} + \left(k_{3i}e_{ai} + k_{2i}e_{vi} + k_{1i}e_{pi}\right)\right) \\
&= \sum_{i=0}^{N-1} s_i \left(-re_{ai} + ru_i + k_{3i}e_{ai} + k_{2i}e_{vi} + k_{1i}e_{pi}\right) \\
&= \sum_{i=0}^{N-1} s_i \left(-re_{ai} + r\left(e_{ai} - \frac{1}{r}\left(k_{3i}e_{ai} + k_{2i}e_{vi} + k_{1i}e_{pi}\right) - \frac{1}{r}\sigma_i \text{sign}(s_i)\right)\right. \\
&\quad \left. + k_{3i}e_{ai} + k_{2i}e_{vi} + k_{1i}e_{pi}\right) \\
&= \sum_{i=0}^{N-1} -\sigma_i \|s_i\|^2
\end{aligned}$$

Therefore, with the help of the controller (21), the error states can arrive at the integral sliding mode surface.

Then, the existence of sliding mode surface is proved in the following. Once the error states go into the integral sliding mode surface, it can be obtained that $s_i = 0$ and $\dot{s}_i = 0$. Then the equality is satisfied

$$\dot{e}_{ai} = -k_{3i}e_{ai} - k_{2i}e_{vi} - k_{1i}e_{pi} \tag{22}$$

The above equality can be rewritten as

$$\dot{e}_i = Ce_i$$

where $e_i = \left[e_{pi}, e_{vi}, e_{ai}\right]^T$, and matrix C is

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -k_{1i} & -k_{2i} & -k_{3i} \end{bmatrix}$$

Choose the values for control gains k_{1i}, k_{2i}, k_{3i} such that the matrix C is Hurwitz. Then error states can converge to zero along with the sliding surface (20).

To integrate the above process, the encirclement control of the target can be realized by using the proposed distributed sliding mode control algorithm for leader-follower's systems. The proof is completed.

5. **Numerical Example.** In this section, an example is presented to show the validity of the proposed target-encirclement control strategy. Leader-follower multiple quadrotors include one leader and four followers. The communication network among them is described in Figure 1.

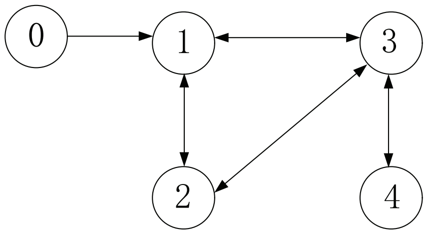

FIGURE 1. Communication topology

Simulation parameters are chosen as follows, where $l = 2$, $\theta_0 = \frac{\pi}{5}$, $k_{1i} = 2$, $k_{2i} = 3$, $k_{3i} = 5$. It can be proved that matrix C is Hurwitz. The observer parameters are $\alpha_{pi} = \alpha_{vi} = \alpha_{ai} = i$, $\beta_{pi} = \beta_{vi} = \beta_{ai} = i$, $\gamma_{pi} = \gamma_{vi} = \gamma_{ai} = 0.5i$, $\kappa_{pi} = \kappa_{vi} = \kappa_{ai} = 0.5i$.

Figure 2 describes the trajectories of the target, leader and followers, respectively. It is noticed that the leader and followers can encircle the target successfully. The observer errors of the target and leader are shown in Figures 3 and 4. With the help of the proposed observers, the information of the target and leader can be estimated within fixed time.

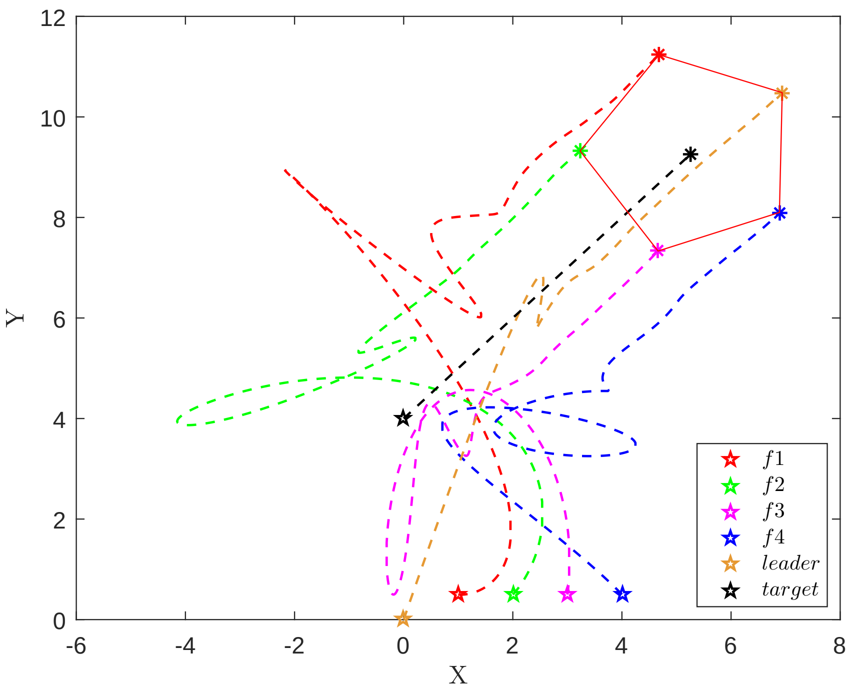

FIGURE 2. (color online) Encirclement trajectories of the multi-agent system

6. **Conclusions.** In this paper, a distributed target-encirclement control problem for multiple quadrotors has been investigated via a leader-follower framework. Fixed-time observers have been proposed for followers to estimate the information of the target and leader, and it is proved that the real values can be estimated accurately within fixed time for any initial states. The integral sliding mode surface and sliding mode control algorithm have been designed for leader and followers such that the target can be surrounded by the leader-follower system. Simulation results have shown the effectiveness of the presented

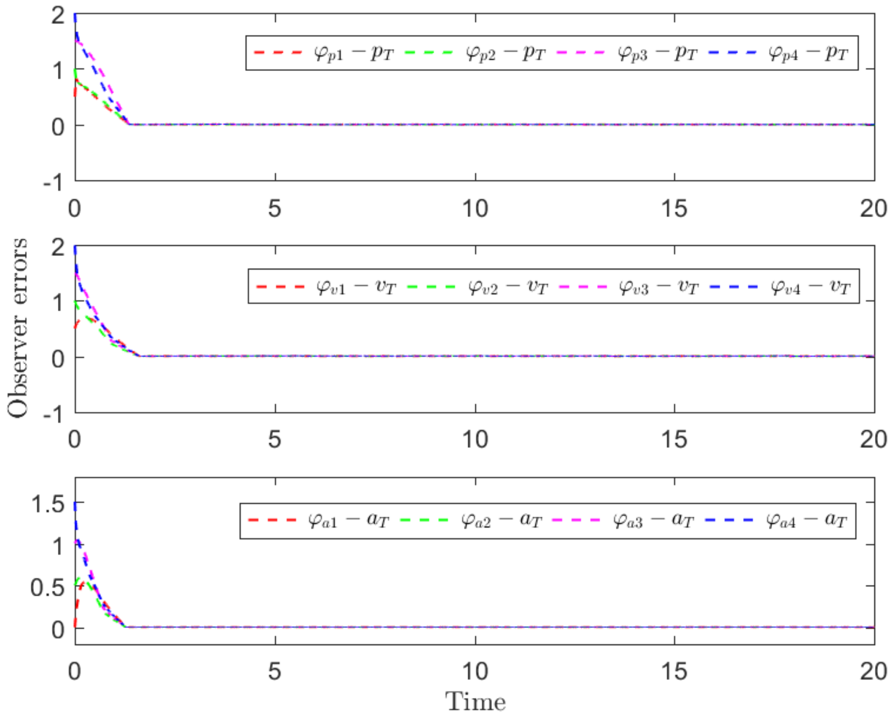

FIGURE 3. (color online) Observer errors of the target

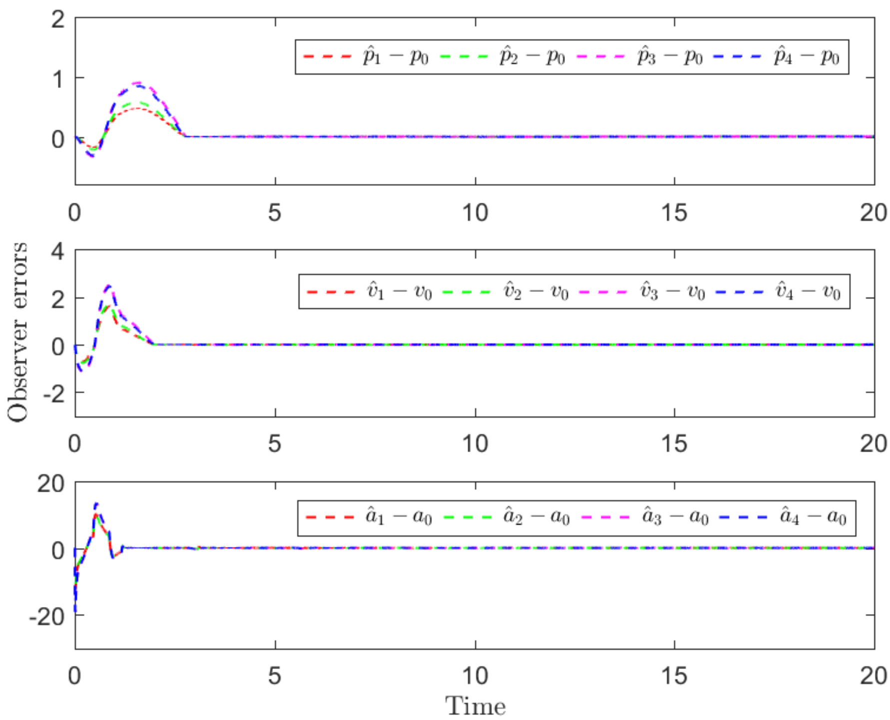

FIGURE 4. (color online) Observer errors of the leader

control strategy. In our future work, the obstacles avoidance problem of the target-encirclement control will be considered such that multiple quadrotors have no collisions during the tracking process.

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