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A DESIGN METHOD OF ADAPTIVE DISTURBANCE OBSERVER BASED ON FRIEDLAND-TACKER EQUATION

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Abstract. *This paper proposes a design method for adaptive disturbance observers. Adaptive disturbance observers are observers that can estimate and control unknown parameters and disturbances. They enable the attainment of smooth robotic motion control even if there exist exogenous disturbances and robotic parameter uncertainties. In this paper, we newly derive a design method for adaptive disturbance observers based on the fundamental design equation called the Friedland-Tacker equation. Furthermore, we consider an estimation problem of a single-link robotic arm and demonstrate the advantage of the proposed observer in comparison with the conventional adaptive observer.*
Keywords: Adaptive observer, Disturbance estimation, Friedland-Tacker equation, Sinusoids, Pendulum robot, Auxiliary observer gain

1. **Introduction.** When parameters such as the position of the center of gravity or the moment of inertia are unknown, the control of a mechanical system such as a robot arm requires control to estimate these parameters. This control is called adaptive control, and the control of mechanical systems is practical when adaptive observers are used. An adaptive observer is an observer that can estimate the values of unknown parameters, even if the system contains them. When controlling mechanical systems, it is often difficult to accurately calculate mechanical parameters such as the center of gravity and moments of inertia. In addition, the differentiation of the position data from encoders or resolvers may be prohibitive because it amplifies the measurement noise. In dealing with those situations, it is promising to employ the adaptive observers, which can estimate the unknown parameters and velocity.

Among the mechanical systems, multi-joint robots are often subject to unknown disturbances such as mutual interference forces between the links and frictional forces at the joints, making it difficult to achieve high-precision movements. The control performance of a robot also depends on its model. Inverse dynamics-based methods are unable to obtain sufficient control performance because it is difficult to accurately calculate all parameters. Therefore, it is essential to establish mechanisms for estimating the unknown disturbances and mechanical parameters simultaneously to achieve better control performance. It is also significant to consider that the motion of the mechanical systems is described by nonlinear differential equations.

Adaptive observers for multi-input-multi-output (MIMO) linear time-varying systems have been developed in [6, 7]. Some results on nonlinear systems have also been reported in [6, 12, 13]. A powerful way of designing nonlinear observers is to linearize a given nonlinear system exactly and then construct a linear observer for the resulting linear system [4, 5]. In previous research [3], a systematic design method of adaptive observers for the linear

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time-varying systems is derived after extending the approach in [7, 9, 11]. The systematic design method of adaptive observers also serves as the basis for the study of nonlinear systems, such as those reported in [12, 13]. This design method has the possibility of being applied to the broad classes of nonlinear systems if combined with various exact linearization methods, but it could not cope with time-dependent disturbances such as sinusoids. In [14], the nonlinear disturbance observer was used to accurately estimate the size of the uncertainty disturbance, further improving the accuracy of the manipulator control system. However, the nonlinear disturbance has not been properly considered using the unknown parameters. In [1, 8], the fundamental design equations developed by Friedland and Tacker are proposed and enable estimation of the unknown constant parameters and the time-variant disturbances for a linear system, but not in a nonlinear system.

Inspired by the aforementioned articles, this paper's designs focus on the previous studies [3, 9] to clarify a design method for adaptive observers having disturbance estimation capability. Specifically, we newly employ a fundamental design equation along [2]. This design equation is named the Friedland-Tacker equation after [1, 8], and enables simultaneous estimation of the unknown constant parameters and time-variant disturbances.

For verification, we apply the proposed design method to an adaptive disturbance estimation problem for the pendulum robot in [9]. We illustrate that the proposed design method is more advantageous than the previous design method [3, 9]. This paper is organized as follows. Section 2 shows how to design the adaptive disturbance observer based on the Friedland-Tacker equation [2]. The relationship with the previous design method [3, 9] is also noted. In Section 3, the proposed design method is applied to the adaptive disturbance estimation for the pendulum robot. The advantage of the proposed adaptive disturbance observer is verified by the numerical simulations. In Section 4, we give concluding remarks.

2. **Adaptive Disturbance Observer.**

2.1. **Problem formulation.**

The state-space equations to be estimated by the adaptive observer are expressed as follows:

$$\dot{x}(t) = A(t)x(t) + B_0(t)r(t) + B_2(t)u(t), \tag{1}$$

$$y(t) = C_2(t)x(t) + D_{20}(t)r(t), \tag{2}$$

where $x(t)$, $u(t)$, and $y(t)$ denote the state, input, and output of the controlled object, respectively. The exogenous input $r(t)$ is introduced to be the vector consisting of constant unknown parameters and time-dependent disturbance. The system matrices $A(t)$, $B_0(t)$, $B_2(t)$, $C_2(t)$, and $D_{20}(t)$ are assumed to be piecewise continuous and bounded in time. Specifically, $r(t)$ is assumed to be generated by the linear time-invariant exosystem

$$\dot{x}_{\mathrm{m}}(t) = A_{\mathrm{m}}x_{\mathrm{m}}(t), \tag{3}$$

$$r(t) = C_{\mathrm{m}}x_{\mathrm{m}}(t). \tag{4}$$

The adaptive observer problem is formulated as estimating $x(t)$ and $x_{\mathrm{m}}(t)$ simultaneously based on the information on $u(t)$ and $y(t)$.

In the previous research [3, 9], $r(t)$ represents only the unknown constant parameter θ. This means that the system matrices A_{m} and C_{m} are restricted to be $A_{\mathrm{m}} = 0$ and $C_{\mathrm{m}} = I$, and therefore, the state-space equations of the controlled object have the following form

$$\dot{x}(t) = A(t)x(t) + B_0(t)\theta + B_2(t)u(t),$$
$$y(t) = C_2(t)x(t) + D_{20}(t)\theta.$$

2.2. **Proposed design method.**

Combining Equations (1)-(2) and (3)-(4), we obtain the augmented system

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{\mathrm{m}}(t) \end{bmatrix} = A^{[\mathrm{m}]}(t) \begin{bmatrix} x(t) \\ x_{\mathrm{m}}(t) \end{bmatrix} + B_2^{[\mathrm{m}]} u(t), \tag{5}$$

$$y(t) = C_2^{[\mathrm{m}]}(t) \begin{bmatrix} x(t) \\ x_{\mathrm{m}}(t) \end{bmatrix}, \tag{6}$$

where the system matrices are constructed by

$$A^{[\mathrm{m}]} := \begin{bmatrix} A(t) & B_0(t)C_{\mathrm{m}} \\ 0 & A_{\mathrm{m}} \end{bmatrix}, \ B_2^{[\mathrm{m}]} := \begin{bmatrix} B_2(t) \\ 0 \end{bmatrix}, \ C_2^{[\mathrm{m}]}(t) := \begin{bmatrix} C_2(t) & D_{20}(t)C_{\mathrm{m}} \end{bmatrix}.$$

The proposed adaptive disturbance observer is derived as the observer for the augmented system presented previously

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_{\mathrm{m}}(t) \end{bmatrix} = A^{[\mathrm{m}]}(t) \begin{bmatrix} x(t) \\ x_{\mathrm{m}}(t) \end{bmatrix} + L^{[\mathrm{m}]}(t) \left(C_2^{[\mathrm{m}]}(t) \begin{bmatrix} x(t) \\ x_{\mathrm{m}}(t) \end{bmatrix} - y(t) \right). \tag{7}$$

To determine the observer gain $L^{[\mathrm{m}]}$ for the augmented system, we employ the following Friedland-Tacker equation [2]:

$$\dot{T}(t) = (A(t) + L(t)C_2(t))\, T(t) - T(t)A_{\mathrm{m}} + (B_0(t) + L(t)D_{20}(t))\, C_{\mathrm{m}}, \tag{8}$$

where $L(t)$ is an arbitrary observer gain stabilizing for the matrix pair $(A(t), C_2(t))$. By using solution $T(t)$ of (8), we define the auxiliary observer gain

$$L_{\mathrm{m}}(t) := -e^{A_{\mathrm{m}}t}\Gamma e^{A_{\mathrm{m}}^{\mathrm{T}}t} \left(C_2(t)T(t) + D_{20}(t)C_{\mathrm{m}}\right)^{\mathrm{T}} \Sigma, \tag{9}$$

where Γ and Σ are positive definite matrices. Finally, the observer gain $L^{[\mathrm{m}]}(t)$ for the augmented system (5)-(6) is constructed as follows:

$$L^{[\mathrm{m}]}(t) = \begin{bmatrix} L(t) + T(t)L_{\mathrm{m}}(t) \\ L_{\mathrm{m}}(t) \end{bmatrix}.$$

When the exogenous input $r(t)$ represents the unknown constant parameter (i.e., $A_{\mathrm{m}} = 0$ and $C_{\mathrm{m}} = I$), the Friedland-Tacker equation (8) reduces to equation

$$\dot{\Upsilon}(t) = (A(t) + L(t)C_2(t))\, \Upsilon(t) + (B_0(t) + L(t)D_{20}(t))$$

for the variable $\Upsilon(t)$, which is employed in the previous research [3, 9]. It is confirmed formally that the solutions $T(t)$ and $\Upsilon(t)$ are related by

$$\Upsilon(t) = T(t)e^{A_{\mathrm{m}}t}.$$

3. **Design Example.**

3.1. **Simulation configuration.**

The pendulum robot considered in [4, 9] is depicted in Figure 1. We illustrate the features of the proposed design method through the adaptive disturbance estimation for the pendulum robot.

The equation of motion of the pendulum robot is given by

$$I\ddot{q}(t) + \frac{1}{2}mgl \sin q = d(t) + u(t), \tag{10}$$

where $q(t)$ is the angle of rotation of the arm, $u(t)$ is the arm torque, m is the arm mass, l is the arm length, I is the moment of inertia, and g is the acceleration of gravity. In contrast with [4, 9], Equation (10) involves the disturbance torque $d(t)$ affecting the pendulum robot, which is assumed to be the sinusoid with the known angular frequency ω. We also assume that the mechanical parameters are not known and denote the unknown parameters by $\theta_1 := mgl/(2I)$, $\theta_2 := 1/I$. By setting $x_1(t) := q(t)$, $x_2(t) = \dot{q}(t)$, and $y(t) = q(t)$, the state-space equations are formulated as follows:

$$\dot{x}_1(t) = x_2(t) \tag{11}$$

$$\dot{x}_2(t) = -\theta_1 \sin y(t) + \theta_2 u(t) \tag{12}$$

$$y(t) = x_1(t) \tag{13}$$

By adding the disturbance torque $d(t)$, we can rearrange the state-space equations as follows:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -\sin y(t) & u(t) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} d(t), \tag{14}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \tag{15}$$

We define the exogenous input $r(t)$ as the vector $r(t) := \begin{bmatrix} \theta_1 \\ \theta_2 \\ d(t) \end{bmatrix}$ of the unknown parameters and disturbance. This $r(t)$ is generated by the exosystem (3)-(4) with the specific system matrices

$$A_{\mathrm{m}} := \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\omega \\ 0 & 0 & \omega & 0 \end{bmatrix}, \ C_{\mathrm{m}} := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

With the presented definition of $r(t)$, it is seen that the state-space equations (14)-(15) are in the form of (1)-(2) with the specific system matrices

$$A(t) := \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \ B_0(t) := \begin{bmatrix} 0 & 0 & 0 \\ -\sin y(t) & u(t) & 1 \end{bmatrix}, \ B_2(t) := \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \ C_2(t) := \begin{bmatrix} 1 & 0 \end{bmatrix},$$
$$D_{20}(t) := \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}.$$

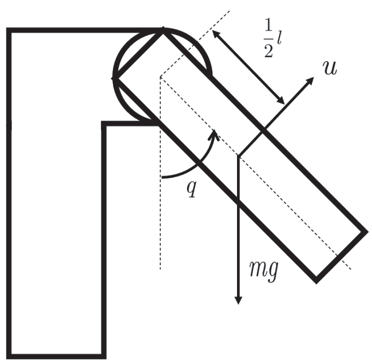
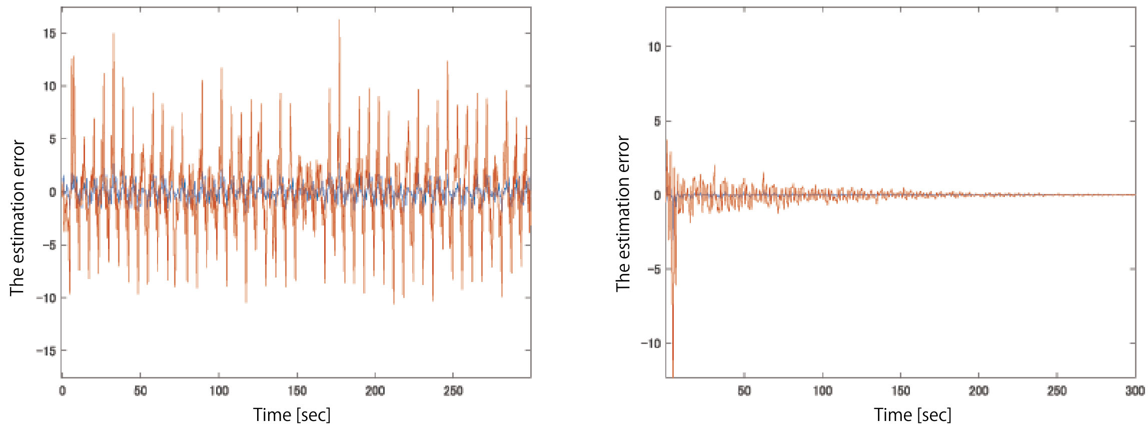


Figure 1. Model of pendulum robot

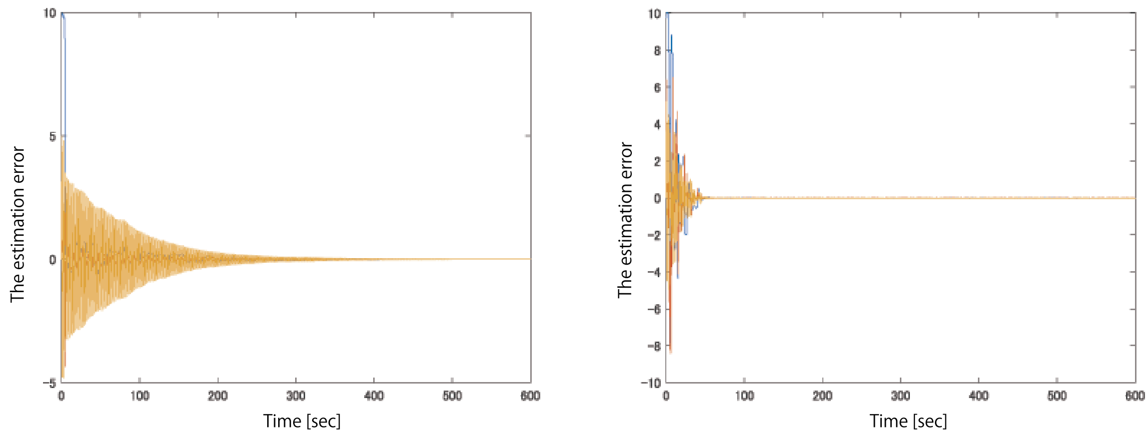
3.2. **Design results.** In this subsection, we tune the design parameters for the adaptive disturbance observer and demonstrate the resulting adaptive disturbance estimation. The mechanical parameters are set to the arm mass $m := 1$(kg), the arm length $l := 1$(m), the moment of inertia $I := 0.5(\mathrm{kg} \cdot \mathrm{m}^2)$ and the acceleration of gravity $g := 9.81(\mathrm{m/s}^2)$. The disturbance $d(t)$ is set to $d(t) := \sin 3t$, and the control input $u(t)$ is set to $u(t) := 5(\sin 2t + \cos 20t)$. The observer gain $L(t)$ for the matrix pair $(A(t), C_2(t))$ is set to $L(t) := -\begin{bmatrix} 1 & 1 \end{bmatrix}$. The weight matrices Γ and Σ are set to $\Gamma := \mathrm{diag}\begin{bmatrix} 2 & 2 & 1 & 1 \end{bmatrix}$ and $\Sigma := 10$, respectively.

First, we compare the proposed design method with the previous design method [9]. The state estimation error resulting from the conventional design method is depicted in Figure 2(a). The amplitudes of the state estimation errors continue to oscillate. This is expected because the conventional design method does not consider the continual perturbation caused by the sinusoidal disturbance $d(t)$. In contrast, as depicted in Figure 2(b), the state estimation error resulting from the proposed design method converges to zero quickly.

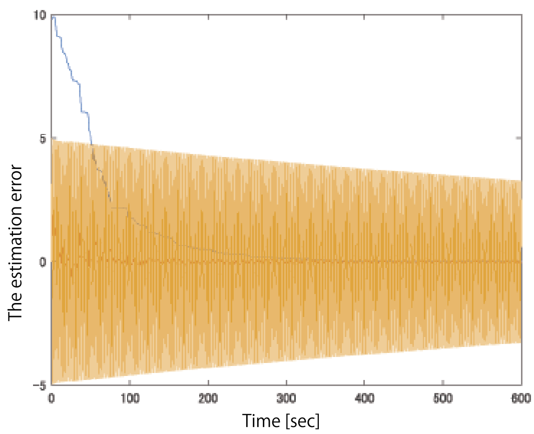


(a) Previous design method [9] (b) Proposed design method

Figure 2. State estimation error of adaptive observers



(a) $\Sigma = 10$ (b) $\Sigma = 1000$



(c) $\Sigma = 0.1$

Figure 3. State estimation error of adaptive observers

This is because the proposed design method estimates the state variables by counteracting the adverse effect of the disturbance.

Second, we examine how the estimation errors of the unknown parameters and disturbance vary according to the weight parameter Σ. By changing the value of Σ as

$\Sigma = 10, 1000, 0.1$, the corresponding estimation errors are depicted in Figures 3(a), 3(b), and 3(c), respectively. By comparison with Figures 3(a) and 3(b), it is observed that the convergence of the estimation errors is accelerated by increasing the value of Σ. On the other hand, by comparison with Figures 3(a) and 3(c), it is observed that the convergence of the estimation errors is slowed by decreasing the value of Σ. We note that the magnitude of the auxiliary observer gain $L_{\mathrm{m}}(t)$ is proportional to Σ. Hence, from those numerical simulations, it is concluded that the larger auxiliary observer gains result in the faster convergence of estimation errors.

4. **Conclusion.** In this paper, we proposed a method for designing an adaptive observer with disturbance estimation capability. In Section 2, it was shown how to design the adaptive disturbance observer based on the Friedland-Tacker equation [2]. The relationship with the previous design method [3, 9] was also noted. In Section 3, the proposed design method was applied to the adaptive disturbance estimation for the pendulum robot. The advantage of the proposed adaptive disturbance observer was verified by the numerical simulations. In addition, a system verification of the observer weight parameter Σ was conducted, and its performance was discussed. The change in the value of the observer weight parameter Σ showed that an increase in the auxiliary observer gain $L_{\mathrm{m}}(t)$ also increases the quick-response characteristics of the system. In future research, we will enhance the proposed design method toward adaptive disturbance estimation for mobile vehicles such as wheelchairs.

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