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# ADAPTIVE COLLABORATIVE FAULT TOLERANCE FOR MULTI-VEHICLE FORMATION SYSTEMS WITH HYBRID ACTUATOR FAULTS AND UNKNOWN DYNAMICS

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Abstract. *In this article, a fault-tolerant control method is developed for a class of first-order nonlinear multi-vehicle formation system with hybrid actuator faults and unknown internal dynamics problem. For unknown internal dynamics problems, the unknown function is efficiently approximated based on neural network techniques. At the same time, the unknown function is combined with the adaptive backstepping technique to design the controller and the adaptive law of the first-order system. Compared with previous research results, this paper presents a novel cooperative fault-tolerant control problem by combining hybrid actuator faults with the multi-vehicle formation problem. Based on the adaptive neural network and the algebraic graph theory, the error model of the multi-vehicle system is established, and the distributed consensus cooperative fault-tolerant controller is designed. Through Lyapunov stability analysis, it is demonstrated that the proposed control method can accomplish the control task.*
**Keywords:** Multi-vehicle formation system, Fault tolerant control (FTC), Neural networks (NNs), Graph theory, Hybrid actuator faults

1. **Introduction.** The subject of fault tolerance control of multi-vehicle formation systems has gotten a lot of attention from researchers in recent years [1]. With the deepening of the complex process and the expanding control scale of the multi-vehicle system, there is inevitably more unknown uncertainty, multivariable characteristics and more frequent system failures, all of which have a negative impact on the high-performance dynamic characteristics of the formation system. Because system failures are common in real-life situations and can even lead to significant losses, the study of failure-related problem is critical [2,3]. Various methods based on current control theory have been utilized to overcome the above challenges, including compensating the system for actuator failures and solving unknown internal dynamics problems. At present, the actuator fault problems are mostly studied in [4-6] based on some fault method.

In [7], the tracking control problem for fractional-order fault systems is investigated and a compensation term is introduced into the conventional control law to compensate for the effects caused by actuator faults. In [8], based on an adaptive nonlinear state feedback method, an adaptive fault-tolerant constrained controller is designed to solve the spacecraft docking relative position control problem in the presence of control input constraints, partial actuator failures and unknown external disturbances, and the designed control method does not require precise knowledge of actuator faults and can reject uncertain fault information. Afterwards, combining fault-tolerant control with reinforcement learning algorithms, a direct adaptive optimal controller is designed [9], which can effectively reduce the computational effort by using fewer learning parameters than

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conventional reinforcement learning algorithms, and an adaptive auxiliary signal is established to compensate for the effect of faults on control performance. In addition, the study of control system uncertainty is a key issue.

The presence of uncertainty can pose significant obstacles to controller design. Uncertainty includes parameter uncertainty, function uncertainty, structure uncertainty, etc. For different uncertainties, the corresponding treatment method is also different. The uncertainty of the parameters is handled by the adaptive techniques [10]. The uncertainty of functions is usually managed by neural network [11], fuzzy logic system [12] or generalized fuzzy hyperbolic models [13], and the structural uncertainty is usually handled using a robust approach.

Compared to the contributions in [4-9], the main features of the proposed cooperative control scheme designed for hybrid actuator failures are summarized in two aspects.

1) In this paper, the unknown uncertainty of the internal function of the system is considered. The ability of adaptive neural network control methods to approximate in nonlinear systems has been demonstrated, so the neural network is used to approximate the unknown functions.

2) This paper also considers the fault problem of hybrid actuators and proposes a cooperative fault-tolerant control scheme under different types of faults. Fault-tolerant fault control is an effective fault handling method, which can design the appropriate controller to ensure that it still maintains the stability and tracking performance of the original performance. However, most of the current research on fault-tolerant control addresses a certain class of faults. Therefore, this paper addresses the problem of hybrid actuator faults by designing a cooperative controller.

The structure of this paper is as follows. Section 2 provides a description and preliminary knowledge of the neural network and graph theory, laying the foundation for the following expansion. Section 3 describes the system and presents the necessary definition, lemma and assumption. In Section 4, the paper shows the design process and stability proof of a fault-tolerant controller for a first-order nonlinear multi-vehicle formation system. Finally, the conclusion is given in Section 5.

## 2. **Preliminary Knowledge.**

### 2.1. **Graph theory.**

In the paper, the multi-vehicle formation system communication network is depicted by an undirected connected graph $G = (A, N, \Xi)$, where $A = [a_{ij}] \in \mathbb{R}^{m\times m}$ is the adjacency matrix whose element $a_{ij} \geq 0$ is the communicated weight between vehicles $i$ and $j$, $N = \{n_1, n_2, \ldots, n_m\}$ is the node set, and $\Xi = N \times N$ is the edge set. If there is a communication from node $n_j$ to node $n_i$, then the edge $\bar{n}_{ij} = (n_i, n_j) \in \Xi$, and the node $n_j$ is said to be a neighbor of the node $n_i$, and $S_i = \{n_j | (n_i, n_j) \in \Xi\}$ denotes the neighbor label set. If $\bar{n}_{ij} \in \Xi$, the corresponding adjacency element $a_{ij} = 1$, and if $\bar{n}_{ij} \notin \Xi$, the corresponding adjacency element $a_{ij} = 0$. When $a_{ij} = a_{ji}$, the graph $G$ is called as an undirected graph. The undirected graph $G$ is called to be connected if there is a path for any pair of distinct nodes $(n_i, n_{i_1}), (n_{i_1}, n_{i_2}), \ldots, (n_{i_i}, n_j)$. Associated with the graph $G$, the Laplacian matrix is

$$L = C - A \tag{1}$$

where $C = diag\left\{\sum_{j=1}^{m} a_{1j}, \ldots, \sum_{j=1}^{m} a_{mj}\right\}$. The communication matrix between the following and lead vehicles is $D = diag\{d_1, d_2, \ldots, d_m\}$, where $d_i$, $i = 1, 2, \ldots, m$ is the communication weight between vehicle $i$ and leader.

If the vehicle $i$ can have the communication with the leader, then $d_i = 1$; otherwise, $d_i = 0$. It is supposed that $d_1 + d_2 + \cdots + d_m \geq 1$, which implies that at least one of vehicles is connected with the leader.

2.2. **Neural networks.** Neural networks (NNs) had been demonstrated to have the universal function approximation ability, and they can approximate a continuous unknown function $f(x) : \mathbb{R}^m \to \mathbb{R}^n$, by the following form over a compact set $\Omega$

$$\hat{f}(x) = \theta^T \varphi(x) \tag{2}$$

where $\theta \in \mathbb{R}^{q\times n}$ is the weight matrix with the neuron number $q$, $\varphi(x) = [\varphi_1(x), \ldots, \varphi_q(x)]^T$ is the basis function vector with $\varphi_i(x) = \exp\left(-\frac{(x-\mu_i)^T(x-\mu_i)}{2\delta_i^2}\right)$, $\mu_i = [\mu_{i1}, \ldots, \mu_{im}]^T$ is the center of receptive field and $\delta_i$ is width.

For the continuous function $f(x)$, there exists the ideal NNs weight $\theta^* \in \mathbb{R}^{q\times n}$ described as

$$\theta^* := \arg \min_{\theta\in\mathbb{R}^{q\times n}} \left\{ \sup_{x\in\Omega} \left\| f(x) - \theta^T\varphi(x) \right\| \right\} \tag{3}$$

so that $f(x)$ can be rewritten as

$$f(x) = \theta^{*T}\varphi(x) + \varepsilon(x) \tag{4}$$

where $\varepsilon(x) \in \mathbb{R}^n$ is approximation error, and $\|\varepsilon(x)\| \le \varepsilon_d$, $\varepsilon_d$ is a positive constant.

In (4), since the ideal weight matrix $\theta^*$ is an unknown constant matrix, it is unavailable for the actual control design. Therefore, the estimation $\hat{\theta}$ of the ideal NNs weight $\theta^*$ is used.

3. **System Descriptions and Assumptions.** The nonlinear multi-agent system is described in the following.

$$\dot{x}_i(t) = u_i + f_i(x_i(t)) \quad i = 1, 2, \ldots, m \tag{5}$$

where $x_i(t) = [x_{i1}, \ldots, x_{in}]^T \in \mathbb{R}^n$, $u_i = [u_{i1}, \ldots, u_{in}]^T \in \mathbb{R}^n$ are the system state and control input, respectively. $f_i(\cdot) \in \mathbb{R}^n$ is the unknown smooth function.

**Assumption 3.1.** [14] $f_i(\cdot) + u_i$ *are Lipschitz continuous.*

**Assumption 3.2.** [14] *The desired trajectory of formation movement* $x_d$ *and its derivative* $\dot{x}_d(t)$ *are bounded.*

To better study the hybrid actuator faults contained in the system, classify the fault model types in the system model formula (5), as shown in Equation (6). Then, the system input can be described as

$$u_i^F(t) = (1 - \rho_i)u_i(t) + \kappa_i\delta_i(t) \tag{6}$$

where $u_i^F(t)$, $u_i(t)$ represent the $i$ vehicle input signal and the actuator input signal, and the $\delta_i(t)$ indicates the input signal of the $i$ actuator failure, which is divided into four categories, see Table 1.

TABLE 1. Actuator fault types

| Type | Number |
|---|---|
| Faultless | $\rho_i = 0$, $\kappa_i = 0$ |
| Partial failure fault | $0 < \rho_i < 1$, $\kappa_i = 0$ |
| Bias fault | $0 < \rho_i < 1$, $\kappa_i = 1$ |
| Stuck fault | $\rho_i = 1$, $\kappa_i = 1$ |

In Table 1, $\rho_i$ and $\kappa_i$ are the fault parameters. When the actuator is biased fault, $\delta_i(t)$ is bounded, $\|\delta_i(t)\| \le \delta_d$, where $\delta_d$ is a known positive constant, and the bias fault is unknown time-varying bounded.

**Lemma 3.1.** [15] *A necessary and sufficient condition that the Laplacian matrix is irreducible is that the undirected graph* $G$ *is connected.*

**Lemma 3.2.** [15] *If the Laplacian matrix $L$ is irreducible, then $\tilde{L} = L + D$ is a positive definite matrix.*

**Lemma 3.3.** [14] *The continuous function $V(t) \geq 0$ is with bounded initial condition. If it holds $\dot{V}(t) \leq -\alpha V(t) + \beta$, where $\alpha, \beta > 0$ are two positive constants, then the following inequality can be held.*

$$\dot{V}(t) \leq V(0)e^{-\alpha t} + \frac{\beta}{\alpha}\left(1 - e^{-\alpha t}\right) \tag{7}$$

**Definition 3.1.** [10] *The first-order leader-follower formation can be achieved if the nonlinear multi-vehicle system (5) satisfies $\lim_{t\to\infty} \|x_i(t) - x_d(t) - \eta_i\| = 0$, $i = 1, \ldots, m$, where $x_d(t) \in \mathbb{R}^n$ denotes the desired trajectory of the relative positions of the formation movement, and $\eta_i = [\eta_{i1}, \eta_{i2}, \ldots, \eta_{in}]^T \in \mathbb{R}^n$ is the relative position between the follower vehicles and the leader vehicle.*

**The Control Objective:** For a first-order nonlinear multi-vehicle queueing system (5) consisting of a single leader vehicle and multiple following vehicles, design an adaptive formation control protocol so that i) all error signals are semi-globally uniformly ultimately bounded (SGUUB); ii) the first-order leader-follower formation control holds.

## 4. Main Results.

4.1. **Controller design.** Consider the actuator faults (6), the nonlinear multi-vehicle system (5) can be expressed as

$$\dot{x}_i(t) = (1 - \rho_i)u_i(t) + \kappa_i\delta_i(t) + f_i(x_i(t)) \quad i = 1, 2, \ldots, m \tag{8}$$

The desired trajectory of formation movement is denoted by a time variable $x_d(t)$, where it and its derivative $\dot{x}_d(t)$ are assumed bounded. The tracking errors are defined as

$$z_i(t) = x_i(t) - x_d(t) - \eta_i \tag{9}$$

where $\eta_i = [\eta_{i1}, \eta_{i2}, \ldots, \eta_{in}]^T \in \mathbb{R}^n$ is the relative position between vehicle $i$ and leader, which depicts the formation pattern.

From Equation (8), the following error dynamics can be generated.

$$\dot{z}_i(t) = f_i(x_i) + (1 - \rho_i)u_i(t) + \kappa_i\delta_i(t) - \dot{x}_d(t) \tag{10}$$

Define the formation errors as

$$e_i(t) = \sum_{j\in N_i} a_{ij}(x_i(t) - \eta_i - x_j(t) + \eta_j) + d_i(x_i(t) - x_d(t) - \eta_i) \tag{11}$$

Using (9), the formation error (11) can be rewritten as

$$e_i(t) = \sum_{j\in N_i} a_{ij}(z_i(t) - z_j(t)) + d_i z_i \tag{12}$$

**Remark 4.1.** *According to Lemma 3.2, the matrix $\tilde{L} = L + D$ is a positive definite matrix. Let $\chi_1, \ldots, \chi_m$ be its eigenvectors associated with the eigenvalues $\lambda_1, \ldots, \lambda_m$, the following inequality is held.*

$$\lambda_{\min}\|e(t)\|^2 \leq z^T(t)\left(\tilde{L} \otimes I_m\right)z(t) \leq \lambda_{\max}\|e(t)\|^2 \tag{13}$$

*where $z(t) = \left[z_1^T(t), \ldots, z_m^T(t)\right]^T \in \mathbb{R}^{mn}$, $e(t) = \left[e_1^T(t), \ldots, e_m^T(t)\right]^T \in \mathbb{R}^{mn}$, $\lambda_{\min}$ and $\lambda_{\max}$ are the minimal and maximal eigenvalues of the matrix, respectively. $M = \left(Q^T\Lambda^{-1}Q\right)\otimes I_m$ with $Q = [\chi_1, \ldots, \chi_m] \in \mathbb{R}^{mn}$, $\Lambda = diag\{\lambda_1, \ldots, \lambda_m\}$.*

The time derivative of formation error $e_i(t)$ is

$$\dot{e}_i(t) = b_i\dot{x}_i - d_i\dot{x}_d - \sum_{j\in N_i} a_{ij}\dot{x}_j \tag{14}$$

where $b_i = \sum_{j\in N_i} a_{ij} + d_i$.

Combining (5) and (10), (14) can be further described as

$$\dot{e}_i(t) = b_i f_i(x_i) + b_i u_i - d_i\dot{x}_d(t) - \sum_{j\in N_i} a_{ij} f_j(x_j) - \sum_{j\in N_i} a_{ij} u_j \tag{15}$$

Design the actual fault tolerant controller as

$$u_i(t) = \frac{1}{1-\rho_i}\left(-\gamma_i e_i(t) - \frac{1}{2}\hat{\theta}_i^T(t)\varphi_i(x_i) - \kappa_i\delta_i(t)\right) \tag{16}$$

where $\gamma_i$ is a constant.

Design the adaptive law as

$$\dot{\hat{\theta}}_i(t) = \Gamma_i\left(\varphi_i(x_i)e_i^T(t) - \sigma_i\hat{\theta}_i(t)\right) \tag{17}$$

where $\Gamma_i$ is a positive definite matrix, and $\sigma_i > 0$ is a design constant.

**Remark 4.2.** *In controller (16), the error term $e_i(t)$ defined in (11) is intended to control the multi-vehicle system keeping the formation pattern. The NNs term $\hat{\theta}_i^T(t)\varphi_i(x_i)$ aims to compensate the unknown dynamic via on-line tuning the NNs weight $\hat{\theta}_i(t)$ using the updating law (17). Since the proposed scheme provides an ideal solution for the basic formation control of first-order nonlinear multi-vehicle systems, it can be applied and extended by combining it with various control techniques, such as the problem of finite-time fault-tolerant control [14].*

4.2. **Stability analysis.** To verify the main results of the designed collaborative fault-tolerant controller, the following theorem is given.

**Theorem 4.1.** *Consider a typical class of nonlinear multi-vehicle formation system (5) with collaborative fault-tolerant. It is guaranteed that in the case of actuator failure, if adaptive formation control (16) with the update law (17) is performed on the multi-intelligent system and the design constants are chosen to satisfy $\gamma_i > 1$, then the following control objective can be achieved by choosing appropriate design parameters.*

*1) All errors are semi-globally uniformly ultimately bounded (SGUUB).*

*2) The multi-agent formation can be achieved for sufficiently smooth movement trajectory.*

**Proof:** Choose the following Lyapunov function candidate

$$V(t) = \frac{1}{2}z^T(t)\left(\tilde{L}\otimes I_m\right)z(t) + \frac{1}{2}\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\Gamma_i^{-1}\tilde{\theta}_i(t)\right\} \tag{18}$$

Then, the first-order derivative of $V(t)$ is

$$\dot{V}(t) = z^T(t)\left(\tilde{L}\otimes I_m\right)\otimes\dot{z}(t) - \sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\left(\varphi_i(x_i)e_i^T(t) - \sigma_i\hat{\theta}_i(t)\right)\right\} \tag{19}$$

where $z(t) = \left[z_1^T(t), \ldots, z_n^T(t)\right]^T \in \mathbb{R}^n$, $\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i^*(t)$.

The time derivative of $V(t)$ along (10) and (17) is

$$\dot{V}(t) = \sum_{i=1}^{m} e_i^T(t)\left(f_i(x_i) - \dot{x}_d(t) + (1-\rho_i)u_i(t) + \kappa_i\delta_i(t)\right)$$

$$-\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\left(\varphi(x)e_i^T(t)-\sigma_i\hat{\theta}_i(t)\right)\right\} \tag{20}$$

Inserting the controller (16) into (20), we have

$$\dot{V}(t)=\sum_{i=1}^{m} e_i^T(t)\left(\theta_i^{*T}(t)\varphi_i(x_i)-\gamma_i e_i(t)-\frac{1}{2}\hat{\theta}_i^T(t)\varphi_i(x_i)-\dot{x}_d(t)+\varepsilon_i(x_i)\right)$$
$$-\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\left(\varphi_i(x_i)e_i^T(t)-\sigma_i\hat{\theta}_i(t)\right)\right\} \tag{21}$$

From Equation (21), we can derive

$$\dot{V}(t)=\sum_{i=1}^{m} e_i^T(t)\left(\frac{1}{2}\hat{\theta}_i^T(t)\varphi_i(x_i)-\gamma_i e_i(t)-\tilde{\theta}_i^T(t)\varphi_i(x_i)-\dot{x}_d(t)+\varepsilon_i(x_i)\right)$$
$$-\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\left(\varphi_i(x_i)e_i^T(t)-\sigma_i\hat{\theta}_i(t)\right)\right\} \tag{22}$$

From Equation (22), we can derive

$$\dot{V}(t)=\sum_{i=1}^{m}\left(\frac{1}{2}e_i^T(t)\hat{\theta}_i(t)\varphi_i(x_i)-\gamma_i e_i^T(t)e_i(t)-e_i^T(t)\tilde{\theta}_i(t)\varphi_i(x_i)-e_i^T(t)\dot{x}_d(t)\right.$$
$$\left.+e_i^T(t)\varepsilon_i(x_i)\right)-\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\left(\varphi_i(x_i)e_i^T(t)-\sigma_i\hat{\theta}_i(t)\right)\right\} \tag{23}$$

According to the property of trace operation, $a^Tb=Tr\left\{ab^T\right\}=Tr\left\{ba^T\right\}$ for $\forall a,b\in\mathbb{R}^n$ we have

$$e_i^T(t)\tilde{\theta}_i^T(t)\varphi_i(x_i)=Tr\left\{\tilde{\theta}_i^T(t)\varphi_i(x_i)e_i^T(t)\right\} \tag{24}$$

Using the above Equation (24), Equation (23) can become the following one.

$$\dot{V}(t)=\sum_{i=1}^{m}\frac{1}{2}e_i^T(t)\hat{\theta}_i(t)\varphi_i(x_i)-\sum_{i=1}^{m}\gamma_i e_i^T(t)e_i(t)-\sum_{i=1}^{m}e_i^T(t)\dot{x}_d(t)+\sum_{i=1}^{m}e_i^T(t)\varepsilon_i(x_i)$$
$$-\sum_{i=1}^{m} Tr\left\{\sigma_i\tilde{\theta}_i^T(t)\hat{\theta}_i(t)\right\} \tag{25}$$

Applying Cauchy-Bchwarz inequality $\left(x^Ty\right)^2\leq\|x\|^2\|y\|^2$, $x,y\in\mathbb{R}^n$ and Young's inequality $ab\leq\frac{1}{2}a^2+\frac{1}{2}b^2$, $a,b\in\mathbb{R}^n$, there are the following facts.

$$-e_i^T(t)\dot{x}_d(t)\leq\frac{1}{2}\left\|e_i(t)\right\|^2+\frac{1}{2}\left\|\dot{x}_d(t)\right\|^2 \tag{26}$$

$$e_i^T(t)\varepsilon_i(x_i)\leq\frac{1}{2}\left\|e_i(t)\right\|^2+\frac{1}{2}\varepsilon_d^2 \tag{27}$$

$$\frac{1}{2}e_i^T(t)\hat{\theta}_i(t)\varphi_i(x_i)\leq\frac{1}{4}\left\|e_i(t)\right\|^2+\frac{1}{4}Tr\left\{\hat{\theta}_i^T(t)\varphi_i(x_i)\varphi_i^T(x_i)\hat{\theta}_i(t)\right\} \tag{28}$$

Inserting Inequalities (26), (27) and (28) into (25) has

$$\dot{V}(t)\leq-\sum_{i=1}^{m}\left(\gamma_i-\frac{5}{4}\right)\left\|e_i(t)\right\|^2+\frac{1}{2}\sum_{i=1}^{m}\left\|\dot{x}_d(t)\right\|^2+\frac{1}{2}\sum_{i=1}^{m}\varepsilon_d^2$$
$$+\frac{1}{4}\sum_{i=1}^{m} Tr\left\{\hat{\theta}_i^T(t)\varphi_i(x_i)\varphi_i^T(x_i)\hat{\theta}_i(t)\right\}-\sum_{i=1}^{m} Tr\left\{\sigma_i\tilde{\theta}_i^T(t)\hat{\theta}_i(t)\right\} \tag{29}$$

Based on the facts $\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i^*(t)$, we have

$$Tr\left\{\sigma_i \tilde{\theta}_i^T(t)\hat{\theta}_i(t)\right\} = \frac{\sigma_i}{2} Tr\left\{\tilde{\theta}_i^T(t)\tilde{\theta}_i(t)\right\} + \frac{\sigma_i}{2} Tr\left\{\hat{\theta}_i^T(t)\hat{\theta}_i(t)\right\} - \frac{\sigma_i}{2} Tr\left\{\theta_i^{*T}(t)\theta_i^*(t)\right\} \quad (30)$$

Substituting (30) into (29) yields

$$\begin{aligned} \dot{V}(t) \leq & -\sum_{i=1}^{m}\left(\gamma_i - \frac{5}{4}\right)\|e_i(t)\|^2 - \sum_{i=1}^{m}\frac{\sigma_i}{2} Tr\left\{\tilde{\theta}_i^T(t)\tilde{\theta}_i(t)\right\} \\ & -\sum_{i=1}^{m}\tau_i Tr\left\{\hat{\theta}_i^T(t)\varphi_i(x_i)\varphi_i^T(x_i)\hat{\theta}_i(t)\right\} + \Delta(t) \end{aligned} \quad (31)$$

where $\Delta(t) = \sum_{i=1}^{m}\frac{\sigma_i}{2} Tr\left\{\theta_i^{*T}\theta_i^*\right\} + \frac{1}{2}\sum_{i=1}^{m}\|\dot{x}_d(t)\|^2 + \frac{1}{2}\sum_{i=1}^{m}\varepsilon_d^2$, because all terms of $\Delta(t)$ are bounded, it satisfies $\|\Delta(t)\| \leq \beta$, where $\beta$ is a constant.

Let $\gamma = \min_{i=1,\ldots,n}\left\{\left(\gamma_i - \frac{5}{4}\right)\right\}$, $\sigma = \min_{i=1,\ldots,n}\left\{\frac{\sigma_i}{\lambda_{\max}^{\Gamma_i^{-1}}}\right\}$, $\tau = \min_{i=1,\ldots,m}\left\{\tau_i\lambda_{\min}^{\varphi_i}\right\}$, $\gamma_i > 1$ and $\lambda_{\max}^{\Gamma_i^{-1}}$ denote the maximal eigenvalue of $\Gamma_i^{-1}$, $\lambda_{\min}^{\varphi_i}$ is the minimal eigenvalue of $\varphi_i(x_i)\varphi_i^T(x_i)$, then Inequality (31) can be rewritten as

$$\begin{aligned} \dot{V}(t) \leq & -\frac{\gamma}{2\lambda_{\max}} z^T(t)\left(\tilde{L}\otimes I_m\right)z(t) - \frac{\sigma}{2}\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\Gamma_i^{-1}\tilde{\theta}_i(t)\right\} \\ & -\frac{\tau}{2}\sum_{i=1}^{m} Tr\left\{\tilde{\theta}_i^T(t)\tilde{\theta}_i(t)\right\} + \beta \end{aligned} \quad (32)$$

Further, let $\alpha = \min\left\{\frac{\gamma}{\lambda_{\max}}, \sigma\right\}$, Inequality (32) can be rewritten as

$$\dot{V}(t) \leq -\alpha V(t) + \beta \quad (33)$$

Applying Lemma 3.3, the following inequality can be held.

$$V(t) \leq e^{-\alpha t}V(0) + \frac{\beta}{\alpha}\left(1 - e^{-\alpha t}\right) \quad (34)$$

Combined with Theorem 4.1, all error signals are SGUUB. At the same time, the tracking errors can obtain the desired accuracy by choosing the design parameters large enough, and it means that the multi-vehicle formation can be achieved.

5. **Conclusions.** In this paper, a collaborative fault-tolerant control method is developed for a class of first-order nonlinear multi-vehicle formation system under hybrid actuator faults and unknown internal dynamics problems. The collaborative fault-tolerant control method can compensate for multiple types of actuator failures and stabilize the system. In the proposed control scheme, NNs is used to approximate the unknown dynamic functions. The method is proposed to compensate for the loss of multiple types of actuator failures without any fault detection and isolation mechanism. The control method in this paper can compensate for multiple types of actuator failures and stabilize the system successfully. In addition, the method ensures bounded output even in the case of an actuator failure. Thus, the goal of effective, cost-efficient and reliable system control design can be achieved.

Finally, it is a future topic to design controllers and adaptive laws to ensure that the output tracks the reference signal for a finite time.

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