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SLIDING MODE CONTROL OF SUPERCAVITATING VEHICLES BASED ON EXTENDED STATE OBSERVER

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Abstract. *The supercavitating vehicles can achieve high speed but they face technical challenges in system stability, nonlinear dynamics and unpredictable external disturbances. In order to design the control system of supercavitating vehicles, a sliding mode control method based on extended state observer is proposed. An auxiliary state is constructed according to the system states to refer to the total disturbance, which includes the uncertain dynamics, external disturbance and nonlinear planing force. The linear extended state observer is introduced to estimate the total disturbance. To obtain differential approximation in dynamical model, the tracking differentiator is designed. The stability of the system is proved by Lyapunov stability theorem. Finally, the tracking performance of the system under disturbance and undisturbance is simulated. The simulation results show the effectiveness of the control method.*

Keywords: Supercavitating vehicle, Sliding mode control, Extended state observer, Tracking differentiator

1. Introduction.

Supercavitating vehicles (SV) are a type of underwater vehicles that can reach extremely high speed by exploiting supercavitation technology. This emerging technology uses the proper design of a cavitator attached to the vehicle nose to create a large gas bubble that envelopes the vehicle body to eliminate skin friction drags, and the gas bubble is called supercavity generated from a sharp edge of the cavitator. Thus, there are only control surfaces such as cavitator and parts of fins and the tail of the hull contacting with the water. The supercavity provides an opportunity for SV to achieve high speed and underwater drag reduction. At the same time, designing a proper control system for SV is still one of the most challenging research areas because of many factors such as nonlinear planing force, highly coupled dynamics, model uncertainties and unpredictable external disturbances [1].

In recent years, control strategies designed for SV have been studied in a series of papers, such as sliding mode control [1, 2, 3], adaptive control [4, 5], robust predictive control [6], and active disturbance rejection control [7]. Among the control algorithms, sliding mode control is considered as the most popular scheme and has high applicability in reality. However, it is attractive to reduce chattering caused by unknown disturbance in sliding mode control. In [1], a fractional-order sliding mode control method is proposed to achieve the depth and pitch attitude tracking control of SV. However, the unknown disturbance in the control system has not been properly dealt with. In [2], a boundary sliding mode controller based on disturbance observer is proposed for the dive plane dynamics of an SV. The simulation results show that the presented sliding mode controllers have shown good performance for both stabilization and tracking responses. However, disturbance observer is transplanted to the control techniques only for external disturbances estimation

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where the internal system uncertainties are not addressed. In [8], two methods of sliding mode controllers combined with extended state observer and adaptive law are proposed. Simulation results show that extended state observer is the only option for disturbance that are absolutely unknown. The SV is subjected to unpredictable disturbances during maneuvering due to the complex underwater environment and the uncertainty of the model. Under this circumstance, a linear extended state observer (LESO) is proposed to solve the above problems. The main strength for LESO is that it can simultaneously address the internal uncertainties and external disturbances in real time for systems in the absence of accurate model information. The essence of LESO is that it treats the inertial uncertainties and external disturbances as the total uncertainties. Then, the total uncertainties, as a newly extended state, are observed by a state observer rather than a disturbance one. With the aid of the linear or nonlinear nonsmooth feedback functions, high precision and robustness of the estimations can be achieved [9].

In this paper, the sliding mode control method was exploited for designing the controller of the SV. The nonlinear planing force and external disturbance of the SV are regarded as the lumped disturbance, and LESO is used to estimate the lumped disturbance. The stability of the system is proved by Lyapunov stability theorem. Finally, simulation results show that this method can reduce the chattering of sliding mode control in the presence of unknown disturbance of SV.

The specific contributions of this paper are as follows.

1) An auxiliary state is constructed according to the system states to refer to the total disturbance, which includes the uncertain dynamics, external disturbances and nonlinear planing force. The LESO is introduced to estimate the total disturbance, such design decreases the computational burden dramatically, while making it very simple and achievable to realize the disturbance compensation.

2) In this paper, a sliding mode control method based on LESO was proposed for SV with consideration of model uncertainty, unpredictable external disturbances and nonlinear planing force. Compared with the existing works, the highlight of the proposed control scheme is that it can simultaneously address the internal uncertainties and external disturbance in real time for systems.

3) The stability of the control system is proved by the Lyapunov theorem.

This paper is organized as follows. In Section 2, the nonlinear model of the SV on vertical plane is briefly introduced. Section 3 introduces the control scheme designed in this paper. Simulation results are given in Section 4 and the conclusion part is presented in Section 5.

2. Modeling of SV on Vertical Plane.

The forces on the vertical plane of the SV are mainly the gravity on the body center of the vehicles, the fluid force on the cavitator, the planing force on the tail of the vehicle and the thrust, etc. Based on [1, 10], the dynamic model of the vehicles in the vertical plane is given as follows:

$$\begin{bmatrix} \dot{z} \\ \dot{v} \\ \dot{\theta} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 & -V & 0 \\ 0 & a_{22} & 0 & a_{24} \\ 0 & 0 & 0 & 1 \\ 0 & a_{42} & 0 & a_{44} \end{bmatrix} \begin{bmatrix} z \\ v \\ \theta \\ q \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_{21} & b_{22} \\ 0 & 0 \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} \delta_c \\ \delta_e \end{bmatrix} + \begin{bmatrix} 0 \\ g \\ 0 \\ 0 \end{bmatrix} + F_P \tag{1}$$

$$a_{22} = \frac{C_1 - C_2}{MV} \qquad a_{24} = \frac{C_1 L_c - C_2 L_e}{MV} \qquad a_{42} = \frac{C_2 L_e - C_1 L_c}{I_{yy}\ V} \qquad a_{44} = \frac{C_1 L_c^2 - C_2 L_e^2}{I_{yy}\ V}$$

$$b_{21} = \frac{C_1}{M} \qquad b_{22} = \frac{C_2}{M} \qquad b_{41} = \frac{-C_1 L_c}{I_{yy}} \qquad b_{42} = \frac{-C_2 L_e}{I_{yy}}$$

$$C_1 = 0.5\pi\rho R_n^2 V^2 C_{x0} \quad C_2 = 0.5n\pi\rho R_n^2 V^2 C_{x0} \quad M = \frac{7}{9} m\rho\pi R^2 L^2$$

where z is the vertical depth, θ is the pitch angle, v is the vertical speed, and q is the pitch rate. δ_c is the cavitator deflection angle, δ_e is the fin deflection angle, I_{yy} is the inertial moment, M is the mass of the SV, and F_P is the planing force. The controller design and simulation in this paper adopt the standard SV model in [11], and the related SV parameters are described in Table 1.

TABLE 1. Model parameters of the SV

Parameters	Value	Unit	Description
C_{x0}	0.82		Lift coefficient
g	9.81	m/s^2	Gravitational acceleration
L	1.8	m	SV length
L_c	$17/28L$	m	Cavitation force arm
L_e	$-11/28L$	m	Fin force arm
m	2		Density ratio ρ_b/ρ
n	0.5		Fin effectiveness
R_n	0.091	m	Cavitator radius
R	0.0508	m	SV radius
V	75	m/s	Body-axis forward speed
ρ	1000	kg/m^3	Density of water
ρ_b	2000	kg/m^3	Uniform density of SV
σ	0.03		Cavitation number

3. Control Design.

3.1. Tracking differentiator design.
To obtain differential approximation, the tracking differentiator is designed. We first give the definition of the tracking differentiator as follows.

Definition 3.1. *The following system can be utilized* [12]*:*

$$\begin{cases} \dot{y}_1(t) = y_2(t) \\ \dot{y}_2(t) = f(y_1(t), y_2(t)) \end{cases} \tag{2}$$

If the solution to this system satisfies the condition $y_1(t)_{t\to\infty} \to 0$, $y_2(t)_{t\to\infty} \to 0$, *then for any constant* $T > 0$ *and bounded integrable function* $v(t)$, *the solution* $x_1(t)$ *of system (3) satisfies* $\lim_{R\to\infty} \int_0^T |x_1(t) - v(t)|dt = 0$.

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = R^2 f(x_1(t) - v(t), x_2(t)/R) \end{cases} \tag{3}$$

The tracking differentiator that satisfies Definition 3.1 is designed as follows:

$$\begin{cases} \dot{y}_1(t) = y_2(t) \\ \dot{y}_2(t) = -R^2\{a_1[(y_1(t) - v(t)) + tanh(y_1(t) - v(t))]\} - R^2\{a_2[y_2(t)/R + tanh(y_2(t)/R)]\} \end{cases} \tag{4}$$

Lemma 3.1. *For the designed tracking differentiator, if the parameters satisfy* $a_1 > 0$, $a_2 > 0$, *then the system is uniformly asymptotically stable at the origin* $(0, 0)$ [13]. *Therefore, for any bounded integrable function* $v(t)$, *the solution of system (4) satisfies* $y_1(t) \to v(t)$, $y_2(t) \to \dot{v}(t)$.

3.2. Sliding mode controller design.
In order to facilitate the controller design, the dynamic model (1) of the supercavitating vehicles can be rewritten as

$$\begin{cases} \dot{X}_1 = A_1X_1 + B_1X_2 \\ \dot{X}_2 = A_2X_2 + B_2u + F_g + \widetilde{d} \\ \widetilde{d} = F_P + f(w,t) \end{cases} \tag{5}$$

where $X_1 = [z,\theta]^T$, $X_2 = [v,q]^T$, $u = [\delta_c,\delta_e]^T$, $F_g = [g,0]^T$, $\widetilde{d}$ contains both parameter uncertainty and external disturbance $f(w,t)$.

The actual state variables of the system (5) are $[X_1,X_2]^T$, assume the reference state variables of the system are $[R_1,R_2]^T$, then the error state variables are written as

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} R_1 \\ R_2 \end{bmatrix} - \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \tag{6}$$

Therefore, the error model is

$$\begin{cases} \dot{E}_1 = A_1E_1 + B_1E_2 - A_1R_1 - B_1R_2 + \dot{R}_1 \\ \dot{E}_2 = A_2E_2 - B_2u - F_g - \widetilde{d} - A_2R_2 + \dot{R}_2 \\ \widetilde{d} = F_P + f(w,t) \end{cases} \tag{7}$$

The switch function is $S = CE_1 + E_2$, and consider the following reaching law [14] is $\dot{S} = -\tau S - \sigma sgn(S)$. Taking the derivative of the sliding manifold, we have

$$\begin{aligned} \dot{S} &= C\dot{E}_1 + \dot{E}_2 \\ &= CA_1\dot{E}_1 + (CB_1 + A_2)\dot{E}_2 - B_2u - \widetilde{d} - F_g + K \\ &= -\tau S - \sigma sgn(S) \end{aligned} \tag{8}$$

where $K = -CA_1R_1 - (CB_1 + A_2)R_2 + C\dot{R}_1 + \dot{R}_2$. Then, solving for $u(t)$ in (8) gives the control law

$$u(t) = B_2^{-1}\left[CA_1E_1 + (CB_1 + A_2)E_2 - \widetilde{d} - F_g + K + \tau S + \sigma sgn(S)\right] \tag{9}$$

3.3. **Linear extended state observer design.** Note that the control law consists of the lumped disturbances, which are not completely known to us. In this subsection, a LESO is proposed to estimate the unknown system uncertainties accurately [15]. Considering that the kinematic and dynamic equation of the supercavitating vehicles contain uncertain parameters and unmodeled characteristics, defining a new extended state variable X_3 to refer to the system's unknown lumped disturbances, then the dynamic model (5) can be extended as the following form

$$\begin{cases} \dot{X}_1 = A_1X_1 + B_1X_2 \\ \dot{X}_2 = A_2X_2 + B_2u + F_g + X_3 \\ \dot{X}_3 = g(t) \end{cases} \tag{10}$$

Remark 3.1. *The unknown lumped disturbances X_3 are differentiable and bounded by an unknown positive constant, and its derivative $g(t)$ is also bounded by a positive constant $\overline{g}(t)$, that is $||g(t)|| \leq \overline{g}(t)$.*

The LESO is designed to estimate the system's unknown lumped disturbances. Namely, based on [16], the LESO model for systems (10) is given by

$$\begin{cases} \dot{Z}_1 = Z_2 - \beta_{01}(Z_1 - X_2) + B_2u + A_2X_2 + F_g \\ \dot{Z}_2 = -\beta_{02}(Z_1 - X_2) \end{cases} \tag{11}$$

where $Z_2 = [z_{21}, z_{22}]^T$ are the estimations of the disturbance $\widetilde{d}$, and β_{01}, β_{02} are the relevant design parameters.

With the disturbance estimated by the LESO, the control law (9) is written as

$$\begin{aligned} &u_{leso}(t) = B_2^{-1}[CA_1E_1 + (CB_1 + A_2)E_2 - Z_2 - F_g + K + \tau S + \sigma sgn(S)] \\ &K = -CA_1R_1 - (CB_1 + A_2)R_2 + C\dot{R}_1 + \dot{R}_2 \end{aligned} \tag{12}$$

Proof: (Lyapunov stability). In order to examine the stability of the closed-loop system, one must develop an expression for the observer error dynamics. Defining the observer error $E_{leso1} = Z_1 - X_2$, $E_{leso2} = Z_2 - X_3 = Z_2 - \widetilde{d}$, the observer error dynamics are expressed as

$$\begin{cases} \dot{E}_{leso1} = E_{leso2} - \beta_{01} E_{leso1} \\ \dot{E}_{leso2} = -g(t) - \beta_{02} E_{leso1} \end{cases} \tag{13}$$

The stability of the LESO has been obtained by selecting the appropriate parameters β_{01} and β_{02} [17]. When the observer is stable, the derivative of vector $\dot{E}_{leso} = \left[\dot{E}_{leso1}, \dot{E}_{leso2}\right]^T = 0$, and then, the errors of estimation can be written as

$$\begin{cases} E_{leso1} = -g(t)/\beta_{02} \\ E_{leso2} = -g(t)\beta_{01}/\beta_{02} \end{cases} \tag{14}$$

From (14), it is clear that the estimation errors are determined by the parameters β_{01} and β_{02}. At the same time, parameter β_{01} should be larger and parameter β_{02} should be small enough. Thus, via tuning these parameters properly, the estimation errors E_{leso1} and E_{leso2} can be limited to be small enough.

The following Lyapunov function candidate for the sliding variable is considered

$$V = \frac{1}{2} S^T S \tag{15}$$

where $S = [s_1, s_2]^T$, which is an energy-like function with positive definite $V > 0$ of the sliding variables.

The time derivative of this function can be given by

$$\dot{V} = S^T \dot{S} \tag{16}$$

Then, substituting Equations (8) and (12) into (16) will lead to

$$\begin{aligned} \dot{V} &= S^T \dot{S} \\ &= S^T \left(-\tau S - \sigma sgn(S) + Z_2 - \widetilde{d}\right) \\ &= -\tau S^2 - \sigma |S| + S^T \left(Z_2 - \widetilde{d}\right) \\ &= -\tau S^2 - \sigma |S| - S^T E_{leso2} \\ &= -\tau S^2 - \sigma |S| + S^T (g(t)\beta_{01}/\beta_{02}) \\ &\le -\tau S^2 - \sigma |S| + S^T (\overline{g}(t)\beta_{01}/\beta_{02}) \end{aligned} \tag{17}$$

where $\overline{g}(t)$ is an unknown positive constant. Appropriate β_{01}, β_{02}, τ, σ can be selected such that $\dot{V} < 0$.

Remark 3.2. *In (17), the boundary layer of the sliding surface is affected by the estimation error of the LESO. Thus, parameters selection of the LESO is more important because it not only determines the performance of the LESO observing the lumped disturbance but also impacts the behavior of the sliding surface.*

4. **Simulation.** In this section, simulations are conducted for the supercavitating vehicle to illustrate the performance of the proposed controller. The controlled vehicle variable is the vertical depth z.

4.1. **Case1: Depth tracking without external disturbance.** In this subsection, the step response in the vertical position is carried out with a 1 m step input introduced for the depth z, the corresponding responses for vehicle states are given in Figure 1 for references. It can be observed that the vehicle reaches the set point command with no overshoot as illustrated, and the transition motions are very smooth.

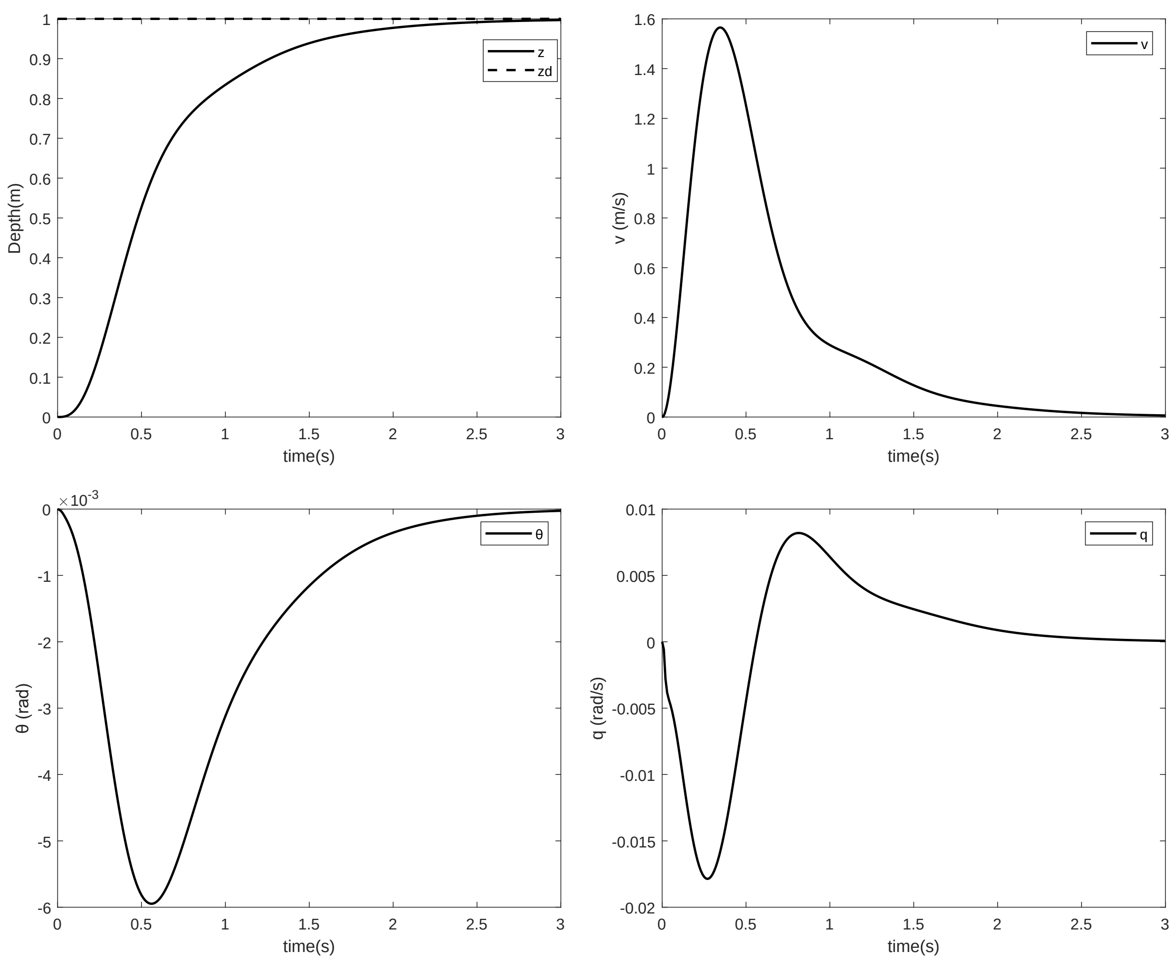

FIGURE 1. Tracking responses due to depth step input

The corresponding control actions are plotted in Figure 2. It can be observed that these signals are smooth enough with almost no chattering and no saturation.

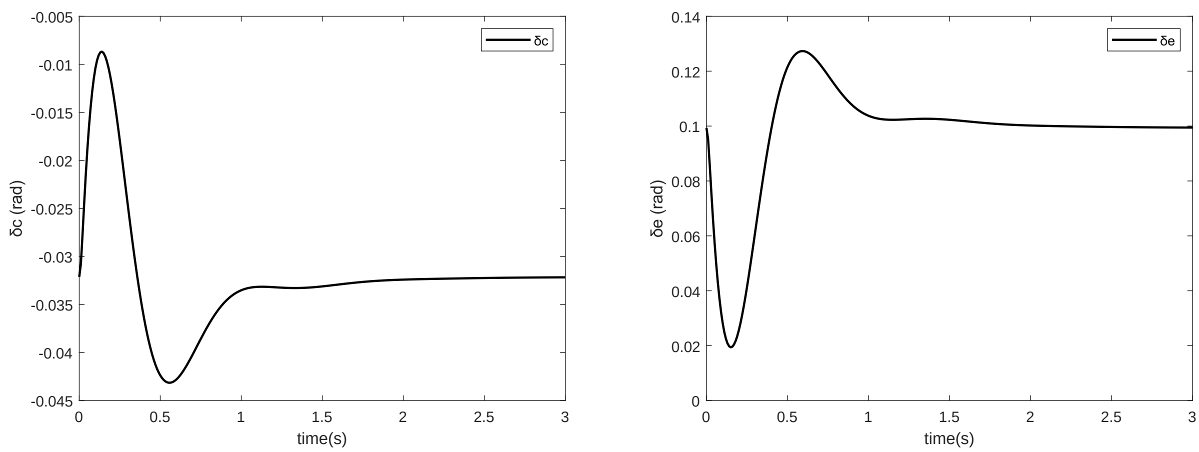

FIGURE 2. Control actions due to depth step input

4.2. **Case2: Depth tracking with external disturbance $\widetilde{d} = [2, 0]^T$.** The disturbance attenuation is a crucialability of the robust controller about underwater maneuvering motions. A disturbance is introduced at 0.5 s which tends to produce 2 m/s vertical speed of the vehicle. Figure 3 shows that the controller can successfully fight against the external

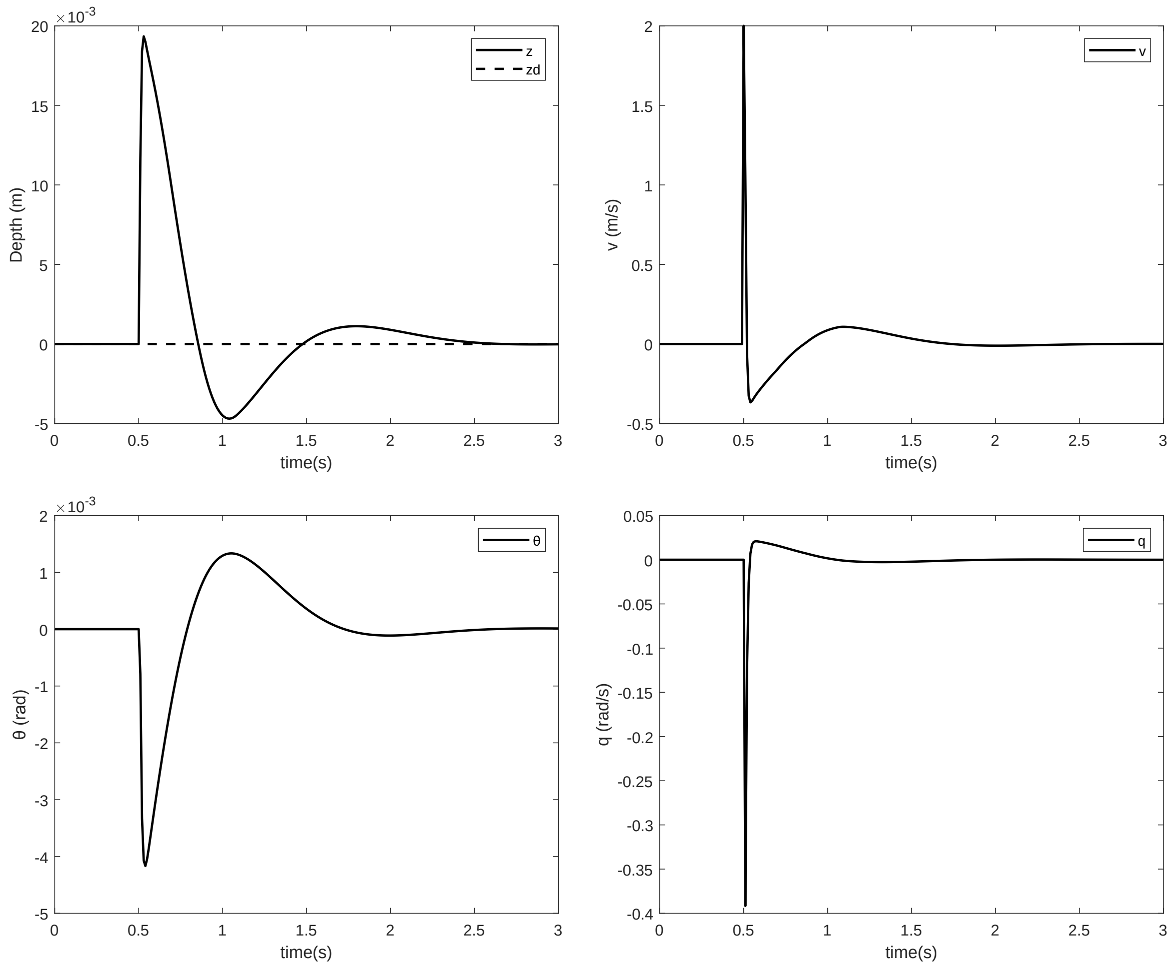

Figure 3. Tracking responses with external disturbance

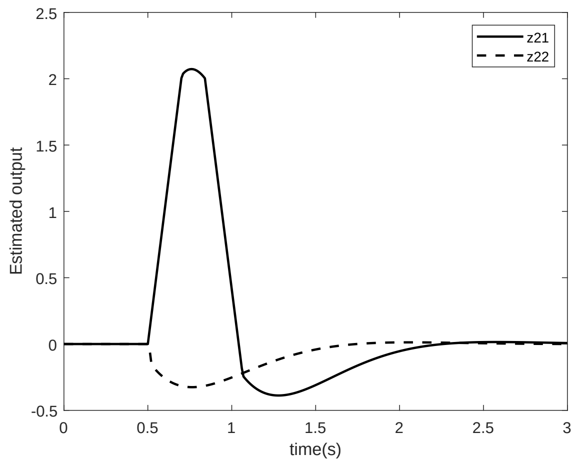

Figure 4. Estimation of disturbances via LESO

disturbance and finally can drive the vehicle back to the reference depth within 2 s. It can be seen from Figure 4 that the unknown disturbance can be accurately estimated by the designed LESO.

Because of the change of vertical speed, the vehicle aft will be immersed into water and the planing force occurs as shown in Figure 5. And it can be seen that the planing force disappears rapidly under the action of the controller.

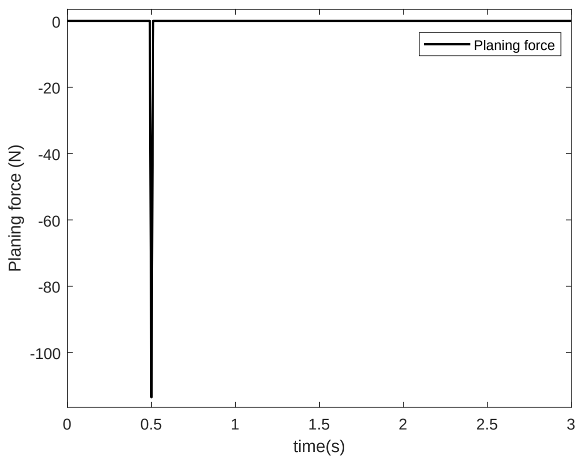

FIGURE 5. Planing force

5. **Conclusion.** In this paper, a method combining sliding mode control and linear extended state observer has been presented for the tracking control of supercavitating vehicles under the interior and external uncertainties. And nonlinear planing force as disturbance of system to simplify the control model. Then, the linear extended state observer is designed to estimate the lumped disturbance and tracking differentiator is developed to obtain differential approximation. And due to the approximation characteristics of the tracking differentiator, the actuator reactions are more smooth. Simulation results show that this method can improve system robustness against unknown disturbance and reduce the chattering of sliding mode control. In the future, the effectiveness of the control scheme designed in this paper will be further verified in actual projects.

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