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OBSERVER-BASED ROBUST ADAPTIVE FUZZY CONTROL FOR NONLINEAR CPSS UNDER DECEPTION ATTACKS

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Abstract. *This paper studies the adaptive fuzzy output feedback control problem for nonlinear cyber-physical systems (CPSs) under deception attacks. In the control design, fuzzy logic systems (FLSs) are used to approximate unknown nonlinear dynamics, using the Nussbaum function, the controller protected from time-varying gains caused by deception attacks. Combined with backstepping technique, a novel robust adaptive fuzzy output feedback control scheme is developed. Based on Lyapunov stability theory, it is proved that all closed-loop signals are semi-globally uniformly ultimately bounded (SGUUB).*
Keywords: Fuzzy control, Nonlinear CPSs, Deception attacks, Nussbaum function

1. **Introduction.** Cyber-physical systems (CPSs), as the next-generation mainstream system, have been widely researched in various fields. Many practical engineering cases are mostly related to CPSs, such as smart grids [1] and environmental control [2]. However, due to the embedded networked control technique, CPSs become more vulnerable and consequent cyber attacks have become one of the major threats to CPSs. For example, Stuxnet achieved this goal by inserting code into an executive loop to hijack or fully control the sensor [3]. In [4], adversaries compromised the Dtrack variant corrupted the management network. In the case of Brazil power services, adversaries attacked with the Sodinokibi ransomware to cause damage [5]. Therefore, the secure problems of the systems are much worthy of our research for CPSs.

In the articles published in recent years, many scholars have turned their focus to how to improve the safety of CPSs [6-10]. The authors in [6,7] have achieved the secure state estimation (SSE) and control for CPSs, respectively. Subsequently, the authors provided the conditions for SSE in [8] and proposed a control loop for secure local to improve the resilience of system. Different from the existing literature [8], to reduce unnecessary network resource transmission, the event-triggered control algorithms have been developed in [9]. Although the above studies are abundant, most of the mentioned results are all for linear system, and algorithms relevant for linear systems may not be suitable for nonlinear system.

As we all know, the nonlinear system model is more complicated than linear system model due to the relationship of nonlinear dynamics, and the corresponding control method is also different from linear system model. According to a recent study report, because of the stronger approximated capacity to unknown nonlinear dynamics, FLSs/radial basis function neural networks (RBFNNs) are important tools for solving nonlinear system control problems; many scholars have proposed fuzzy/neural network control schemes for secure control problems [10]. The authors in [10] have presented an adaptive switching mechanism for a large-scale class of nonlinear system under DoS (denial of service) attacks. In [11], to mitigate the effects caused by the considered attacks, a novel coordinate

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transformation is developed in the backstepping control design, and then a state feedback control design has been achieved for nonlinear CPSs under deception attacks. However, no results are available yet for output feedback control of nonlinear CPSs with deception attacks. Therefore, the secure problems of the output feedback control are much worthy of our research for nonlinear CPSs.

Inspired by the results of the above discussion, adaptive fuzzy secure control design is studied for nonlinear system under deception attacks and unmeasured states in this paper. FLSs are used to approximate unknown nonlinear function along with an observer designed for unmeasured states. In addition, to deal with the multiple unknown time-varying gains caused by the deception attacks, the new types of Nussbaum functions are introduced in the adaptive control. Based on these algorithms, we design a novel adaptive output feedback control algorithm based on the backstepping method. Compared with [11], its main contributions are summarized as the following.

1) This paper proposes a fuzzy adaptive state secure control method for nonlinear CPSs with unmeasured states under deception attacks. Although [11] studies nonlinear CPSs under deception attacks, it is based on state feedback, but the developed control algorithm in [11] cannot be adopted to solve the output feedback control issue.

2) Different from [11], with the help of the approximated technique of FLS, the restrictive assumption that the unknown nonlinear dynamics can be converted to the product of a known function and upper of time-varying gain caused by deception attacks is removed.

2. **Problem Statement and Preliminaries.**

2.1. **System descriptions and assumptions.**

In this paper, consider the nonlinear system as

$$\begin{aligned} &\dot{x}_i = x_{i+1} + f_i\left(\bar{x}_i\right) \\ &\dot{x}_n = u + f_n\left(\bar{x}_n\right) \\ &y = x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, \ldots, x_i]^T \in \mathbb{R}^i$ is a state vector and it is assumed that $\bar{x}_i$ are unmeasured, u and y denote the control input and output of system, respectively. $f_i\left(\bar{x}_i\right)$ is an unknown smooth nonlinear function and satisfies $f_i(0) = 0$.

Remark 2.1. *The system (1) is a common strict-feedback nonlinear system. In practice, many real-world system can be modeled as the above nonlinear strict-feedback system, such as unmanned aerial vehicles system [10] and marine surface vehicles.*

Then, we can rewrite system (1) as

$$\begin{aligned} &\dot{x} = Ax + Ky + \sum_{i=1}^{n} I_i f_i\left(\bar{x}_i\right) + Bu \\ &y = Cx \end{aligned} \tag{2}$$

where matrix A is a strict Hurwitz (chooses a vector K such that it holds), and $A = \begin{bmatrix} -k_1 & \cdots & I \\ \vdots & \cdots & \vdots \\ -k_n & \cdots & 0 \end{bmatrix}$, $K = \begin{bmatrix} k_1 \\ \vdots \\ k_n \end{bmatrix}$, $I_i = \begin{bmatrix} \underbrace{0, \ldots, 0}_{i-1}, 1, 0, \ldots, 0 \end{bmatrix}^T$, $B = [0, \ldots, 0, 1]^T$, $C = [1, \ldots, 0, 0]^T$.

2.2. **Deception attacks model.**

According to the definition of deception attacks, we can describe

$$\tilde{x}_i(t) = x_i(t) + \delta_s(x_i(t), t) \tag{3}$$

where $\delta_s(x_i(t),t)$ is the malicious sensor deception attack signal. Similar to [11], assume that the attack is parametric and state-dependent $\delta_s(x_i(t),t) = w(t)x_i(t)$ with unknown time-varying weight $w(t)$.

Assumption 2.1. [11] *With $w(t)$, $1+w(t) \neq 0$. Assume $|w(t)| \leq \bar{w}$ and $|\dot{w}(t)| \leq \bar{\dot{w}}$ with unknown positive constants $\bar{w}$ and $\bar{\dot{w}}$, and define the following function*

$$\lambda = \frac{1}{1+w(t)} \tag{4}$$

where $\underline{\lambda} \leq \lambda \leq \bar{\lambda}$ and $\underline{\dot{\lambda}} \leq \dot{\lambda} \leq \bar{\dot{\lambda}}$, $\underline{\lambda}$, $\bar{\lambda}$, $\underline{\dot{\lambda}}$ and $\bar{\dot{\lambda}}$ are unknown constants.

According to (3) and (4), one has

$$x_i(t) = \lambda \tilde{x}_i(t) \tag{5}$$

2.3. **Nussbaum function properties.**

Definition 2.1. [11] *The continuous known Nussbaum-type function $\mathcal{N}(\varsigma)$, and*

$$\lim_{k\to+\infty} \sup \frac{1}{k}\int_0^k \mathcal{N}(\varsigma)d\varsigma = +\infty \tag{6}$$

$$\lim_{k\to-\infty} \inf \frac{1}{k}\int_0^k \mathcal{N}(\varsigma)d\varsigma = -\infty \tag{7}$$

Lemma 2.1. [11] *Smooth functions $V(x,t) \geq 0$ and $\varsigma : \mathbb{R}^+ \to \mathbb{R}$, which is defined on $[0,t_f)$. $\mathcal{N}(\varsigma)$ holds $0 < |m(x,t)| \leq m' < \infty$, and a constant m'. Let n_0 choose the appropriate constant, and $P(t)$ is real-valued continuous and $P(0)=0$; we can get*

$$V(x,t) \leq n_0 + e^{-cl}\int_0^t \left(m(x,t)\mathcal{N}(\varsigma)\dot{\varsigma} + \dot{\varsigma}\right) e^{cl} dl + P(t) \tag{8}$$

The functions $V(x,t)$, $\varsigma(t)$, $\int_0^t \left(m(x,t)\mathcal{N}(\varsigma)\dot{\varsigma} + \dot{\varsigma}\right) dl$ are bounded on $[0,t_f)$.

3. **State Observer Design.** In this paper, a measurement of a state is attacked, and the state is said to be unavailable. We need to design a state observer to estimate the unavailable states $\bar{x}_i$, $i = 1,\ldots,n$. Similar to [12], $f_i(\bar{x}_i) = \theta_i^T \varphi_i\left(\hat{\bar{x}}_i\right) + \varepsilon_i$, and $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_n]^T$ is the estimation of state x.

For the unavailable state of system (2), design the state observer as

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \sum_{i=1}^n I_i \hat{\theta}_i^T \varphi_i\left(\hat{\bar{x}}_i\right) + Bu \\ \hat{y} &= C\hat{x} \end{aligned} \tag{9}$$

The observation error is defined as $e = x - \hat{x}$. From (2) and (9), we can get

$$\dot{e} = \dot{x} - \dot{\hat{x}} = Ae + Ky + \varepsilon + \sum_{i=1}^n I_i \tilde{\theta}_i^T \varphi_i\left(\hat{\bar{x}}_i\right) \tag{10}$$

where $e = [e_1, e_2, \ldots, e_n]^T$ and $\tilde{\theta} = \theta - \hat{\theta}$.

Choosing the Lyapunov function candidate as $V_0 = \frac{1}{2}e^T Pe$, from (10), it can be obtained that V_0 along the time derivative satisfies

$$\dot{V}_0 = \frac{1}{2}e^T\left(A^T P + PA\right)e + e^T P\left(Ky + \varepsilon + \sum_{i=1}^n I_i \tilde{\theta}_i^T \varphi_i\left(\hat{\bar{x}}_i\right)\right) \tag{11}$$

By employing the Young's inequality, we can get

$$e^T P\left(Ky + \varepsilon + \sum_{i=1}^n I_i \tilde{\theta}_i^T \varphi_i\left(\hat{\bar{x}}_i\right)\right)$$

$$\leq \frac{1}{2}\|e\|^2\|P\|^2\left(\|K\|^2+1+n\right)+\frac{1}{2}\bar{\lambda}^2\tilde{x}_1^2+\frac{1}{2}\|\varepsilon_0\|^2+\frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i \tag{12}$$

Substituting (12) into (11) yields

$$\dot{V}_0 \leq -\vartheta_0\|e\|^2+\frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i+\frac{1}{2}\bar{\lambda}^2\tilde{x}_1^2+D_0 \tag{13}$$

where $\vartheta_0=-\lambda_{\min}(Q)-\frac{1}{2}\|P\|^2\|K\|^2-\frac{1}{2}\|P\|^2$ and $D_0=\frac{1}{2}\|\varepsilon_0\|^2$.

4. **Control Design.** In this section, an observer-based fuzzy adaptive output feedback control strategy using adaptive backstepping technique is proposed. First, coordinate transformation is

$$\begin{aligned} z_1 &= \tilde{x}_1 \\ z_i &= \hat{x}_i - v_i \\ \chi_i &= v_i - \alpha_{i-1} \end{aligned} \tag{14}$$

where z_i is the error surface, α_{i-1} is a virtual control function to be designed in each step, and v_i is a corresponding first-order filter virtual control. χ_i is the output error of the first-order filter, and the first-order filter is designed as

$$w_i\dot{v}_i+v_i=\alpha_{i-1}, \quad v_i(0)=\alpha_{i-1}(0) \tag{15}$$

where w_i, $i=1,\ldots,n$ are positive constants.

Step 1: From (5) and (14), it can be obtained that z_1 along the time derivative satisfies

$$\dot{z}_1=\dot{\tilde{x}}_1=\frac{\dot{x}_1}{\lambda}-\frac{\dot{\lambda}z_1}{\lambda} \tag{16}$$

Choosing the Lyapunov candidate function $V_1=V_0+\frac{1}{2}z_1^2+\frac{1}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1+\frac{1}{2\tau_1}\tilde{\lambda}_1^2+\frac{1}{2\tau_2}\tilde{\lambda}_2^2$, according to (16), it can be obtained that V_1 along the time derivative satisfies

$$\begin{aligned} \dot{V}_1 \leq{}& \dot{V}_0+z_1\left[\frac{1}{\lambda}\left(e_2+z_2+\theta_1^T\varphi_1(x_1)+\varepsilon_1+\chi_2+\alpha_1\right)-\frac{\dot{\lambda}z_1}{\lambda}\right] \\ &-\frac{1}{\gamma_1}\tilde{\theta}_1^T\dot{\hat{\theta}}_1-\frac{1}{\tau_1}\tilde{\lambda}_1\dot{\hat{\lambda}}_1-\frac{1}{\tau_2}\tilde{\lambda}_2\dot{\hat{\lambda}}_2 \end{aligned} \tag{17}$$

By employing the Young's inequality, we can get

$$\begin{aligned} &z_1\frac{1}{\lambda}\left(e_2+\theta_1^T\varphi_1(x_1)+z_2+\varepsilon_1+\chi_2\right) \\ &\leq \frac{5}{2\underline{\lambda}^2}z_1^2+\frac{1}{2}\|e\|^2+\frac{1}{2}z_2^2+\frac{1}{2}\|\theta_1\|^2+\frac{1}{2}\varepsilon_{10}^2+\frac{1}{2}\chi_2^2 \end{aligned} \tag{18}$$

According to (13) and (18), rewrite (18) as

$$\begin{aligned} \dot{V}_1 \leq{}& -\left(\vartheta_0-\frac{1}{2}\right)\|e\|^2+\frac{1}{2}z_2^2+z_1\left[\frac{1}{\lambda}\alpha_1+\frac{1}{2}\hat{\lambda}_1z_1+\hat{\lambda}_2z_1+\hat{\theta}_1^T\varphi_1\left(\hat{x}_1\right)\right] \\ &+\frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i-z_1\theta_1^T\varphi_1\left(\hat{x}_1\right)+\frac{1}{\gamma_1}\tilde{\theta}_1^T\left(\dot{\hat{\theta}}_1-\gamma_1z_1\varphi_1\left(\hat{x}_1\right)\right) \\ &-\frac{1}{\tau_1}\tilde{\lambda}_1\left(\dot{\hat{\lambda}}_1-\frac{1}{2}\tau_1z_1^2\right)-\frac{1}{\tau_2}\tilde{\lambda}_2\left(\dot{\hat{\lambda}}_2-\tau_2z_1^2\right)+\frac{1}{2}\left(\varepsilon_{10}^2+\|\theta_1\|^2+\chi_2^2\right)+D_0 \end{aligned} \tag{19}$$

where $\lambda_1=\frac{5}{\underline{\lambda}^2}+\bar{\lambda}^2$ and $\lambda_2=\frac{\bar{\dot{\lambda}}}{\underline{\lambda}}$.

By employing the Young's inequality, we can get

$$-z_1\theta_1^T\varphi_1\left(\hat{x}_1\right)\leq\frac{1}{2}z_1^2+\frac{1}{2}\|\theta_1\|^2 \tag{20}$$

Substituting (20) into (19) yields

$$\dot{V}_1 \leq -\left(\vartheta_0 - \frac{1}{2}\right)\|e\|^2 + \frac{1}{2}\left(z_1^2 + z_2^2\right) + \frac{1}{2}\sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i + \frac{1}{2}\chi_2^2 + z_1\left[\frac{1}{\lambda}\alpha_1 + \frac{1}{2}\hat{\lambda}_1 z_1 + \hat{\lambda}_2 z_1 + \hat{\theta}_1^T\varphi_1\left(\hat{x}_1\right)\right] + \frac{1}{\gamma_1}\tilde{\theta}_1^T\left(\dot{\hat{\theta}}_1 - \gamma_1 z_1\varphi_1\left(\hat{x}_1\right)\right) - \frac{1}{\tau_1}\tilde{\lambda}_1\left(\dot{\hat{\lambda}}_1 - \frac{1}{2}\tau_1 z_1^2\right) - \frac{1}{\tau_2}\tilde{\lambda}_2\left(\dot{\hat{\lambda}}_2 - \tau_2 z_1^2\right) + D_0 + \frac{1}{2}\varepsilon_{10}^2 + \|\theta_1\|^2 \tag{21}$$

According to Definition 2.1, α_1 is a virtual control function, constructed as follows:

$$\alpha_1 = \mathcal{N}(\varsigma_i)\bar{\alpha}_1 \tag{22}$$

The auxiliary controller $\bar{\alpha}_1$ is designed in (24), and define $\dot{\varsigma}_i$ as

$$\dot{\varsigma}_i = z_1\bar{\alpha}_1 \tag{23}$$

$$\bar{\alpha}_1 = c_1 z_1 + \hat{\lambda}_1 z_1 + \hat{\lambda}_2 z_1 + \frac{1}{2}z_1 + \hat{\theta}_1^T\varphi_1\left(\hat{x}_1\right) \tag{24}$$

where $c_1 > 0$ is design parameters.

Substituting (22)-(24) into (21) yields

$$\dot{V}_1 \leq -\left(\vartheta_0 - \frac{1}{2}\right)\|e\|^2 + c_1 z_1^2 + \frac{1}{2}\left(z_2^2 + \chi_2^2 + \sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i\right) + D_0 + \frac{1}{2}\varepsilon_{10}^2 + \|\theta_1\|^2 + \left(\frac{1}{\lambda}\mathcal{N}(\varsigma_i) + 1\right)\dot{\varsigma}_i + \frac{1}{\gamma_1}\tilde{\theta}_1^T\left(\dot{\hat{\theta}}_1 - \gamma_1 z_1\varphi_1\left(\hat{x}_1\right)\right) - \sum_{k=1}^{2}\left(\frac{1}{\tau_k}\tilde{\lambda}_k\left(\dot{\hat{\lambda}}_k - \frac{1}{2}\tau_k z_k^2\right)\right) \tag{25}$$

Then, we can design the parameter adaptive laws $\dot{\hat{\theta}}_1$, $\dot{\hat{\lambda}}_1$, $\dot{\hat{\lambda}}_2$

$$\dot{\hat{\theta}}_1 = \gamma_1 z_1\varphi_1(\hat{x}_1) - \sigma_1\hat{\theta}_1 \tag{26}$$

$$\dot{\hat{\lambda}}_1 = \frac{1}{2}\tau_1 z_1^2 - \bar{\sigma}_1\hat{\lambda}_1 \tag{27}$$

$$\dot{\hat{\lambda}}_2 = \tau_2 z_1^2 - \bar{\sigma}_2\hat{\lambda}_2 \tag{28}$$

where $\bar{\sigma}_1 > 0$ and $\bar{\sigma}_2 > 0$ are design parameters.

Substituting (26)-(28) into (25) yields

$$\dot{V}_1 \leq -\vartheta_1\|e\|^2 + c_1 z_1^2 + \frac{1}{2}\left(z_2^2 + \chi_2^2 + \sum_{i=1}^{n}\tilde{\theta}_i^T\tilde{\theta}_i\right) + \left(\frac{1}{\lambda}\mathcal{N}(\varsigma_i) + 1\right)\dot{\varsigma}_i + \frac{\sigma_1}{\gamma_1}\tilde{\theta}_1^T\hat{\theta}_1 + \sum_{k=1}^{2}\frac{\bar{\sigma}_k}{\tau_k}\tilde{\lambda}_k\hat{\lambda}_k + D_1 \tag{29}$$

where $\vartheta_1 = \vartheta_0 - \frac{1}{2}$ and $D_1 = D_0 + \|\theta_1\|^2 + \frac{1}{2}\varepsilon_{10}^2$.

Step i ($i \in [2, n]$): From z_i in (14), we have

$$\dot{z}_i = \dot{\hat{x}}_i - \dot{v}_i = z_{i+1} + \chi_{i+1} + \alpha_i + \hat{\theta}_i^T\varphi_i\left(\hat{\bar{x}}_i\right) - \dot{v}_i \tag{30}$$

Choosing the Lyapunov candidate function as $V_i = V_{i-1} + \frac{1}{2}z_i^2 + \frac{1}{2\gamma_i}\tilde{\theta}_i^T\tilde{\theta}_i + \chi_i^2$, according to (30), it can be obtained that V_i along the time derivative satisfies

$$\dot{V}_i = \dot{V}_{i-1} + z_i\left[z_{i+1} + \chi_{i+1} + \alpha_i + \theta_i^T\varphi_i\left(\hat{\bar{x}}_i\right) - \dot{v}_i\right] + \frac{1}{\gamma_1}\tilde{\theta}_i^T\left(-z_i\gamma_1\varphi_i\left(\hat{\bar{x}}_i\right) - \dot{\hat{\theta}}_i\right) + \chi_i\left(\dot{v}_i - \dot{\alpha}_{i-1}\right) \tag{31}$$

where $N_i(\cdot)$ is a known continuous function on the bounded closed-loop set $\Omega_i =: \Big\{ \Big(e_i, z_i, \tilde{\theta}_i, \chi_i, \tilde{\lambda}_1, \tilde{\lambda}_2 \Big) \Big| \Big(e_i^T P_i e_i + z_i^2 + \tilde{\theta}_i^T \tilde{\theta}_i + \chi_i^2 + \tilde{\lambda}_1^2 + \tilde{\lambda}_2^2 \Big) \le 2\xi \Big\}$ with $\xi > 0$. There exists a constant N_i with $|N_i(\cdot)| \le N_i$ on Ω_i.

By employing the Young's inequality, we can get

$$z_i \left(\chi_i + \theta_i^T \varphi_i \left(\hat{\bar{x}}_i \right) \right) - \chi_i \left(\frac{1}{w_i} \chi_i + \dot{\alpha}_{i-1} \right) \le z_i^2 + \frac{1}{2} \chi_i^2 + \frac{1}{2} \|\theta_i\|^2 - \left(\frac{1}{w_i} - N_i^2 \right) \chi_i^2 \tag{32}$$

Substituting (32) into (31) yields

$$\dot{V}_i \le \dot{V}_{i-1} + z_i \left[z_i + z_{i+1} + \alpha_i - \dot{v}_i \right] + \frac{1}{\gamma_1} \tilde{\theta}_i^T \left(-z_i \gamma_1 \varphi_i \left(\hat{\bar{x}}_i \right) - \dot{\hat{\theta}}_i \right) - \left(\frac{1}{w_i} - N_i^2 - \frac{1}{2} \right) \chi_i^2 + \frac{1}{2} \|\theta_i\|^2 \tag{33}$$

Design the virtual control function α_i and the parameter adaptive law of $\dot{\hat{\theta}}_i$ as follows:

$$\alpha_i = -\left(-z_{i-1} + c_i z_i + z_i - \dot{v}_i \right) \tag{34}$$

$$\dot{\hat{\theta}}_i = -\gamma_1 z_i \varphi_i \left(\hat{\bar{x}}_i \right) - \sigma_1 \hat{\theta}_i \tag{35}$$

where $\alpha_i = u$, $i = n$.

According to (34) and (35), we can rewrite (33) as

$$\begin{aligned} \dot{V}_i \le & -\vartheta_1 \|e\|^2 + \sum_{k=1}^{i} c_k z_k^2 + \frac{1}{2} z_{i+1}^2 + \frac{1}{2} \sum_{i=1}^{n} \tilde{\theta}_i^T \tilde{\theta}_i - \frac{1}{2} \sum_{k=2}^{i} \left(\frac{1}{w_k} - N_k^2 - \frac{1}{2} \right) \chi_k^2 \\ & + \left(\frac{1}{\lambda} \mathcal{N}(\varsigma_i) + 1 \right) \dot{\varsigma}_i + \frac{\sigma_1}{\gamma_1} \sum_{k=1}^{i} \tilde{\theta}_k^T \hat{\theta}_k + \sum_{k=1}^{2} \frac{\bar{\sigma}_k}{\tau_k} \tilde{\lambda}_k \hat{\lambda}_k + D_i \end{aligned} \tag{36}$$

where $D_i = D_{i-1} + \frac{1}{2} \|\theta_i\|^2$.

By employing the Young's inequality, we can get

$$\begin{aligned} & \frac{\sigma_1}{\gamma_1} \sum_{i=1}^{n} \tilde{\theta}_i^T \hat{\theta}_i + \sum_{k=1}^{2} \frac{\bar{\sigma}_k}{\tau_k} \tilde{\lambda}_k \hat{\lambda}_k \\ & \le - \sum_{\substack{i=1,\dots,n \\ k=1,2}} \left(\frac{\sigma_1}{2\gamma_i} \left\| \tilde{\theta}_i \right\|^2 + \frac{\bar{\sigma}_1}{2\tau_k} \left\| \tilde{\lambda}_k \right\|^2 \right) + \sum_{\substack{i=1,\dots,n \\ k=1,2}} \left(\frac{\sigma_1}{2\gamma_i} \|\theta_i\|^2 + \frac{\bar{\sigma}_1}{2\tau_k} \|\lambda_k\|^2 \right) \end{aligned} \tag{37}$$

Substituting (37) into (36) yields

$$\begin{aligned} \dot{V}_n \le & -\vartheta_1 \|e\|^2 - \sum_{i=1}^{n} c_i z_i^2 - \left(\frac{\sigma_1}{2\gamma_1} - \frac{1}{2} \right) \sum_{i=1}^{n} \left\| \tilde{\theta}_i \right\|^2 - \frac{1}{2} \sum_{i=2}^{n} \left(\frac{1}{w_i} - N_i^2 \right) \chi_i^2 \\ & + \left(\frac{1}{\lambda} \mathcal{N}(\varsigma_i) + 1 \right) \dot{\varsigma}_i - \frac{\bar{\sigma}_1}{2\tau_1} \left\| \tilde{\lambda}_1 \right\|^2 - \frac{\bar{\sigma}_2}{2\tau_2} \left\| \tilde{\lambda}_2 \right\|^2 + D_n \end{aligned} \tag{38}$$

where $D_n = D_n' + \frac{\sigma_1}{2\gamma_1} \sum_{i=1}^{n} \|\theta_i\|^2 + \frac{\bar{\sigma}_1}{2\tau_1} \|\lambda_1\|^2 + \frac{\bar{\sigma}_1}{2\tau_1} \|\lambda_2\|^2$.

Theorem 4.1. *According to Assumption 2.1 for the system (1), the controller (34), the fuzzy adaptive state observer (9), the virtual control functions (22) and (34), and the parameter adaptive laws (26)-(28), and (35), the proposed control method can ensure that the observe and tracking errors converge to a small neighborhood of the origin, and all the signals in closed-loop system are bounded.*

Proof: Let

$$C = \min\left\{\vartheta_1, c_i, \frac{\sigma_1}{2\gamma_1}, \frac{1}{2}\left(\frac{1}{w_i} - N_i^2\right), \frac{\bar{\sigma}_1}{2\tau_1}, \frac{\bar{\sigma}_2}{2\tau_2}\right\}, \quad i = 1, \ldots, n \tag{39}$$

Then, (38) can be rewritten as

$$\dot{V} \le -CV + D_n + \left(\frac{1}{\lambda}\mathcal{N}(\varsigma_i) + 1\right)\dot{\varsigma}_i \tag{40}$$

According to Lemma 2.1, we known that $V(t)$, $\varsigma_i(t)$ and $\int_0^t \left(\frac{1}{\lambda}\mathcal{N}(\varsigma_i) + 1\right)\dot{\varsigma} dl$ are bounded on $[0, t]$. Define $\bar{D}_1 = \max_{t\in[0,t]} \left(\frac{1}{\lambda}\mathcal{N}(\varsigma_i) + 1\right)\dot{\varsigma}_i$. Integrating (40) over $[0, t]$, one has

$$0 \le V(t) \le \frac{D}{C} + \left[V(0) - \frac{D}{C}\right] e^{-Ct} \tag{41}$$

where $D = D_n + D_1 + \bar{D}_1$ and based on the above formula, we can get that e_i, z_i, $\tilde{\lambda}_i$, $\tilde{\theta}_i$ can be shown that all signals in the closed-loop system are bounded. In addition, the state observer errors satisfy that $\lim_{t\to\infty}\|e_i\| = \sqrt{D/(C\lambda_{\min}(P))}$, $i = 1, \ldots, n$.

5. **Simulation Results.** An example is simulated to validate the effectiveness of the proposed novel robust adaptive fuzzy output feedback control scheme against deception attacks.

Example 5.1. *Let us consider the following second-order nonlinear system*

$$\dot{x}_1 = x_2 - x_1^4 e^{-x_1^4} - x_1 \tag{42}$$

$$\dot{x}_2 = u + \sin(x_2)\sin(x_1) - 4x_1x_2 \tag{43}$$

$$\tilde{x}_1(t) = x_1(t) + \delta_s(x_1(t), t) \tag{44}$$

The attack signal given with $\delta_s(x_i(t), t) = \big(-1.3 + 0.5\cos(t)^2\big)x(1)$ from $t = 0$ s. The initial states are $x(0) = [1, 0.9]^T$. The parameters are chosen as $c_1 = 20$, $c_2 = 70$, $k_1 = 9$, $k_2 = 7$, $\gamma_1 = 0.02$, $\sigma_1 = \bar{\sigma}_1 = \bar{\sigma}_2 = 0.001$, $\tau_1 = 1$, $\tau_2 = 0.3$. And the Nussbaum-type function is chosen as $\mathcal{N}(\varsigma_i) = e^{\hat{\theta}_1^2}\sin\left(\hat{\theta}_1\right)$.

According to the adaptive control scheme designed in Section 3, the control laws and parameter adaptation laws are given as follows:

$$\alpha_1 = \mathcal{N}(\varsigma_i)\left(c_1z_1 + \hat{\lambda}_1z_1 + \hat{\lambda}_2z_1 + 0.5z_1 + \hat{\theta}_1^T\varphi_1(\hat{x}_1)\right) \tag{45}$$

$$u = z_1 - c_2z_2 - z_2 + \dot{v}_2 \tag{46}$$

with

$$\dot{\hat{\theta}}_1 = \gamma_1z_1\varphi_1(\hat{x}_1) - \sigma_1\hat{\theta}_1 \tag{47}$$

$$\dot{\hat{\lambda}}_1 = \frac{1}{2}\tau_1z_1^2 - \bar{\sigma}_1\hat{\lambda}_1 \tag{48}$$

$$\dot{\hat{\lambda}}_2 = \tau_2z_1^2 - \bar{\sigma}_2\hat{\lambda}_2 \tag{49}$$

$$\dot{\hat{\theta}}_2 = -\gamma_1z_2\varphi_2\left(\hat{\bar{x}}_2\right) - \sigma_1\hat{\theta}_2 \tag{50}$$

The state variables and the control input are plotted in Figures 1 and 2, respectively. Figures 1 and 2 show the boundedness of the signals of the closed-loop system, which demonstrate the validity of the algorithm.

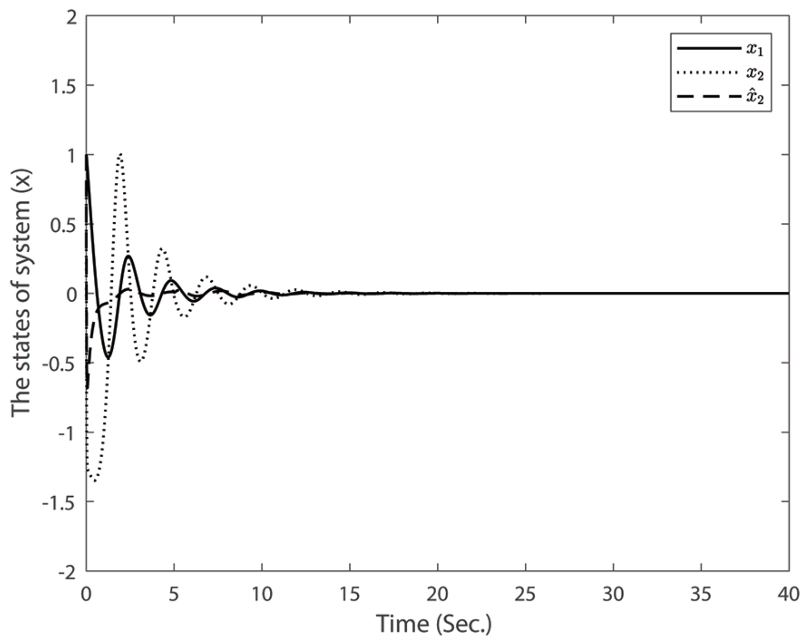

FIGURE 1. The system states

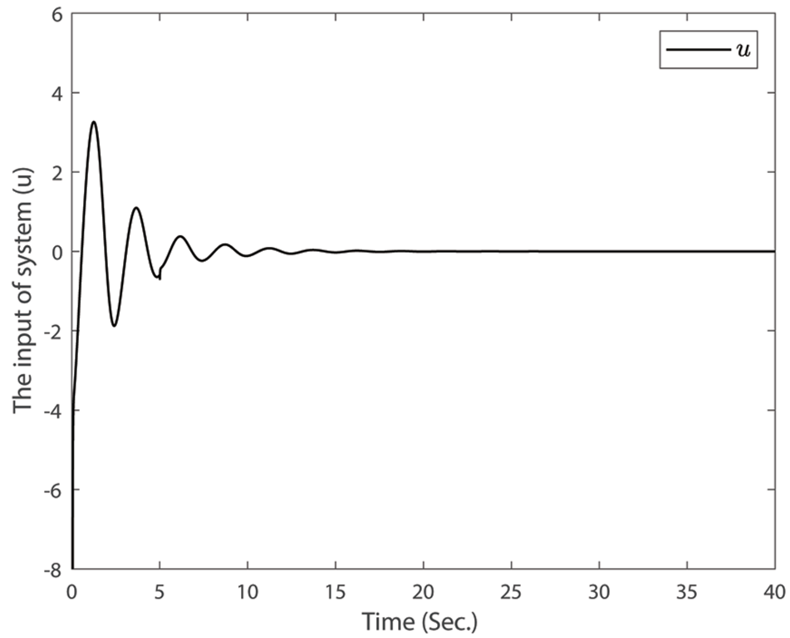

FIGURE 2. The control input

6. **Conclusions.** In this paper, a novel robust adaptive fuzzy output feedback control scheme has been developed for a class of nonlinear CPSs under deception attacks. Nussbaum function is introduced to keep the controller free of the time-varying gain caused by deception attacks. Combined with backstepping technique, an observer-based controller is constructed successfully. Based on Lyapunov stability theory, it is proved that all closed-loop signals are SGUUB.

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