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FUZZY ADAPTIVE LEADER-FOLLOWING FORMATION CONTROL FOR ELECTRICALLY DRIVEN UNMANNED NONHOLONOMIC MOBILE ROBOTS

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Abstract. *In many existing systems of leader-following formation, motor dynamics and external disturbance are not considered. In this paper, the formation tracking problem is studied by a practical unified error transformation control approach for multi-agent systems with motor dynamics and external disturbances. To simplify the controller design process, the potential functions are removed in this paper and an error function is constructed by using the relative distance and angle. It is assumed that the parameters in kinematics and dynamics of the robot and the motor are uncertain. Then a fuzzy adaptive dynamic surface tracking control method is developed which ensures formation tracking, maintaining connectivity and avoiding collisions for electrically driven unmanned nonholonomic mobile robots. Finally, the stability of the closed-loop system is analyzed under Lyapunov function theory sense.*
Keywords: Motor dynamics, Fuzzy adaptive control, Distributed formation tracking, Connectivity-maintaining, Collision-avoiding

1. **Introduction.** In recent years, the unmanned nonholonomic mobile multi-robot system is a typical underactuated nonlinear system. General smooth feedback control law is disabled for this system, especially influenced by other external factors such as load change, resistance interference, and the friction of wheels against the ground, it is more difficult to find a general and effective control method to achieve its motion control. Therefore, the control problem of uncertain nonholonomic mobile multi-robot system has attracted great attention in the control field [1,2]. Due to the rapid development of new energy vehicles, cars driven by electricity are becoming more and more widely. Most of formation systems have no regard for the dynamics deriving from electric motors which need to implement the mobile robots in practice; however, the motor dynamics equation is not considered in the above controller design process, and the kinematics and dynamics equation of the robot is only considered. As we all know, motor dynamics is a significance section for the nonholonomic mobile robots; for example, in the case of high-speed motion and highly variable load, electric drive is particularly important. It can not only save resources but also protect the environment. Therefore, we need to take the motor equation into consideration in the design of the actual controller.

There are many methods of formation control, including leader-follower [3], behavior-based method [4], virtual structure method [5], etc. Leader-follower method is the most simply and effective method among these methods and has been widely applied in the research process [6-9]. The potential function is a common method in formation controller design, but when selecting the designing value to figure out the maintaining connectivity, collision avoidance and trajectory tracking control for multi-agent systems, the potential

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function may lead to conflicts. [10] developed a connectivity maintaining and collision avoidance formation tracking scheme which can overcome the drawback of using the potential function, but ignoring motor dynamics as well. In [11], the problem of electric drive for a single robot system is studied. As far as we know, at present, there are no available results to study electrically driven robots with external disturbances for unmanned nonholonomic mobile robots. All unknown parameters come from motor dynamics and the external disturbances. We use fuzzy logic systems (FLS) in [12] to approximate unknown functions. Compared to the previous research, this article mainly has the following two contributions.

1) This paper proposes a fuzzy adaptive formation control algorithm for unmanned nonholonomic mobile robots incorporating motor dynamics and the external disturbances; compared with the related works [13-15], this paper removes the related conflicts raised by any potential functions; by this way the trajectory tracking of formation, connectivity-maintaining and collision avoiding can be effectively achieved.

2) In this article, to simplify the multi-agent systems control design process, we employ the DSC technique to structure an error function by the information of relative distance and angle, and then all signals tend to be stable from the closed-loop system by the Lyapunov stability theorem.

The rest of this paper is organized as follows. In Section 2, the kinematic and dynamic model are introduced and the problem is formulated. In Section 3, a unified error transformation approach and fuzzy logic system are presented. Control design is discussed in Section 4. Finally, Section 5 concludes this paper.

2. Problem Statement and Preliminaries.

2.1. Model of electrically driven unmanned nonholonomic mobile robots.

We consider a pair of leader-follower formation, which contains one leader and N followers, leader is labeled as 0, and followers are labeled as $i = 1, \ldots, N$; the dynamics and kinematics of the ith unmanned nonholonomic robots are denoted as follows:

$$\begin{aligned} \dot{q}_i &= P_i\left(q_i\right) v_i, \\ \dot{v}_i &= M_i^{-1}[NK_T i_i - A_i\left(\dot{q}_i\right) v_i - D_i v_i - F_{id}], \\ \dot{i}_i &= \mathrm{L}_i^{-1}(u_i - R_{ai} i_i - NK_E v_i - u_{id}), \end{aligned} \tag{1}$$

with $i = 1, \ldots, N$ and the position of center for the mobile robot's two wheels can be represented by (x_i, y_i), the heading angle of the center of the robot's two wheels can be represented by θ_i, $q_i = [x_i, y_i, \theta_i] \in \mathbb{R}^3 : (x_i, y_i)$ is the state of the center of the robot's two wheels, $v_i = [v_{i,1}, v_{i,2}] \in \mathbb{R}^2$; $v_{i,1}, v_{i,2}$ are the angular velocities of the mobile robot wheels, independently and F_{id} is a vector of disturbances including unmodeled dynamics. K_T, K_E are the motor constants, $i_i = [i_{i,1}, i_{i,2}] \in \mathbb{R}^2$ is the electric current of the ith robot, u_i is the input voltage of the ith robot and $u_i = R_{ai} i_i + \mathrm{L}_i \dot{i}_i + K_E \dot{\theta}_{im}$, the resistance is R_{ai}, and the inductance is L_i.

Assumption 2.1. [15] *The matrices M_i, $A_i\left(\dot{q}_i\right)$ and D_i are unknown in the dynamics of (1), but the parameters r_i and R_i are known and the system matrix M_i is symmetric and positive definite. The disturbances F_{id}, u_{id} are bounded as $\|F_{id}\| \le \bar{F}_{id}$, $\|u_{id}\| \le \bar{u}_{id}$, $\bar{F}_{id} > 0$, $\bar{u}_{id} > 0$ are constants and $\dot{F}_{id}$, $\dot{u}_{id}$ are existing where $i = 1, \ldots, N$.*

2.2. Problem statement and formation model.

The distance $l_{i,j}$ is from the ith robot to the jth robot and the relative angle $\phi_{i,j}$ for the ith robot to the jth robot are represented as follows:

$$\begin{aligned} l_{i,j} &= \sqrt{\left(x_j - x_i\right)^2 + \left(y_j - y_i\right)^2}, \\ \phi_{i,j} &= \arctan 2\left(y_j - y_i, x_j - x_i\right). \end{aligned} \tag{2}$$

The formation model can be represented as

$$\begin{aligned} \dot{l}_{i,j} &= \lambda_1 v_{j,1} + \lambda_2 v_{j,2} - \lambda_3 v_{i,1} - \lambda_4 v_{i,2}, \\ \dot{\phi}_{i,j} &= l_{i,j}^{-1}(\lambda_5 v_{j,1} + \lambda_6 v_{j,2} - \lambda_7 v_{i,1} - \lambda_8 v_{i,2}), \end{aligned} \tag{3}$$

with $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$.

A directed graph $G \triangleq (\nu, \varepsilon)$, $\nu \triangleq \{1, 2, \ldots, M\}$ is introduced to represent the communications of robots which have unequal communication ranges. If $l_{i,j} < L_{\max}$ where $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$, the ith mobile robot and the jth mobile robot are neighbors. The set of neighbour nodes for the ith mobile robot is $N_i(t) = \{j \,|l_{i,j}(t) < L_{\max}\}$. $\varepsilon \subseteq \nu \times \nu$ denotes the collection of edges and $(j, i) \in \varepsilon$ is that the ith mobile robot can receive signal from the jth mobile robot but not send messages back; the detailed introduction can be found in [14].

Assumption 2.2. [14] *$G(t)$ is a single direction graph. $v_L \leq \bar{v}_L$ and $\omega_L \leq \bar{\omega}_L$ which are reasonable only for the ith robots satisfying $0 \in N_i(0)$, $i = 1, \ldots, N$.*

The purpose of this paper is designing a controller F_i for the ith robot under graph which is from the ith mobile robot to the jth mobile robot to achieve three objectives:

i) when $t \geq 0$, if $L_{\min} < l_{i,j}(0) < L_{\max}$, get $L_{\min} < l_{i,j}(t) < L_{\max}$;

ii) when $t \geq 0$, if $l_{i,j}(0) \geq L_{\max}$, get $L_{\min} < l_{i,j}(t)$;

iii) $\lim_{t\to\infty} |l_{i,j}(t) - l_{i,j,d}| \leq \zeta_1$ and $\lim_{t\to\infty} |\phi_{i,j}(t) - \phi_{i,j,d}| \leq \zeta_2$

$i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$. $\zeta_1 > 0$, $\zeta_2 > 0$ are the positive small constants that can be made arbitrarily, $l_{i,j,d}$ is the desired relative distance from the ith robot to the jth robot, and $\phi_{i,j,d}$ is the desired angle for the ith robot.

Assumption 2.3. [14] *$L_{\min} < l_{i,j,d} < L_{\max}$ is the range of $l_{i,j,d}$ and $-\pi \leq \phi_{i,j,d} < \pi$ is the range of $\phi_{i,j,d}$, $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$.*

Remark 2.1. *Compared to the previous related works for unmanned nonholonomic mobile robot systems, this article designs a novel error function of formation trajectory tracking incorporating motor dynamics to resolve formation trajectory tracking issues, avoid collision meanwhile maintaining connectivity.*

3. Main Results.

3.1. A unified error transformation approach.

In this part, we propose a unified error transformation approach incorporating motor dynamics for multi-agent systems to achieve distributed formation control issues. We apply the following transformation error

$$s_{i,j} = \ln\left(\frac{\delta_{i,j} + p_{i,j,1}}{\delta_{i,j}(1 - p_{i,j,1})}\right), \tag{4}$$

with $p_{i,j,1} = (l_{i,j} - l_{i,j,d})/(L_{\max} - l_{i,j,d})$, $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$, and $\delta_{i,j} = (l_{i,j} - L_{\min})/(L_{\max} - l_{i,j,d})$. Design the error surfaces additional motor dynamics for the formation controller as follows:

$$\begin{aligned} &Z_{i,1} = \sum_{j=0, j\neq i}^{M} a_{i,j}\eta_{i,j}^T \eta_{i,j}, \quad Z_{i,2} = \theta_i - \theta_{i,v}, \\ &Z_{i,3} = z_i - \bar{\alpha}_i, \quad Z_{i,4} = i_i - \bar{i}_i, \quad E_{i,1} = \bar{\alpha}_i - \alpha_{i,v}, \quad E_{i,2} = \bar{i}_i - i_{i,v}, \end{aligned} \tag{5}$$

where $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$. The $Z_{i,1}$ denote the errors of the distance and bearing angle, $\eta_{i,j} = [s_{i,j}, p_{i,j,2}]^T$, $p_{i,j,2} = \phi_{i,j} - \phi_{i,j,d}$, $Z_{i,2}$ denote the errors of the heading angle to solve underactuated problems, $Z_{i,3} = [Z_{i,3,1}, Z_{i,3,2}]^T$, $Z_{i,4} = [Z_{i,4,1}, Z_{i,4,2}]^T$, $E_{i,1} = [E_{i,1,1}, E_{i,1,2}]^T$, $E_{i,2} = [E_{i,2,1}, E_{i,2,2}]^T$ are the boundary layer errors, and $a_{i,j}$ is the weight of information transfer between the ith robot and jth robot given by

$$a_{i,j}=\begin{cases}\bar{a}_{i,j}, & l_{i,j}(0)<L_{\max},\\ \underline{a}_{i,j}, & l_{i,j}(0)\geq L_{\max},\end{cases}$$
$$\bar{a}_{i,j}=\begin{cases}1, & l_{i,j}(t)<L_{\max}\text{ for }t\geq 0,\\ 0, & \text{otherwise},\end{cases}$$
$$\underline{a}_{i,j}=\begin{cases}1, & l_{i,j}(t)<L_{p}\text{ for }t\geq 0,\\ 0, & \text{otherwise},\end{cases} \tag{6}$$

with design constants L_p denoting the distance of preparing to avoid collision, and $L_{\min} < L_p < L_{\max}$. The virtual control laws are denoted by $\alpha_{i,v} = [\alpha_{i,v,1}, \alpha_{i,v,2}]^T$ and $i_{i,v} = [i_{i,v,1}, i_{i,v,2}]^T$ which will become $\bar{\alpha}_i = [\bar{\alpha}_{i,1}, \bar{\alpha}_{i,2}]^T$, $\bar{i}_i = [\bar{i}_{i,1}, \bar{i}_{i,2}]^T$ when they pass the first-order low-pass filters, satisfying $\iota_{i\alpha_i,l}\dot{\bar{\alpha}}_{i,v,l} + \bar{\alpha}_{i,v,l} = \alpha_{i,v,l}$, $\iota_{ii_i,l}\dot{\bar{i}}_{i,v,l} + \bar{i}_{i,v,l} = i_{i,v,l}$, $\bar{\alpha}_{i,v,l}(0) = \alpha_{i,v,l}(0)$ and $\bar{i}_{i,v,l}(0) = i_{i,v,l}(0)$, the small constants $\iota_{i\alpha_i,l} > 0$ and $\iota_{ii_i,l} > 0$ $(i = 1, 2)$. Virtual heading angles $\theta_{i,v}$ are designed as follows:

$$\dot{\theta}_{i,v}=\frac{r_i}{2R_i}\left(\alpha_{i,v,1}-\alpha_{i,v,2}\right)+k_{i,2}\left(\theta_i-\theta_{i,v}\right), \tag{7}$$

where $k_{i,2} > 0$ is designed small constant. Here, the relative angles are within bounds $Z_{i,2}$, $p_{i,j,2} \in [-\pi, \pi)$ which is closer to the real angle problem in practice.

Remark 3.1. *By using nonlinearly error transformation approach, we need to prove the boundedness of $Z_{i,1}$ to guarantee connectivity maintenance and collision avoidance of mobile robots, $Z_{i,1}$ will be proved in Theorem 4.1. The $Z_{i,2}$ is employed to define the heading angles of robots for formation trajectory tracking. The $Z_{i,3}$, $Z_{i,4}$ and $E_{i,1}$, $E_{i,2}$ are the boundary layer errors and they are designed by the technique of dynamic surface. When all errors converge to a neighborhood of the origin, the goals will be achieved.*

3.2. **Fuzzy logic system.** Defining a continuous function $f(x)$ on a compact set Ω, with $\varepsilon > 0$, there exists the fuzzy logic system such as

$$f(x)=W^{*T}\Theta_i+\gamma_i. \tag{8}$$

4. **Control Design.** The controller of the ith robot for networked electrically driven unmanned nonholonomic mobile system (1) is designed as

$$\alpha_{i,v}=\bar{\mathrm{X}}_{i,1}^{-1}\left(k_{i,1}\eta_i+\mathrm{X}_{i,2}\right), \tag{9}$$
$$u=R_i\left(NK_T\right)^{-1}\left(-k_{i,3}Z_{i,3}-\hat{\Xi}_i\left(\vartheta_i\middle|\hat{W}_i\right)\right)-\hat{\Gamma}_i\left(\vartheta_i\middle|\hat{\mathrm{K}}_i\right)-k_{i,4}Z_{i,4}, \tag{10}$$

with $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$, $k_{i,m} > 0$, $m = 1, 2, 3, 4$, are parameters designed by designer, $\bar{\mathrm{X}}_{i,1}^{-1} = \mathrm{X}_{i,1}^T\left(\mathrm{X}_{i,1}\mathrm{X}_{i,1}^T\right)^{-1}$, $\eta_i = \left[\eta_{i,h_{i,1}}^T, \ldots, \eta_{i,h_{i,B_i}}^T\right]^T$, $\mathrm{X}_{i,1}$ and $\mathrm{X}_{i,2}$ are known constants and $\bar{\mathrm{X}}_{i,1}^{-1}$ in (9) is well defined in [14] where $b_i = 1, \ldots, B_i$, the subscripts $h_{i,1}, \ldots, h_{i,B_i}$ present the elements of the set $h_i = \{h_{i,1}, \ldots, h_{i,B_i}\} = \{j|a_{i,j} \neq 0\}$.

Adaptive laws of $\hat{W}_{i,l}$, $\hat{\mathrm{K}}_{i,l}$ are chosen as follows:

$$\dot{\hat{W}}_{i,l}=\xi_{i,l}Z_{i,3,l}\Theta_{i,l}-\xi_{i,l}\sigma_{iW}\hat{W}_{i,l},\quad \dot{\hat{\mathrm{K}}}_{i,l}=\xi_{i,l}Z_{i,4,l}\Theta_{i,l}-\xi_{i,l}\sigma_{iK}\hat{\mathrm{K}}_{i,l}, \tag{11}$$

where $l = 1, 2$, $\xi_{i,l} > 0$ are tuning gains and the σ_i is an adjustable parameter.

Construct the Lyapunov function as follows:

$$V=V_1+V_2+(1/2)\sum_{i=1}^{M}\left[\sum_{m=1}^{2}E_{i,1,m}^2+E_{i,2,m}^2+tr\left(\tilde{W}_i^T\xi_i\tilde{W}_i\right)+tr\left(\tilde{\mathrm{K}}_i^T\xi_i\tilde{\mathrm{K}}_i\right)\right], \tag{12}$$

where $V_1 = (1/2)\sum_{i=1}^M Z_{i,1} + Z_{i,2}^2$, $V_2 = (1/2)\sum_{i=1}^M Z_{i,3}^T M_i Z_{i,3} + Z_{i,4}^T L_i Z_{i,4}$, $\bar{V}_2 = V_1 + V_2$, $\xi_i = diag\,[\xi_{i,1}, \xi_{i,2}]$.

Theorem 4.1. *For networked electrically driven unmanned nonholonomic mobile robots (1), under Assumptions 2.1-2.3, for constant κ, $V(0) \leq \kappa$ which denotes the initial value of Lyapunov function is bounded, the proposed controller (9)-(11) can achieve three objectives as follows:*

(i) when $t \geq 0$, if $L_{\min} < l_{i,j}(0) < L_{\max}$, get $L_{\min} < l_{i,j}(t) < L_{\max}$;

(ii) when $t \geq 0$, if $l_{i,j}(0) \geq L_{\max}$, get $L_{\min} < l_{i,j}(t)$ for $t \geq 0$;

(iii) $\lim_{t\to\infty} |l_{i,j}(t) - l_{i,j,d}| \leq \zeta_1$ and $\lim_{t\to\infty} |\phi_{i,j}(t) - \phi_{i,j,d}| \leq \zeta_2$ where $i = 1, \ldots, N$, $j = 0, \ldots, N$, $i \neq j$. The positive constants $\zeta_1 > 0$, $\zeta_2 > 0$ are which can be made arbitrarily small, $l_{i,j,d}$ is the desired distance from the ith robot to the jth robot, and $\phi_{i,j,d}$ is the desired angle for the ith robot.

Proof: Combining (3) and (5) yields

$$\dot{Z}_{i,1} = 2\eta_{i,j}^T \left\{ -\mathrm{X}_{i,1} \left(Z_{i,3} + E_{i,1} + \alpha_{i,v} \right) + \mathrm{X}_{i,2} \right\}, \tag{13}$$

$$\dot{Z}_{i,2} = 0.5 r_i R_i^{-1} \left(Z_{i,3,1} + E_{i,1,1} + \alpha_{i,v,1} - Z_{i,3,2} - Z_{i,1,2} - \alpha_{i,v,2} \right) - \dot{\theta}_{i,v}. \tag{14}$$

From (1), (3) and (5) yield

$$M_i \dot{Z}_{i,3} = N K_T i_i - A_i(\dot{q}_i) v_i - D_i v_i - M_i \dot{\bar{\alpha}}_i - F_{id}, \tag{15}$$

$$L_i \dot{Z}_{i,4} = u_i - R_{ai} i_i - N K_E v_i - L_i \dot{\bar{i}} - u_{id}. \tag{16}$$

Step 1: Choosing the following Lyapunov function $V_1 = (1/2) \sum_{i=1}^{M} \left(Z_{i,1} + Z_{i,2}^2 \right)$, substituting (7) and (9) into (13) and (14), the time derivative of V_1 becomes

$$\begin{aligned} \dot{V}_1 = \sum_{i=1}^{M} \big[& -k_{i,1} Z_{i,1} - k_{i,2} Z_{i,2}^2 - \eta_i^T \mathrm{X}_{i,1} \left(Z_{i,3} + E_{i,1} \right) + \bar{R}_i Z_{i,2} \left(Z_{i,3,1} + E_{i,1,1} \right. \\ & \left. - Z_{i,3,2} - E_{i,1,2} \right) \big], \end{aligned} \tag{17}$$

where $\bar{R}_i = 0.5 B_i r_i R_i^{-1}$.

Step 2: Consider $\bar{V}_2 = V_1 + V_2$, combining (3) and (5) yields (15), (16) and (17), and the time derivative of $\bar{V}_2$ is

$$\begin{aligned} \dot{\bar{V}}_2 = \sum_{i=1}^{M} \Big[& -k_{i,1} Z_{i,1} - k_{i,2} Z_{i,2}^2 - \eta_i^T \mathrm{X}_{i,1} \left(Z_{i,3} + E_{i,1} \right) \\ & + \bar{R}_i Z_{i,2} \left(Z_{i,3,1} + E_{i,1,2} - Z_{i,3,2} - E_{i,1,2} \right) \\ & + Z_{i,3}^T \left(N K_T i_i - A_i(\dot{q}_i) v_i - D_i v_i - M_i \dot{\bar{\alpha}}_i - F_{id} \right) \\ & + Z_{i,4}^T \Big(u_i - R_{ai} i_i - N K_E v_i - L_i \dot{\bar{i}} - u_{id} \Big) \Big]. \end{aligned} \tag{18}$$

Note that

$$-\eta_i^T \mathrm{X}_{i,1} Z_{i,3} + \bar{R}_i Z_{i,2} \left(Z_{i,3,1} - Z_{i,3,2} \right) = Z_{i,3}^T \Psi_i, \tag{19}$$

where $\Psi_i = [\Psi_{i,1}, \Psi_{i,2}]^T = -\mathrm{X}_{i,1}^T \eta_i + \bar{R}_i Z_{i,2} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Using Young's inequality and (19), substitute (10) into (18) to get

$$\begin{aligned} \dot{\bar{V}}_2 \leq \Bigg[& \sum_{i=1}^{M} -k_{i,1} Z_{i,1} - k_{i,2} Z_{i,2}^2 - k_{i,3} Z_{i,3}^T Z_{i,3} - k_{i,4} Z_{i,4}^T Z_{i,4} - \eta_i^T \mathrm{X}_{i,1} E_{i,1} \\ & + \bar{R}_i Z_{i,2} \left(E_{i,1,1} - E_{i,1,2} \right) + \frac{\chi_{i\Xi} + \chi_{i\Gamma}}{2} + Z_{i,3}^T \left(\Xi_i(\vartheta_i) - \hat{\Xi}_i \left(\vartheta_i \middle| \hat{W}_i \right) \right) \\ & + Z_{i,4}^T \left(\Gamma_i(\vartheta_i) - \hat{\Gamma}_i \left(\vartheta_i \middle| \hat{\mathrm{K}}_i \right) \right) \Bigg], \end{aligned} \tag{20}$$

where $\chi_i = \chi_{i\Xi} + \chi_{i\Gamma}$, $\chi_i > 0$ is a constant, $\Xi_i(\vartheta_i) = \Psi_i - A_i(\dot{q}_i) v_i - D_i v_i - H_i \dot{\bar{\alpha}}_i + \frac{\bar{F}_{id}^2 Z_{i,3}}{2\chi_{i\Xi}}$, $\Gamma_i(\vartheta_i) = -NK_E v_i - L_i \dot{\bar{i}} + \frac{\bar{u}_{id}^2 Z_{i,4}}{2\chi_{i\Gamma}}$, we will use FLSs to estimate the unknown nonlinear function using $\Xi_i(\vartheta_i) = \hat{\Xi}_i\left(\vartheta_i \middle| \hat{W}_i\right) + \tilde{W}_i^T \Theta_i + \gamma_{i\Xi}$, $\Gamma_i(\vartheta_i) = \hat{\Gamma}_i\left(\vartheta_i \middle| \hat{\mathrm{K}}_i\right) + \tilde{\mathrm{K}}_i^T \Theta_i + \gamma_{i\Gamma} : \mathbb{R}^q \mapsto \mathbb{R}^2$, with $\tilde{W}_i = W_i^* - \hat{W}_i$, $\tilde{\mathrm{K}}_i = \mathrm{K}_i^* - \hat{\mathrm{K}}_i$, $i = 1, \ldots, N$, $\vartheta_i \in \kappa_{v_i} \subset \mathbb{R}^q$ are the input vectors of the function approximators.

Step 3: The time derivative of V along (11) and (20) is

$$\begin{aligned}\dot{V} \leq & \left[\sum_{i=1}^{M} -k_{i,1} Z_{i,1} - k_{i,2} Z_{i,2}^2 - k_{i,3} Z_{i,3}^T Z_{i,3} - k_{i,4} Z_{i,4}^T Z_{i,4} - \eta_i^T \mathrm{X}_{i,1} E_{i,1}\right. \\ & + \bar{R}_i Z_{i,2}\left(E_{i,1,1} - E_{i,1,2}\right) + \frac{\chi_i}{2} + \|Z_{i,3}\| \bar{\gamma}_{i\Xi} + \|Z_{i,4}\| \bar{\gamma}_{i\Gamma} \\ & + \sum_{m=1}^{2}\left(-\frac{E_{i,1,m}^2}{\iota_{i\alpha_i,m}} - E_{i,1,m}\Pi_{i,1,m}\right) + \sum_{m=1}^{2}\left(-\frac{E_{i,2,m}^2}{\iota_{ii_i,m}} - E_{i,2,m}\Pi_{i,2,m}\right) \\ & \left. + \sigma_{iW} tr\left(\tilde{W}_i^T \hat{W}_i\right) + \sigma_{iK} tr\left(\tilde{\mathrm{K}}_i^T \hat{\mathrm{K}}_i\right)\right], \end{aligned} \tag{21}$$

where $\Pi_{i,1,m}$, $\Pi_{i,2,m}$ are continuous functions which are related to the derivative of the virtual controllers $\alpha_{i,v}$, $i_{i,v}$, respectively.

Then, by using $-\eta_i^T \mathrm{X}_{i,1} E_{i,1} + \bar{R}_i Z_{i,2}(E_{i,1,1} - E_{i,1,2}) = \Psi_{i,1} E_{i,1,1} + \Psi_{i,2} E_{i,1,2}$, we can get

$$\begin{aligned}\dot{V} \leq \sum_{i=1}^{M} & \left[-k_{i,1} Z_{i,1} - k_{i,2} Z_{i,2}^2 - k_{i,3} Z_{i,3}^T Z_{i,3} - k_{i,4} Z_{i,4}^T Z_{i,4}\right. \\ & - \sum_{m=1}^{2}\left(-\frac{E_{i,1,m}^2}{\iota_{i\alpha_i,m}} - \frac{E_{i,1,m}^2 \bar{\Pi}_{i,1,m}^2}{2\chi_{i\Xi}}\right) - \sum_{m=1}^{2}\left(-\frac{E_{i,2,m}^2}{\iota_{ii_i,m}} - \frac{E_{i,2,m}^2 \Pi_{i,2,m}^2}{2\chi_{i\Gamma}}\right) \\ & \left. + \frac{Z_{i,3}^T Z_{i,3}}{2} + \frac{Z_{i,4}^T Z_{i,4}}{2} + \frac{\bar{\gamma}_{i\Xi}^2}{2} + \frac{\bar{\gamma}_{i\Gamma}^2}{2} + \sigma_{iW} tr\left(\tilde{W}_i^T \hat{W}_i\right) + \sigma_{iK} tr\left(\tilde{\mathrm{K}}_i^T \hat{\mathrm{K}}_i\right) + \frac{3}{2}\chi_i\right], \end{aligned} \tag{22}$$

where $\bar{\Pi}_{i,1,m} = \Pi_{i,1,m} + \Psi_{i,m}$. Consider the sets $O := \left\{q_0^T q_0 + \dot{q}_0^T \dot{q}_0 \leq 2\kappa\right\}$ and $\ell := \left\{\sum_{j=1}^{M} Z_{j,1} + Z_{j,2}^2 + Z_{j,3}^T Z_{j,3} + Z_{j,4}^T Z_{j,4} + E_{j,1,m} + E_{j,2,m}\right\}$, where $i = 1, \ldots, N$, $m = 1, 2$. Here, $\ell \times O$, is compact in R^{6M+6} and thus there exist constants $g_{i,1,m} > 0$ and $g_{i,2,m} > 0$ such that $\bar{\Pi}_{i,1,m} < g_{i,1,m}$ and $\Pi_{i,2,m} < g_{i,2,m}$ on $\ell \times O$. The $k_{i,3} = \frac{1}{2} + k_{i,3}^*$, $k_{i,4} = \frac{1}{2} + k_{i,4}^*$, $\frac{1}{\iota_{i\alpha_i,m}} = \frac{g_{i,1,m}^2}{2\chi_{i\Xi}}$, $\frac{1}{\iota_{ii_i,m}} = \frac{g_{i,2,m}^2}{2\chi_{i\Gamma}}$ with positive constants $k_{i,3}^*$, $k_{i,4}^*$, $\iota_{i\alpha_i,m}$ and $\iota_{ii_i,m}$ yields

$$\begin{aligned}\dot{V} \leq & \left[\sum_{i=1}^{M} - k_{i,1} Z_{i,1} - k_{i,2} Z_{i,2}^2 - k_{i,3}^* Z_{i,3}^T Z_{i,3} - k_{i,4}^* Z_{i,4}^T Z_{i,4} - \frac{\sigma_{iW}}{2}\left\|\tilde{W}_i\right\|_F^2 - \frac{\sigma_{iK}}{2}\left\|\tilde{\mathrm{K}}_i\right\|_F^2\right. \\ & - \sum_{m=1}^{2}\left(\iota_{i\alpha_i,m}^* E_{i,1,m}^2 + \left(1 - \frac{\bar{\Pi}_{i,1,m}^2}{g_{i,1,m}^2}\right)\frac{E_{i,1,m}^2 g_{i,1,m}^2}{2\chi_{i\Gamma}}\right) \\ & \left. - \sum_{m=1}^{2}\left(\iota_{ii_i,m}^* E_{i,2,m}^2 + \left(1 - \frac{\bar{\Pi}_{i,2,m}^2}{g_{i,2,m}^2}\right)\frac{E_{i,2,m}^2 g_{i,2,m}^2}{2\chi_{i\Gamma}}\right)\right] + \bar{\chi}, \end{aligned} \tag{23}$$

where $\bar{\chi} = \sum_{i=1}^{M}\left(\frac{3\chi_i}{2} + \frac{\sigma_{iW}\bar{W}_i^2}{2} + \frac{\sigma_{iK}\bar{\mathrm{K}}_i^2}{2}\frac{\bar{\gamma}_{i\Xi}^2}{2} + \frac{\bar{\gamma}_{i\Gamma}^2}{2}\right)$. When $V = \kappa$, $\bar{\Pi}_{i,1,m} < g_{i,1,m}$, $\Pi_{i,2,m} < g_{i,2,m}$ is satisfied, it holds that

$$\dot{V} \leq -CV + \bar{\chi}, \tag{24}$$

where C is chosen as $\min < 0 < C \leq \min\left[2k_{i,1}, 2k_{i,2}, \frac{2k^*_{i,3}}{M_{i,M}}, \frac{2k^*_{i,4}}{L_{i,M}}, \sigma_{iW}\xi_i, \sigma_{iK}\xi_i, 2\iota^*_{i\alpha_i,m}, 2\iota^*_{ii_i,m}\right]$. $M_{i,M}$, $L_{i,M}$ are the maximum eigenvalues of M_i, L_i, respectively. By this inequality, we can know that $\dot{V} < 0$ on $V = \kappa$ when $C > \frac{\bar{\chi}}{\kappa}$. So, $Z_{i,1}$ is bounded. And by Assumption 2.3 and (9), $-\delta_{i,j} < p_{i,j,1}(t) < 1$ to get $L_{\min} < l_{i,j}(t) < L_{\max}$ for all $t \geq 0$. Goal (i) is fully proven. The proof of goal (ii) is the same as the proof of goal (i).

From $\dot{V} \leq -CV + \bar{\chi}$, $Z_1 = [Z_{1,1}, \ldots, Z_{M,1}]^T$ exponentially converges to the compact set $\Omega = \left\{Z_1 \,\middle|\, \|Z_1\| \leq \frac{2\bar{\chi}}{C}\right\}$, we can adjust C to make Ω to be arbitrarily small, and then $\eta_{i,j}$ will be smaller arbitrarily. So $\lim_{t\to\infty} |l_{i,j}(t) - l_{i,j,d}| \leq \zeta_1$ and $\lim_{t\to\infty} |\phi_{i,j}(t) - \phi_{i,j,d}| \leq \zeta_2$ can be achieved. Goal (iii) is fully proven.

5. **Conclusions.** This paper has studied the formation trajectory tracking problem by a practical unified error transformation control approach for multi-agent systems with motor dynamics and external disturbances. The potential functions have been removed in this paper to simplify the controller design process by employing an error function using the relative distance and angle. Although the parameters in kinematics and dynamics of the mobile robot and the motor are uncertain, it is not effective for the design of the controller. Finally, the stability of the closed-loop system has been analyzed by Lyapunov theory. Input saturation can be further considered in future controller designs.

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