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TRENDS OF PAYOFFS OF AGENTS IN BAKERY GAME UNDER NON-COOPERATIVE ENVIRONMENT

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Abstract. *Strategic form game (SFG) has been used widely to model inter-related decision making. Generally, researchers work on a particular game, specified by certain actions and corresponding payoffs. In real world, the situation can be much more complex, and a particular game may not be enough. Furthermore, the actions and payoffs are not known a priori. Here, we consider a more realistic environment, where payoffs are to be optimally computed from given resources and be used by agent for making decision. We are interested in wider spectrum of outcomes in games, where payoffs can vary within a trend such that the agents' strategies remain unchanged. The results show that there exist certain ranges of resources that agents do not change their strategies. Hence, agents receive fair payoffs. Furthermore, taking into account additional computations that normally take place in real world environments does not affect the acceptable computation time for agent payoffs.*
Keywords: Fair payoff, Non-cooperative game, Payoff trends

1. **Introduction.** Multiagent system (MAS) is a very interesting area of research in artificial intelligence. MAS offers extensive potential for creating a decision support system in real world business domains, including supply chains, logistics, pricing, etc. Agents in these systems can be modeled as representatives of decision making units, e.g., persons, and companies. This helps us understand complex situations and make better decisions. As an underpinning mechanism of MAS, game theory (GT) [1] provides a solid basis to study agents' behaviors when their decisions are inter-related to each other, i.e., an agent's decision will affect other agents' decisions. GT can be divided into cooperative game and non-cooperative game. While the former allows for communications and negotiations among agents, the latter enforces agents to make decisions on their own. In reality, non-cooperative game has been studied extensively because it suits real world business very well. The most widely known, studied and adopted is strategic form game (SFG), where a set of actions and their associated payoffs are assigned to agents. The particular actions that agents finally decide to take and payoffs that they finally achieve are the outcome. The challenge in SFG research is to find whether the outcome is in the equilibrium [2], i.e., a stable state that agents will reach their optimal actions such that none of them can deviate and be better off.

Although SFG has been successfully used for making decisions in real world, there are important limits that prohibit us from potentially benefiting even much more with MAS and GT. While game theory assumes existing certain payoffs, we have to collect data and compute them quickly for payoffs in real world. Furthermore, there are a lot

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of uncontrollable factors that affect directly to the value of payoffs. Lastly, we need to analyze many "what-if" scenarios by hands, limited only to a handful of cases, while the possibilities can be millions cases more.

This research investigates such a setting, namely, Bakery Game [3], where a certain amount of resources are given to the agents, with the same technology matrix and price function but different amounts of resource. We investigate how much the resources can vary so that the agents' actions remain unchanged and their payoffs are affected minimally.

2. **Literature Review.** Having been used for modeling complex decision making for decades, STF has been extended [4] to be a little more flexible for managing modern organazation. Pricing strategy is a very challanging and important task in business. Agents are used to dynamically plan for pricing [5] interchangably. They observe the results of their on-line sales and adapt accordingly. Agents can be used to monitor other competitors that play their pricing strategies from on-line systems [6]. The observed data will be forwarded to further analysis. Game theory can also be used to model pricing strategy for port container facilitator [7].

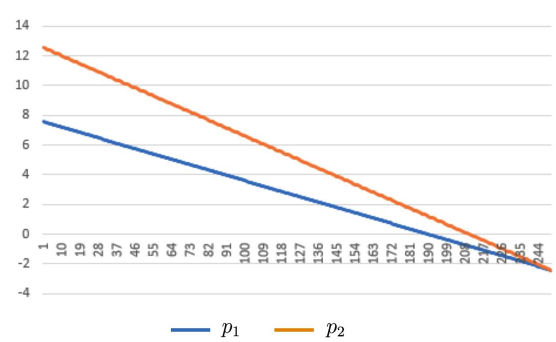
With similar ideas, the following works have deployed agents in supply chain management. To extensively understand complex decision making in supplying chain, the TAC supply chain management (TAC/SCM) [8] game provides a platform for experimenting strategies and consequential results. Bullwhip effect is a well known situation that occurs randomly but severely. Agents [9] are used to intellectually make decision by means of simulation. In modern business model, seeking cooperation among competitors in the same business sectors has also been studied. Strategic agents help seek social welfare [10] for conflicting parties. Li and Zhang [11] suggest a new model under which the shipping forwarders have admitted an opportunity to purchase shipping capacity from each other. Dobson and Chakraborty [12] model collaboration possibility amongst producers controlling the supply of essential complementary components that go into the assembly of competitively produced composite finished goods. Schleich et al. [13] assess collaborative performance on inventory adjustment among firms in business network. It is found that more collaboration can help increase their performance. Yan et al. [14] propose a solution that the manufacturer and e-tailer can agree to introduce the marketplace channel by considering dual upstream disadvantages. Allender et al. [15] enable targeted, or "personalized", pricing strategies by strategically obfuscate their prices so that direct interpersonal comparisons are more difficult. Liu et al. [16] offer fairness-efficiency solution for both stakeholders for urban renewal in China. Wu et al. [17] deal with the problem of allocation retrofit task and incentive among multiple stakeholders in China's energy suppliers. They are satisfied with stable and efficient strategies of incentive/task allocation. Game theory can be used to help increase the channel profits of participants in green supply chains [18], where participants are motivated to act promptly. For supply chains in niche markets, game theory can be used to help reach equilibrium [19].

As we can see here, making decision for modern business has been increasingly using agents and game theory.

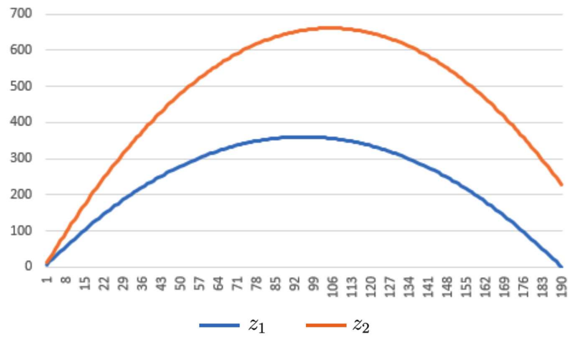
3. **Bakery Game.** Bakery game is an extension of linear production game (LPG) [20]. LPG illustrates a cooperative situation under superadditive environment where agents combine their resources to produce good and distribute the profit among themselves. The set of m agents is denoted by $A = \{a_1, a_2, \ldots, a_{\mathbf{m}}\}$. Resources, denoted by $R = \{r_1, r_2, \ldots, r_{\mathbf{n}}\}$, are distributed to agents. Goods, denoted by $G = \{g_1, g_2, \ldots, g_{\mathbf{o}}\}$, are produced by combining resources of agents because each unit of resources cannot be sold by itself. The *linear technology matrix*, denoted by $L = [\alpha_{i,j}]_{\mathbf{n}\times\mathbf{o}}$, where $\alpha_{i,j} \in \mathbb{Z}^+$, $1 \leq i \leq \mathbf{n}$ and $1 \leq j \leq \mathbf{o}$, defines units of each resource required for producing a unit of good. The price vector, denoted by $P = [p_j]_{1\times\mathbf{o}}$, specifies the unit price of each good. Lastly, the

number for resources given to each agent is defined by vector $B = [\beta_{i,k}]_{\mathbf{n}\times\mathbf{m}}$. Once agents form a coalition, $S \subseteq A$, the bundle resource ith is defined by $b_i = \sum_{1\leq k\leq \mathbf{m}} \beta_{i,k}$. The coalition S can use all these resources to produce any vector $x = \langle x_1, x_2, \ldots, x_{\mathbf{o}}\rangle$ of goods under these constraints: $\alpha_{1,1}x_1 + \alpha_{1,2}x_2 + \cdots + \alpha_{\mathbf{o},1}x_{\mathbf{o}} \leq b_1, \alpha_{2,1}x_1 + \alpha_{2,2}x_2 + \cdots + \alpha_{\mathbf{o},2}x_{\mathbf{o}} \leq b_2, \ldots, \alpha_{\mathbf{m},1}x_1 + \alpha_{\mathbf{m},2}x_2 + \cdots + \alpha_{\mathbf{o},\mathbf{m}}x_{\mathbf{o}} \leq b_{\mathbf{m}}$ and $x_1, x_2, \ldots, x_{\mathbf{o}} \geq 0$.

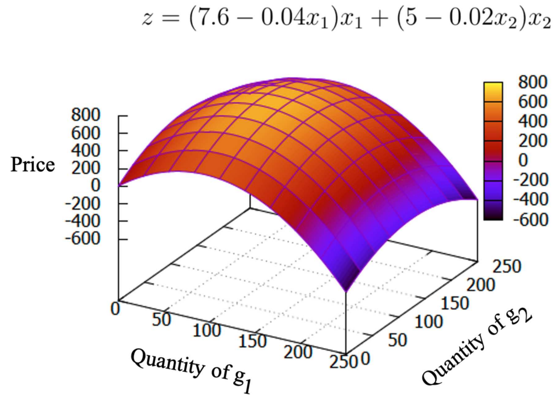
3.1. **Dynamic of prices.** The first price function is given as $d_1 = 190 - 25p_1$, where d_1 is the demand and p_1 is the price of g_1, respectively. We change the relation by focusing on how the changes of demand affect the price as the following: $25p_1 = 190 - d_1$, $p_1 = \frac{190-d_1}{25}$, and $p_1 = (7.6 - 0.04d_1)$. Similarly, the second price function is given as $d_2 = 250 - 50p_2$, where d_2 is the demand and p_2 is the price for g_2, respectively. We change the relation by focusing on how the changes of demand affect the price as the following: $50p_2 = 250 - d_2$, $p_2 = \frac{250-d_2}{50}$, and $p_2 = (5 - 0.02d_2)$. The dynamic behaviours of prices are shown in Figure 1(a), by increasing the number of goods one by one, both prices drop constantly. Note that the highest price of p_1 is 7.56 and the highest price of p_2 is 4.98. When the number of x_1 is 190, the price of p_1 becomes 0. When the number of x_2 is 250, the price of p_2 becomes 0. Therefore, it is commonly known to each agent that over supply can be harmful.



(a) Increasing number of goods lowers their prices



(b) Optimal plans of producing both goods individually



(c) Optimal plan for producing both goods combinedly

FIGURE 1. Dynamics of combined prices

3.2. **Dynamics of combined prices and profits.** Bakery game also brings in more complexity. While the price of each good drops constantly, the profits of both goods behave differently from prices. When the production is within requirements, $x_1 \leq D_1$ and $x_2 \leq D_2$, the products will be sold out. This implies that the profits of producing either g_1, denoted by z_1, and g_2, denoted by z_2, alone, are $x_1 + 25p_1 \leq 190$ and $x_2 + 50p_2 \leq 250$, respectively. Figure 1(b) shows such behaviors, rising to their peaks and drop because of over supply. When $g_1 = 95$ and $p_1 = 3.8$, good g_1's maximal profit, z_1, is 95. When

$g_2 = 125$ and $p_2 = 2.5$, good g_2's maximal profit, z_2, is 312.5. Both p_1 and p_2 drop to 0 when $g_1 = 190$ and $g_2 = 250$, respectively. Since an agent can produce both goods at the same time, the objective function is to maximize profit $z = p_1x_1 + p_2x_2$. The objective of all agents is to maximize their profit:

$$z = (7.6 - 0.04x_1)x_1 + (5 - 0.02x_2)x_2. \tag{1}$$

As shown in Figure 1(c), the maximal profit of 673.5 can be achieved. This figure is the global profit for all agents in the same market.

3.3. **Cost and net profit.** In reality, cooperating incurs costs, e.g., transportation cost. The cooperation cost among agents is specified by the matrix $C = [c_{k,l}]_{m\times m}$, which assigns a cooperation cost between each pair (a_k, a_l) of agents such that $c_{k,l} \in \mathbb{Z}^+$ if $k \neq l$, or $c_{k,l} \in \{0\}$ if $k = l$. We assume that all of the resources of agents are pooled at one location, which can be the location of any agent in the coalition. The total cost for cooperation incurred by a coalition will be taken to be the sum of the pairwise cooperation costs between the agent at whose location coalition resources are pooled, and the other members of coalition. For a coalition, there is at least one agent, a_k, such that $\sum_{k'=1}^{m} c_{kk'} \leq \sum_{l'=1}^{m} c_{ll'}$ for all $a_l \in S$. We shall call a coalition member a_k who yields the minimal cooperation cost for the coalition a *coalition center*. Agents in the coalition S have to find a vector x to maximize the revenue accruing to a coalition. Let $P_S = \sum_{l=1}^{o} p_l x_l$ be the maximal revenue the coalition can generate. Let $C_S = \sum_{l\in S} c_{kl}$ be the minimal cooperation cost for the coalition (obtained by selecting the optimal coalition center). Obviously, the ultimate objective of agents in the coalition is to maximize profit, i.e., the coalition value v_S, where

$$v_S = P_S - C_S. \tag{2}$$

This incurring cost decreases the net profit or the coalition value and makes the environment of the game becoming non-superadditive.

4. **Experiments, Results and Discussions.** In this section, we explore further to see the behavior of payoffs for agents in the aforementioned bakery game.

4.1. **Design of experiment.** Bakery game of non-cooperative agents forces agents not trying to produce too many goods. However, we need to assure we provide enough resources for all possible ranges of goods to be produced. Therefore, agents will have enough resources to maximize profits. This will allow us to see all possible outcomes of the games within certain trends. Trends allow us to observe certain outcomes consistently. Note that the price functions and other figures provided there and in other relations are merely a representation of similar situations in many environments. The principles studies in this work can be applied in those circumstances without loss of generality. In our experiments, we deliberately cover millions of cases but we choose only the extreme and obvious cases that satisfies our purposes for presenting in this research. Therefore, our setting must satisfy a number of requirements. i) The number of resources provided to agents must favor both products evenly. ii) The number of resources provided to agents must favor product p_1. iii) The number of resources provided to agents must favor product p_2.

4.2. **Resources and trends setting.** In this research, we generate number of resources, increasing one by one, and capture only extreme and obvious cases to present the behavior of agents. As shown in Table 1, we provide 10 units of r_1 and 20 units of r_2 that will favor both products evenly. We provide 17 units of r_1 and 5 units of r_2 that will favor product p_1 as required. Lastly, we provide 3 units of r_1 and 35 units of r_2 that will favor product p_2. Therefore, the ranges of resources are $r_1 = \{3, 10, 17\}$ and $r_2 = \{5, 20, 35\}$. The intervals of r_1 and r_2 are 7 and 15, respectively. Then we arrange them as combinations of five trends and three agents. The combinations are shown in Table 1. Note that Trend

TABLE 1. Resources provided to agents

			Trends					Changes			
								T-1 → T-2		T-1 → T-4	
			T-1	T-2	T-3	T-4	T-5	No.	%	No.	%
Resources	a_1	r_1	10	3	17	3	17	−7	−70	−7	70
		r_2	20	35	5	5	35	15	75	−15	−75
	a_2	r_1	10	10	10	10	10	0	0	0	0
		r_2	20	20	20	20	20	0	0	0	0
	a_3	r_1	10	17	3	17	3	7	70	7	70
		r_2	20	5	35	35	5	−15	−75	15	75

T-1 is the reference of Trend T-2 and Trend T-4. While Trend T-3 and Trend T-5 are the reverse of Trend T-2 and Trend T-5, respectively.

4.3. **Games and strategies setting.** As it was shown in [3] that the grand coalition's value is the same for all trends, it is the best strategy for all agents. The coalition value of a_2 in all trends varies despite the same number of resources in all trends because of number of goods produced by other agents. Also, there is a diagonal similarity between Trend T-2, Trend T-3 and Trend T-4, Trend T-5. Based on the given resources in each game, there can be many possible plans for producing goods for each agent. Each of these plans can be considered a strategy of each agent in each game. Since there are so many possibilities, we consider only three strategies for each agent in each game. The first strategy is to produce only g_1. The second strategy is to optimally produce both g_1 and g_2. The third strategy is to produce only g_2. Hence, there are strategies $\{s_{1,1}, s_{1,2}, s_{1,3}\}$, $\{s_{2,1}, s_{2,2}, s_{2,3}\}$ and $\{s_{3,1}, s_{3,2}, s_{3,3}\}$ for agents a_1, a_2 and a_3, respectively. For each strategic profile, the optimal plan for each agent will be computed and the actual payoff will also be calculated taking account of the total amount of goods and respective unit prices. Note that the actual payoff for each agent may be less than its expected value. Given a strategic profile, the payoffs for all agents are the *payoff vector* (v_1, v_2, v_3). There are twenty-seven strategic profiles for each game. Strategic profile $(s_{1,1}, s_{2,1}, s_{3,1})$, indicating that agent a_1 plays $s_{1,1}$, a_2 plays $s_{2,1}$, a_3 plays $s_{3,1}$, is associated with payoff vector (v_1, v_2, v_3), indicating that payoffs for agents a_1, a_2 and a_3 are v_1, v_2 and v_3, respectively.

4.4. **Trend T-1 result and discussion.** Given resources in trend T-1, we deliberately compute for the best plan for each agent as a sole seller and compute for agents' payoff in strategic form game. Figure 2 shows both agents' expected profits as sole sellers in the market and agents' payoffs as players in game of trend T-1. As a sole seller in the market, a_1, a_2, a_3 expect to produce 18 units of g_1, 17 units of g_2 and receive profit of 203.6. Since they are all in the market, we have to carefully consider the outcome of the game. Let us consider agent a_3. Assuming, agent a_1 plays $s_{1,1}$ and agent a_2 plays $s_{2,1}$, agent a_3's best strategy is $s_{3,1}$, receiving the highest payoff 115. This remains the same when agent a_2 plays $s_{2,2}$ or $s_{2,3}$. If a_1 plays $s_{1,2}$, strategy $s_{3,1}$ remains the best choice for a_3, no matter what a_2 plays. In other words, agent a_3 always plays $s_{3,1}$. By similarly analyzing the situation, the strategic profile $(s_{1,1}, s_{2,1}, s_{3,1})$ is the outcome and is also in NE. Their actual payoffs drop to 115 each.

4.5. **Trend T-2 result and discussion.** In trend T-2, the results are different from trend T-1, as shown in Figure 3. In case of sole seller, a_1 expects to receive profit of 105 by producing only 15 units of g_1, a_2 expects to receive profit of 165 by producing only 25 units of g_1, and a_3 expects to receive profit of 47.88 by producing only 16 units of g_2. In case of game trend T-2, we carefully analyze and receive the same outcome $(s_{1,1}, s_{2,1}, s_{3,1})$. However, the payoff vector is $(86.4, 144, 34.56)$.

Sole Seller Trend T-1

Strategies		g_1	g_2	total
a_1	$s_{1,1}$	50	0	165
	$s_{1,2}$	18	17	203.06
	$s_{1,3}$	0	25	112.5
a_2	$s_{2,1}$	25	0	165
	$s_{2,2}$	18	17	203.6
	$s_{2,3}$	0	25	112.5
a_3	$s_{3,1}$	25	0	165
	$s_{3,2}$	18	17	203.6
	$s_{3,3}$	0	25	112.5

Game Trend T-1

				a_3		
				$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1	$s_{1,1}$	a_2	$s_{2,1}$	(**115**, 115, *115*)	122, 122, *87.84*	140, 140, *0*
			$s_{2,2}$	122, 87.84, 122	129, 92.88, 92.88	147, 105.84, 0
			$s_{2,3}$	140, 0, 140	147, 0, 105.84	165, 0, 0
	$s_{1,2}$	a_2	$s_{2,1}$	**87.84,** 122, 122	92.88, 129, 92.88	113.04, 157, 0
			$s_{2,2}$	92.88, 92.88, 129	97.92, 97.92, 97.92	110.88, 110.88, 0
			$s_{2,3}$	105.84, 0, 147	110.88, 0, 110.88	123.84, 0, 0
	$s_{1,3}$	a_2	$s_{2,1}$	**0**, 105.84, 147	0, 110.88, 110.88	0, 123.84, 0
			$s_{2,2}$	0, 0, 165	0, 0, 123.84	0, 0, 0
			$s_{2,3}$	0, 0, 165	0, 0, 123.84	0, 0, 0

FIGURE 2. Agents' expected profits as sole sellers in the market (left) and agents' payoffs as players in game (right) of trend T-1

Sole Seller Trend T-2

Strategies		g_1	g_2	total
a_1	$s_{1,1}$	15	0	105
	$s_{1,2}$	8	7	92.26
	$s_{1,3}$	0	15	70.5
a_2	$s_{2,1}$	25	0	165
	$s_{2,2}$	13	12	149.16
	$s_{2,3}$	0	25	112.5
a_3	$s_{3,1}$	6	0	44.16
	$s_{3,2}$	3	2	32.36
	$s_{3,3}$	0	16	47.88

Game Trend T-2

				a_3		
				$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1	$s_{1,1}$	a_2	$s_{2,1}$	(**86.4**, 144, *34.56*)	88.2, 147, *17.64*	90, 150, *0*
			$s_{2,2}$	93.6, 81.12, 37.44	95.4, 82.68, 19.08	97.2, 84.24, 0
			$s_{2,3}$	101.4, 0, 40.56	103.2, 0, 20.64	105, 0, 0
	$s_{1,2}$	a_2	$s_{2,1}$	**48.32,** 151, 36.24	49.28, 154, 18.48	50.24, 157, 0
			$s_{2,2}$	52.16, 84.76, 39.12	53.12, 86.32, 19.92	54.08, 87.88, 0
			$s_{2,3}$	56.32, 0, 42.24	57.28, 0, 21.48	58.24, 0, 0
	$s_{1,3}$	a_2	$s_{2,1}$	**0**, 159, 38.16	0, 162, 19.44	0, 165, 0
			$s_{2,2}$	0, 81.12, 37.44	0, 90.48, 20.88	0, 92.04, 0
			$s_{2,3}$	0, 0, 44.16	0, 0, 22.44	0, 0, 0

FIGURE 3. Agents' expected profits as sole sellers in the market (left) and agents' payoffs as players in game (right) of trend T-2

TABLE 2. Comparison of trend T-1 and trend T-2

Goods								Sole seller				Game			
g_1		Changes		g_2		Changes		Profit		Changes		Payoff		Changes	
T-1	T-2	Amt	%	T-1	T-2	Amt	%	T-1	T-2	Amt	%	T-1	T-2	Amt	%
18	8	−10	−55.56	17	7	−10	−58.82	203.06	92	−110.8	−55	155	86.4	−68.6	−44
18	13	−5	−27.78	17	12	−5	−29.41	203.06	149	−53.9	−27	155	144	−11	−7.1
18	0	−18	−100	17	16	−1	−5.88	203.06	48	−155.18	−76	155	34.56	−120	−78

4.6. **Comparison of trend T-1 and trend T-2.** The results of trend T-1 and trend T-2 are carefully computed and compared, as shown in Table 2. From trend T-1 to trend T-2, the number of a_1's resource r_1 decreases by 70%, but the number of a_1's resource r_2 increases by 75%. By considering as a sole seller, the number of g_1 decreases by 56%, 28%, and 100% for a_1, a_2 and a_3, respectively. The number of g_2 decreases by 58.824%, 29.412%, and 5.8824% for a_1, a_2 and a_3, respectively. However, if we consider the situation from game theory perspective, we have different views. In trend T-1, the best plan is to produce only product g_1 for 25 units. In trend T-2, the best plan is to produce only product g_1 for 15 units, in which the number of products is reduced by 40%. Given the best plan, if a_1 is the only seller in the market, it will be able to make profit of up to 165$ in trend T-1, but the profit will be decreased to 105$ in trend T-2, or profit decreases by 36.36%. However, we consider a game composed of three agents, a_1, a_2, a_3, whose outcome is in NE. Agent a_1's payoff decreases from 115$ in trend T-1 to 86$ in trend T-2, or payoff decreases by 25.22%. In trend T-1, agent a_1's payoff decreases from 165$ in the noncompetitive market to 115$ in the competitive market, or payoff decreases by 30.30%. In trend T-2, considering a_1's payoff, profit decreases from 105$ in the noncompetitive market, to 86$ in the competitive market, or payoff decreases by 22.09%. Since the potential to produce good of a_1 in trend T-1 under noncompetitive market decreases by 36.36% and decreases by 25.22% in competitive market, agent a_1's payoff decreases 30.30% in trend T-1. However, it decreases merely 22.09% in trend T-2, or a_1's payoff is 8.21% higher.

4.7. **Trend T-3 result and discussion.** In trend T-3, the results are reverse version of trend T-2, as shown in Figure 4. In case of sole seller, a_3 expects to receive profit of 105 by producing only 15 units of g_1, a_2 expects to receive profit of 165 by producing only 25 units of g_1, and a_1 expects to receive profit of 74.88 by producing only 16 units of g_2. In case of game trend T-3, we carefully analyze and receive the same outcome $(s_{1,1}, s_{2,1}, s_{3,1})$. However, the payoff vector is $(34.56, 144, 86.4)$.

Sole Seller Trend T-3

Strategies		g_1	g_2	total
a_1	$s_{1,1}$	6	0	44.16
	$s_{1,2}$	3	2	32.36
	$s_{1,3}$	0	16	74.88
a_2	$s_{2,1}$	25	0	165
	$s_{2,2}$	13	12	149.16
	$s_{2,3}$	0	25	112.5
a_3	$s_{3,1}$	15	0	105
	$s_{3,2}$	8	7	92.26
	$s_{3,3}$	0	15	70.5

Game Trend T-3

				a_3		
				$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1	$s_{1,1}$	a_2	$s_{2,1}$	(**34.56**, 144, *86.4*)	36.24, 151, *48.32*	38.16, 159, *0*
			$s_{2,2}$	37.44, 81.12, 93.6	39.12, 84.76, 52.16	41.04, 88.92, 0
			$s_{2,3}$	40.56, 0, 101.4	42.24, 0, 56.32	44.16, 0, 0
	$s_{1,2}$	a_2	$s_{2,1}$	**17.64**, 147, 88.2	18.48, 154, 49.28	19.44, 162, 0
			$s_{2,2}$	19.08, 82.68, 95.4	19.92, 86.32, 53.12	20.88, 90.48, 0
			$s_{2,3}$	20.64, 0, 103.2	21.48, 0, 57.28	22.44, 0, 0
	$s_{1,3}$	a_2	$s_{2,1}$	**0**, 150, 90	0, 157, 50.24	0, 165, 0
			$s_{2,2}$	0, 84.24, 97.2	0, 87.88, 54.08	0, 92.04, 0
			$s_{2,3}$	0, 0, 105	0, 0, 58.24	0, 0, 0

FIGURE 4. Agents' expected profits as sole sellers in the market (left) and agents' payoffs as players in game (right) of trend T-3

4.8. **Trend T-4 result and discussion.** In trend T-4, the results are different from trend T-1, trend T-2 and trend T-3, as shown in Figure 5. In case of sole seller, a_1 expects to receive profit of 70.5 by producing only 16 units of g_2, a_2 expects to receive profit of 200 by producing only 50 units of g_2, and a_3 expects to receive profit of 339.24 by producing 11 units of g_1 and 74 units of g_2. In case of game trend T-4, we carefully analyze and receive the same outcome $(s_{1,1}, s_{2,1}, s_{3,1})$. However, the payoff vector is $(27.84, 116, 199.52)$.

Sole Seller Trend T-4

Strategies		g_1	g_2	total
a_1	$s_{1,1}$	6	0	44.16
	$s_{1,2}$	3	2	32.36
	$s_{1,3}$	0	16	70.5
a_2	$s_{2,1}$	25	0	165
	$s_{2,2}$	13	12	149.16
	$s_{2,3}$	0	50	200
a_3	$s_{3,1}$	43	2	262.76
	$s_{3,2}$	22	21	244.02
	$s_{3,3}$	11	74	339.24

Game Trend T-4

				a_3		
				$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1	$s_{1,1}$	a_2	$s_{2,1}$	(**27.84**, 116, *199.52*)	32.88, 137, *120.56*	35.52, 148, *65.12*
			$s_{2,2}$	30.72, 66.56, 220.16	35.76, 77.48, 131.12	38.4, 83.2, 70.4
			$s_{2,3}$	33.84, 0, 242.52	38.88, 0, 142.56	41.52, 0, 76.12
	$s_{1,2}$	a_2	$s_{2,1}$	**17.64**, 147, 252.84	16.8, 140, 123.2	18.12, 151, 66.44
			$s_{2,2}$	15.12, 65.52, 216.72	18.24, 79.04, 133.76	19.56, 84.76, 71.72
			$s_{2,3}$	17.28, 0, 247.68	19.8, 0, 145.2	21.12, 0, 77.44
	$s_{1,3}$	a_2	$s_{2,1}$	**0**, 122, 209.84	0, 143, 125.84	0, 154, 67.76
			$s_{2,2}$	0, 69.68, 230.48	0, 80.6, 136.4	0, 86.32, 73.04
			$s_{2,3}$	0, 0, 88.2	0, 0, 53.76	0, 0, 78.76

FIGURE 5. Agents' expected profits as sole sellers in the market (left) and agents' payoffs as players in game (right) of trend T-4

4.9. **Comparison of trend T-1 and trend T-4.** The results of trend T-1 and trend T-4 are carefully computed and compared, as shown in Table 3. From trend T-1 to trend T-4, the number of a_1's resource r_1 increases by 70%, but the number of a_1's resources r_2 decreases by 75%. By considering as a sole seller alone, the number of g_1 decreases by 100%, increases by 139%, and decreases by 39% for a_1, a_2 and a_3, respectively. The number of g_2 decreases by 5.8824%, decreases by 88.235%, and increases by 335.29% for a_1, a_2 and a_3, respectively. However, if we consider the situation from game theory perspective, we have different views. In trend T-4, the best plan is to produce only product

TABLE 3. Comparison of trend T-1 and trend T-4

Goods								Sole seller				Game			
g_1		Changes		g_2		Changes		Profit		Changes		Payoff		Changes	
T-1	T-4	Amt	%	T-1	T-4	Amt	%	T-1	T-4	Amt	%	T-1	T-4	Amt	%
18	0	−18	−100	17	16	−1	−5.88	203.06	71	−132.56	−65	155	27.84	−127	−82
18	43	25	138.9	17	2	−15	−88.24	203.06	149	−53.9	−27	155	116	−39	−25
18	11	−7	−38.89	17	74	57	335.29	203.06	339	136.18	67.1	155	199.52	44.52	29

g_1 for 6 units. In trend T-1, the best plan is to produce only product g_1 for 25 units, in which the number of products increases by 76%. Given the best plan, if a_1 is the only seller in the market, it will be able to make profit of up to 42$ in trend T-4. However, the profit will be increased to 165$ in trend T-1, or profit increases by 74.55%. However, we consider a game of agents a_1, a_2, a_3 whose outcome is in NE, agent a_1's payoff increases from 28$ in trend T-4 to 115$ in trend T-1, or payoff increases by 75.65%. In trend T-4, considering agent a_1's payoff decreases from 42$ in the noncompetitive market to 28$ in the competitive market, or payoff decreases by 50%. In trend T-1, considering a_1's payoff, profit decreases from 165$ in the noncompetitive market to 115$ in the competitive market, or payoff decreases by 30.30%. Since the potential to produce good of agent a_1 in trend T-4 under noncompetitive market decreases by 74.55% and decreases by 75.65% in competitive market, agent a_1's payoff decreases 30.30% in trend T-4. However, it decreases merely 30.30% in trend T-4 or agent a_1's payoff is 19.70% higher.

4.10. **Trend T-5 results and discussion.** In trend T-5, the results are reverse version of trend T-4, as shown in Figure 6. In case of sole seller, a_3 expects to receive profit of 70.5 by producing only 15 units of g_2, a_2 expects to receive profit of 200 by producing only 50 units of g_2, and a_1 expects to receive profit of 339.24 by producing 11 units of g_1 and 74 units of g_2. In case of game trend T-4, we carefully analyze and receive the same outcome $(s_{1,1}, s_{2,1}, s_{3,1})$. However, the payoff vector is $(199.52, 116, 27.84)$.

Sole Seller Trend T-5

Strategies		g_1	g_2	total
a_1	$s_{1,1}$	43	2	262.76
	$s_{1,2}$	22	21	244.02
	$s_{1,3}$	11	74	339.24
a_2	$s_{2,1}$	25	0	165
	$s_{2,2}$	13	12	149.16
	$s_{2,3}$	0	50	200
a_3	$s_{3,1}$	6	0	44.16
	$s_{3,2}$	3	2	32.36
	$s_{3,3}$	0	15	70.5

Game Trend T-5

				a_3		
				$s_{3,1}$	$s_{3,2}$	$s_{3,3}$
a_1	$s_{1,1}$	a_2	$s_{2,1}$	(**199.52**, <u>116</u>, *27.84*)	204.68, 119, *14.28*	209.84, 122, *0*
			$s_{2,2}$	220.16, <u>66.56</u>, 30.72	225.32, 68.12, 15.72	230.48, 69.68, 0
			$s_{2,3}$	242.52, <u>0</u>, 33.84	247.68, 0, 17.28	252.84, 0, 0
	$s_{1,2}$	a_2	$s_{2,1}$	**120.56**, 137, 32.88	123.2, 140, 16.8	125.84, 143, 0
			$s_{2,2}$	131.12, 77.48, 35.76	133.76, 79.04, 18.24	136.4, 80.6, 0
			$s_{2,3}$	142.56, 0, 38.88	145.2, 0, 19.8	147.84, 0, 0
	$s_{1,3}$	a_2	$s_{2,1}$	**65.12**, 148, 35.52	66.44, 151, 18.12	67.76, 154, 0
			$s_{2,2}$	70.4, 83.2, 38.4	71.72, 84.76, 19.56	73.04, 86.32, 0
			$s_{2,3}$	76.12, 0, 41.52	77.44, 0, 21.12	78.76, 0, 0

FIGURE 6. Agents' expected profits as sole sellers in the market (left) and agents' payoffs as players in game (right) of trend T-5

5. **Conclusion.** We study non-cooperative bakery game. A wide range of amount of resources is divided into 5 trends. T-1 is used as a reference. Trends (T-2, T-3) and (T-4, T-5) are diagonally similar. Given certain technology matrix and price functions, we find that within our settings agents' strategies remain unchanged even though resources vary up to 75%. Furthermore, agents' payoffs change relatively small. In the future, this research can be extended to consider more complex situations with more details. While a small number of agents and actions are used in this research, there should be more agents and actions involved. Furthermore, there could be algorithms working on other aspects, including efficiency, etc.

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