ICIC Express Letters
Volume **17**, Number **4**, April **2023**

ICIC International ©2023 ISSN 1881-803X
pp. 381–387

BIPOLAR FUZZY ALMOST INTERIOR IDEALS IN SEMIGROUPS

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Received April 2022; accepted July 2022

Abstract. *In this paper, we define bipolar fuzzy almost interior ideals and bipolar fuzzy weakly almost interior ideals in semigroups. We study the basic properties of bipolar fuzzy almost interior ideals and bipolar fuzzy weakly almost interior ideals in semigroups. Finally, we give some relationship between almost interior ideals (weakly almost interior ideals) and bipilar fuzzy almost interior ideals (bipolar fuzzy weakly almost interior ideals) of semigroups.*
Keywords: Almost interior ideal, Bipolar fuzzy almost interior ideal in semigroups, Bipolar fuzzy weakly almost interior ideal

1. **Introduction.** Fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague, which was introduced by Zadeh in 1965 [1]. In 1994, Zhang [2] extended the concept of the fuzzy set to bipolar fuzzy sets, which is an extension of fuzzy sets whose membership degree range is $[-1, 0] \cup [0, 1]$. A bipolar fuzzy set is the membership degree of an element which means that the element is irrelevant to the corresponding property, the membership degree of an element indicates that the element somewhat satisfies the property, and the membership degree of an element indicates that the element somewhat satisfies the implicit counter-property. The ideals introduced by her are still central concepts in ring theory. The notion of a one-sided ideal of any algebraic structure is a generalization of the idea of an ideal. The almost ideal theory in semigroups was studied by Grosek and Satko in 1980 [3]. In 1981, Bogdanovic, [4] established definitions of almost bi-ideals in semigroups and studied properties of almost bi-ideals in semigroups. Later in [5], Chinram et al. gave definitions of the types of almost ideals in semigroups such that almost quasi-ideal, almost i-ideal, (m, n)-almost ideal. Krishna and Rao gave a definition of the bi-interior ideal in semigroups in 2018 [6]. In 2020, Kaopusek et al. [7] introduced almost interior ideal and weakly almost interior ideals in semigroups and studied the relationship between almost interior ideals and weakly almost interior ideals in semigroups. The work of almost ideals was sutudied in semihypergroups such that in 2021, Muangdoo et al. [11] studied almost bi-hyperideals and their fuzzification of semihypergroups. Rao et al. [12] studied A-Γ-hyperideals and (m, n)-Γ-hyperfilters in ordered Γ-semihypergroups. Nakkhasen et al. [13] discussed fuzzy almost interior ideal hyperideals of semihypergroups.

In this paper, we give a definition of bipolar fuzzy almost interior ideal and bipolar fuzzy weakly almost interior ideal in semigroups. We investigate the basic properties of bipolar fuzzy almost interior ideal and bipolar fuzzy weakly almost interior ideal in semigroups. Finally, we give some relationship between almost interior ideals (weakly almost interior ideals) and bipilar fuzzy almost interior ideals (bipolar fuzzy weakly almost interior ideals) of semigroups.

DOI: 10.24507/icicel.17.04.381

2. **Preliminaries.** In this section, we give some concepts and results, which will be helpful in later sections. A *subsemigroup* of a semigroup E is a non-empty subset K of E such that $KK \subseteq K$. A *left (right) ideal* of a semigroup E is a non-empty subset K of E such that $EK \subseteq K$ ($KE \subseteq K$). By an *ideal* K of a semigroup E, we mean a non-empty set of E which is both a left and a right ideal of E. A subsemigroup K of a semigroup E is called an *interior ideal* of S if $EKE \subseteq K$. An *almost ideal* of a semigroup E is a non-empty subset K of E such that $tK \cap K \neq \emptyset$ and $Kr \cap K \neq \emptyset$ for all $t, r \in E$. An *almost interior ideal* of a semigroup E is a non-empty subset K of E such that $tKr \cap K \neq \emptyset$ for all $t, r \in E$. A *weakly almost ideal* K of a semigroup E is a non-empty subset K of E such that $tKt \cap K \neq \emptyset$ and for all $t \in E$ [7].

Theorem 2.1. [7] *Let K be non-empty subset of a semigroup E. Then the following statments hold.*

(1) Every interior ideal K of E is an almost interior ideal of E.
(2) Every alomst interior ideal K of E is a weakly almost interior ideal of E.

For any $h_i \in [0,1]$, $i \in \mathcal{F}$, define

$$\underset{i\in\mathcal{F}}{\vee} h_i := \sup_{i\in\mathcal{F}}\{h_i\} \text{ and } \underset{i\in\mathcal{F}}{\wedge} h_i := \inf_{i\in\mathcal{F}}\{h_i\}.$$

We see that for any $h, r \in [0,1]$, we have

$$h \vee r = \max\{h, r\} \text{ and } h \wedge r = \min\{h, r\}.$$

A **fuzzy set (fuzzy subset)** of a non-empty set E is a function $\vartheta : E \to [0,1]$.

For any two fuzzy sets ϑ and ξ of a non-empty set E, define the symbol as follows:

(1) $\vartheta \geq \xi \Leftrightarrow \vartheta(h) \geq \xi(h)$ for all $h \in E$,
(2) $\vartheta = \xi \Leftrightarrow \vartheta \geq \xi$ and $\xi \geq \vartheta$,
(3) $(\vartheta \wedge \xi)(h) = \min\{\vartheta(h), \xi(h)\} = \vartheta(h) \wedge \xi(h)$ for all $h \in E$,
(4) $(\vartheta \vee \xi)(h) = \max\{\vartheta(h), \xi(h)\} = \vartheta(h) \vee \xi(h)$ for all $h \in E$,
(5) $\vartheta \subseteq \xi$ if $\vartheta(h) \leq \xi(h)$,
(6) the *support* of ϑ instead of $\mathrm{supp}(\vartheta) = \{h \in E \mid \vartheta(h) \neq 0\}$. For the symbol $\vartheta \leq \xi$, we mean $\xi \geq \vartheta$.

Definition 2.1. [8] *A **bipolar fuzzy set** (BF set) ϑ on a non-empty set E is an object having the form*

$$\vartheta := \{(h, \vartheta^p(h), \vartheta^n(h)) \mid h \in E\},$$

where $\vartheta^p : E \to [0,1]$ and $\vartheta^n : E \to [-1,0]$.

Remark 2.1. *For the sake of simplicity, we shall use the symbol $\vartheta = (E; \vartheta^p, \vartheta^n)$ for the BF set $\vartheta = \{(h, \vartheta^p(h), \vartheta^n(h)) \mid h \in E\}$.*

The following is an example of a BF set.

Example 2.1. *Let $E = \{41, 42, 43, \ldots\}$. Define $\vartheta^p : S \to [0,1]$ as a function*

$$\vartheta^p(u) = \begin{cases} 0 & \text{if } h \text{ is odd number} \\ 1 & \text{if } h \text{ is even number} \end{cases}$$

and $\vartheta^n : S \to [-1,0]$ as a function

$$\vartheta^n(u) = \begin{cases} -1 & \text{if } h \text{ is odd number} \\ 0 & \text{if } h \text{ is even number} \end{cases}.$$

Then, $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF set.

For $h \in E$, define $F_h = \{(h_1, h_2) \in E \times E \mid h = h_1 h_2\}$.

Define products $\vartheta^p \circ \xi^p$ and $\vartheta^n \circ \xi^n$ as follows: For $h \in E$

$$(\vartheta^p \circ \xi^p)(h) = \begin{cases} \bigvee_{(h_1,h_2)\in F_h} \{\vartheta^p(h_1) \wedge \xi^p(h_2)\} & \text{if } h = h_1h_2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$(\vartheta^n \circ \xi^n)(h) = \begin{cases} \bigwedge_{(h_1,h_2)\in F_h} \{\vartheta^n(h_1) \vee \xi^n(h_2)\} & \text{if } h = h_1h_2 \\ 0 & \text{otherwise} \end{cases}.$$

Definition 2.2. [8] *Let K be a non-empty set of a semigroup E. A* ***positive characteristic function and a negative characteristic function*** *are respectively defined by*

$$\lambda_K^p : E \to [0,1], \quad h \mapsto \lambda_K^p(h) := \begin{cases} 1 & h \in K \\ 0 & h \notin K \end{cases}$$

and

$$\lambda_K^n : E \to [-1,0], \quad h \mapsto \lambda_K^n(h) := \begin{cases} -1 & h \in K \\ 0 & h \notin K \end{cases}.$$

Remark 2.2. *For the sake of simplicity, we shall use the symbol $\lambda_K = (E; \lambda_K^p, \lambda_K^n)$ for the BF set $\lambda_K := \{(h, \lambda_K^p(h), \lambda_K^n(h)) \mid h \in K\}$.*

For $h \in E$ and $(t,s) \in [0,1] \times [-1,0]$, a BF point $h_{(t,s)} = (E; x_t^p, x_s^n)$ of a set E is a bipolar set of E defined by

$$x_t^p(h) = \begin{cases} t & \text{if } h = x \\ 0 & \text{if } h \neq x \end{cases}$$

and

$$x_s^n(h) = \begin{cases} s & \text{if } h = x \\ 0 & \text{if } h \neq x \end{cases}.$$

Definition 2.3. [9] *A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a* ***BF subsemigroup*** *on E if it satisfies the following conditions: $\vartheta^p(hr) \geq \vartheta^p(h) \wedge \vartheta^p(r)$ and $\vartheta^n(hr) \leq \vartheta^n(h) \vee \vartheta^n(r)$ for all $h, r \in E$.*

The following is an example of a BF subsemigroup.

Example 2.2. *Let E be a semigroup defined by the following table:*

·	a	b	c	d	e
a	a	a	a	a	a
a	a	a	a	a	a
c	a	a	c	c	e
d	a	a	c	d	e
e	a	a	c	c	e

Define a BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on E as follows:

S	a	b	c	d	e
ϑ^p	*0.9*	*0.8*	*0.5*	*0.3*	*0.3*
ϑ^n	−0.8	−0.8	−0.6	−0.5	−0.3

Then, $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF subsemigroup.

Definition 2.4. [9] *A BF set $\vartheta = (E; \vartheta_p, \vartheta_n)$ on a semigroup E is called a* ***BF left (right) ideal*** *on E if it satisfies the following conditions: $\vartheta^p(hr) \geq \vartheta^p(r)$ ($\vartheta^p(hr) \geq \vartheta^p(h)$) and $\vartheta^n(hr) \leq \vartheta^n(r)$ ($\vartheta^n(hr) \leq \vartheta^n(h)$) for all $h, r \in E$.*

Definition 2.5. [9] *A BF subsemigroup $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a* ***BF interior ideal*** *on E if $\vartheta^p(hre) \geq \vartheta^p(r)$ and $\vartheta^n(hre) \leq \vartheta^n(r)$ for all $h, r, e \in E$.*

Definition 2.6. [10] *A BF subset $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is said to be a* ***BF weakly interior ideal*** *of E if $\vartheta^p(hre) \geq \vartheta^p(r)$ and $\vartheta^n(hre) \leq \vartheta^n(r)$ for all $h, r, e \in E$.*

Remark 2.3. *Every BF almost interior ideal of a semigroup is a BF weakly almost interior ideal of a semigroup. However, the converse of this statement is not true.*

Theorem 2.2. [10] *In left (right) regular, regular, intra-regular, semisimple or weakly regular semigroup E, the BF weakly interior ideals and BF interior ideals coincide.*

Theorem 2.3. [10] *Every BF left (right) ideal of a semigroup E is a BF weakly interior ideal of E.*

3. **Main Results.** In this section, we define the bipolar fuzzy almost interior ideals and bipolar fuzzy almost weakly interior ideals in semigroups.

Definition 3.1. *A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a* BF almost interior ideal *of E if $\left(x_t^p \circ \vartheta^p \circ y_{t'}^p\right) \wedge \vartheta^p \neq 0$ and $\left(x_s^n \circ \vartheta^n \circ y_{s'}^n\right) \vee \vartheta^n \neq 0$ for any BF point $x_t^p, y_{t'}^p, x_s^n, y_{s'}^n \in E$.*

Definition 3.2. *A BF set $\vartheta = (E; \vartheta^p, \vartheta^n)$ on a semigroup E is called a* BF weakly almost interior ideal *of E if $\left(x_t^p \circ \vartheta^p \circ x_{t'}^p\right) \wedge \vartheta^p \neq 0$ and $\left(x_s^n \circ \vartheta^n \circ x_{s'}^n\right) \vee \vartheta^n \neq 0$ for any BF point $x_t^p, x_{t'}^p, x_s^n, x_{s'}^n \in E$.*

It is clear that every BF almost interior ideal of a semigroup E is a BF weakly almost interior ideal of E.

Theorem 3.1. *If $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost interior ideal (weakly almost interior ideal) of a semigroup E and $\xi = (E; \xi^p, \xi^n)$ is a BF subset of E such that $\vartheta \subseteq \xi$, then ξ is a BF almost interior ideal (weakly almost interior ideal) of E.*

Proof: Suppose that $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost interior ideal of a semigroup E and $\xi = (E; \xi^p, \xi^n)$ is a BF subset of E such that $\vartheta \subseteq \xi$. Then for any BF points $x_t^p, y_{t'}^p, x_s^n, y_{s'}^n \in E$, we obtain that $\left(x_t^p \circ \vartheta^p \circ y_{t'}^p\right) \wedge \vartheta^p \neq 0$ and $\left(x_s^n \circ \vartheta^n \circ y_{s'}^n\right) \vee \vartheta^n \neq 0$. Thus,

$$\left(x_t^p \circ \vartheta^p \circ y_{t'}^p\right) \wedge \vartheta^p \subseteq \left(x_t^p \circ \xi^p \circ y_{t'}^p\right) \wedge \xi^p \neq 0$$

and

$$\left(x_s^n \circ \vartheta^n \circ y_{s'}^n\right) \vee \vartheta^n \subseteq \left(x_s^n \circ \xi^n \circ y_{s'}^n\right) \vee \xi^n \neq 0.$$

Hence, $\left(x_t^p \circ \xi^p \circ y_{t'}^p\right) \wedge \xi^p \neq 0$ and $\left(x_s^n \circ \xi^n \circ y_{s'}^n\right) \vee \xi^n \neq 0$. Therefore, ξ is a BF almost interior ideal of E. □

Theorem 3.2. *Let $\vartheta = (E; \vartheta^p, \vartheta^n)$ and $\xi = (E; \xi^p, \xi^n)$ be BF almost interior ideals (weakly almost interior ideals) of a semigroup E. Then $\vartheta \vee \xi$ is also a BF almost interior ideal (weakly almost interior ideal) of E.*

Proof: Since $\vartheta \subseteq \vartheta \vee \xi$, by Theorem 3.1, $\vartheta \vee \xi$ is also a BF almost interior ideal of E. □

Theorem 3.3. *Let K be a nonempty subset of a semigroup E. Then K is an almost interior ideal (weakly almost interior ideal) of E if and only if $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a BF almost interior ideal (weakly almost interior ideal) of E.*

Proof: Suppose that K is an almost interior ideal of a semigroup E. Then $xKy \wedge K \neq \emptyset$ for all $x, y \in E$. Thus, there exists $c \in E$ such that $c \in xKy$ and $c \in K$. Let $x, y \in E$ and $t, t' \in (0,1]$ and $s, s' \in [-1, 0)$. Then $\left(x_t^p \circ \lambda_K^p \circ y_{t'}^p\right)(c) \neq 0$, $\left(x_s^n \circ \lambda_K^n \circ y_{s'}^n\right)(c) \neq 0$ and $\lambda_K^p(c) = 1$ and $\lambda_K^n(c) = -1$. Thus, $\left(x_t^p \circ \lambda_K^p \circ y_{t'}^p\right) \wedge \lambda_K^p \neq 0$, $\left(x_s^n \circ \lambda_K^n \circ y_{s'}^n\right) \vee \lambda_K^n \neq 0$. Hence, $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a BF almost interior ideal of E.

Conversely, suppose that $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a BF almost interior ideal of E and let $x, y \in E$ and $t, t' \in (0,1]$ and $s, s' \in [-1, 0)$. Then $\left(x_t^p \circ \lambda_K^p \circ y_{t'}^p\right) \wedge \lambda_K^p \neq 0$ and $\left(x_s^n \circ \lambda_K^n \circ y_{s'}^n\right) \vee \lambda_K^n \neq 0$. Thus, there exists $c \in E$ such that $\left(\left(x_t^p \circ \lambda_K^p \circ y_{t'}^p\right) \wedge \lambda_K^p\right)(c) \neq 0$ and $\left(\left(x_s^n \circ \lambda_K^n \circ y_{s'}^n\right) \vee \lambda_K^n\right)(c) \neq 0$. Hence, $c \in xKy \wedge K$. Therefore, $xKy \wedge K \neq \emptyset$. We conclude that K is an almost interior ideal of E. □

Theorem 3.4. *Let $\vartheta = (E; \vartheta^p, \vartheta^n)$ be a fuzzy subset of a semigroup E. Then $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost interior ideal (weakly almost interior ideal) of E if and only if* $\operatorname{supp}(\vartheta)$ *is an almost interior ideal (weakly almost interior ideal) of E.*

Proof: Assume that $\vartheta = (E; \vartheta^p, \vartheta^n)$ is a BF almost interior ideal of a semigroup E and let $x, y \in E$ and $t, t' \in (0,1]$ and $s, s' \in [-1, 0)$. Then $\left(x_t^p \circ \vartheta^p \circ y_{t'}^p\right) \wedge \vartheta^p \neq 0$ and $\left(x_s^n \circ \vartheta^n \circ y_{s'}^n\right) \vee \vartheta^n \neq 0$. Thus, there exists $z \in E$ such that $\left(\left(x_t^p \circ \vartheta^p \circ y_{t'}^p\right) \wedge \vartheta^p\right)(z) \neq 0$ and $\left(\left(x_s^n \circ \vartheta^n \circ y_{s'}^n\right) \vee \vartheta^n\right)(z) \neq 0$. So, $\vartheta^p(z) \neq 0$ and $\vartheta^n(z) \neq 0$ there exists $w \in E$ such that $z = xwy$ and $\vartheta^p(w) \neq 0$, and $\vartheta^n(w) \neq 0$. Thus, $\left(\left(x_t^p \circ \lambda_{\operatorname{supp}(\vartheta)}^p \circ y_{t'}^p\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^p\right)(z) \neq 0$ and $\left(\left(x_s^n \circ \lambda_{\operatorname{supp}(\vartheta)}^n \circ y_{s'}^n\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^n\right)(z) \neq 0$. Hence, $\left(x_t^p \circ \lambda_{\operatorname{supp}(\vartheta)}^p \circ y_{t'}^p\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^p \neq 0$ and $\left(x_s^n \circ \lambda_{\operatorname{supp}(\vartheta)}^n \circ y_{s'}^n\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^n \neq 0$. Therefore, $\lambda_{\operatorname{supp}(\vartheta)}^p$ is a BF almost interior ideal of E. By Theorem 3.3, $\operatorname{supp}(\vartheta)$ is an almost interior ideal of E.

Conversely, suppose that $\operatorname{supp}(\vartheta)$ is an almost interior ideal of E. By Theorem 3.3, $\lambda_{\operatorname{supp}(\vartheta)}^p$ is a BF almost interior ideal of E. Then for any BF points $x_t^p, y_{t'}^p, x_s^n, y_{s'}^n \in E$, we have $\left(x_t^p \circ \lambda_{\operatorname{supp}(\vartheta)}^p \circ y_{t'}^p\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^p \neq 0$ and $\left(x_s^n \circ \lambda_{\operatorname{supp}(\vartheta)}^n \circ y_{s'}^n\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^n \neq 0$. Thus, there exists $c \in E$ such that $\left(\left(x_t^p \circ \lambda_{\operatorname{supp}(\vartheta)}^p \circ y_{t'}^p\right) \wedge \lambda_{\operatorname{supp}(\vartheta)}^p\right)(c) \neq 0$ and $\left(\left(x_s^n \circ \lambda_{\operatorname{supp}(\vartheta)}^n \circ y_{s'}^n\right) \vee \lambda_{\operatorname{supp}(\vartheta)}^n\right)(c) \neq 0$. Hence, $\left(x_t^p \circ \lambda_{\operatorname{supp}(\vartheta)}^p \circ y_{t'}^p\right)(c) = 0$, $\lambda_{\operatorname{supp}(\vartheta)}^p(c) \neq 0$ and $\left(x_s^n \circ \lambda_{\operatorname{supp}(\vartheta)}^n \circ y_{s'}^n\right)(c) = 0$, $\lambda_{\operatorname{supp}(\vartheta)}^n(c) \neq 0$. Then there exists $b \in \operatorname{supp}(\vartheta)$ such that $c = xby$. Thus, $\vartheta^p(c) \neq 0$, $\vartheta^p(b) \neq 0$ and $\vartheta^n(c) \neq 0$, $\vartheta^n(b) \neq 0$. So, $\left(x_t^p \circ \vartheta^p \circ y_{t'}^p\right) \wedge \vartheta^p \neq 0$, $\left(x_s^n \circ \vartheta^n \circ y_{s'}^n\right) \vee \vartheta^n \neq 0$. Therefore, ϑ is a BF almost interior ideal of E. □

Next, we investigate minimal BF almost interior ideals (weakly almost interior ideals) in semigroups and study relationships between minimal almost interior ideals (weakly almost interior ideals) and minimal BF almost interior ideals (weakly almost interior ideals) of semigroups.

Definition 3.3. *An almost interior ideal (weakly almost interior ideal) K of a semigroup E is called* minimal *if for any almost interior ideal (weakly almost interior ideal) M of E if whenever $M \subseteq K$, then $M = K$.*

Definition 3.4. *A BF almost interior ideal (weakly almost interior ideal) $\vartheta = (E; \vartheta^p, \vartheta^n)$ of a semigroup E is called* minimal *if for any BF almost interior ideal (weakly almost interior ideal) $\xi = (E; \xi^p, \xi^n)$ of E if whenever $\xi \subseteq \vartheta$, then $sup(\xi) = sup(\vartheta)$.*

Theorem 3.5. *Let K be a nonempty subset of a semigroup E. Then K is a minimal almost interior ideal (weakly almost interior ideal) of E if and only if $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a minimal BF almost interior ideal (weakly almost interior ideal) of E.*

Proof: Assume that K is a minimal almost interior ideal of E. Then K is an almost interior ideal of E. Thus, by Theorem 3.3, $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a BF almost ideal of

E. Let $\xi = (E; \xi^p, \xi^n)$ be a BF almost interior ideal of E such that $\xi \subseteq \lambda_K$. Then $\text{supp}(\xi) \subseteq \text{supp}(\lambda_K) = K$. By Theorem 3.4, $\text{supp}(\xi)$ is an almost interior ideal of E. Since K is minimal, we have $\text{supp}(\xi) = K = \text{supp}(\lambda_K)$. Therefore, $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is minimal BF almost interior ideal of E.

Conversely, suppose that $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a minimal BF almost interior ideal of E. Then $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a BF almost interior ideal of E. Thus, by Theorem 3.3, K is an almost interior ideal of E. Let M be an almost interior ideal of E such that $M \subseteq K$. Then λ_M is a BF almost interior ideal of E such that $\lambda_M \subseteq \lambda_K$. Thus, $\text{supp}(\lambda_M) \subseteq \text{supp}(\lambda_K)$. Since $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a minimal BF almost interior ideal of E, we have $\text{supp}(\lambda_M) = \text{supp}(\lambda_K)$. Thus, $M = \text{supp}(\lambda_M) = \text{supp}(\lambda_K) = K$. Hence, K is minimal almost interior ideal of E. □

Corollary 3.1. *Let E be a semigroup. Then E has no proper almost interior ideal (weakly almost interior ideal) if and only if* $\text{supp}(\vartheta) = E$ *for every BF almost interior ideal (weakly almost interior ideal) $\vartheta = (E; \vartheta^p, \vartheta^n)$ of E.*

Proof: Suppose that E has no proper almost interior ideal and let $\vartheta = (E; \vartheta^p, \vartheta^n)$ be a BF almost interior ideal of E. Then by Theorem 3.4, $\text{supp}(\vartheta)$ is an almost interior ideal of E. By assumption, $\text{supp}(\vartheta) = E$.

Conversely, suppose that $\text{supp}(\vartheta) = E$ and K is a proper almost interior ideal of E. Then by Theorem 3.3, $\lambda_K = \left(E; \lambda_K^p, \lambda_K^n\right)$ is a BF almost interior ideal (weakly almost interior ideal) of E. Thus, $\text{supp}(\lambda_K) = K \neq E$. It is a contradiction. Hence, E has no proper alomst interior ideal. □

4. **Conclusion.** In this paper, we give the concept of BF almost interior ideals (weakly almost interior ideals) in semigroup and we study properties of BF almost interior ideals (weakly almost interior ideals) in semigroups. Moreover, we prove relationship between BF almost interior ideals (weakly almost interior ideals) and almost interior ideals (weakly almost interior ideals). In the future we extend to study other kinds of almost ideals and interval valued fuzzy set or class of kinds of fuzzy sets.

Acknowledgment. This work is partially supported by School of Science, University of Phayao. The author also gratefully acknowledge the helpful comments and suggestions of the reviewers, which have improved the presentation.

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