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ON COVERED LEFT IDEALS OF PARTIALLY ORDERED TERNARY SEMIGROUPS

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Abstract. *The concept of ideal theory plays a vital role in algebraic structures. In this paper, we investigate the concept of covered one-sided ideals of a partially ordered ternary semigroup. We show that the union of two covered left ideals of a partially ordered ternary semigroup T is also a covered left ideal of T if it is a proper subset of T and the intersection of two covered left ideals of a partially ordered ternary semigroup T is also a covered left ideal of T if it is nonempty. The main result is a characterization of a partially ordered ternary semigroup containing covered left ideals. The concept of covered right ideals of a partially ordered ternary semigroup can be defined dually and the results for partially ordered ternary semigroups containing covered right ideals are left-right dual.*

Keywords: Partially ordered ternary semigroups, Left ideals, Covered left ideals, Maximal left ideals, Greatest left ideals

1. **Introduction.** The theory of ternary algebraic systems was studied by Lehmer [1] in 1932, but before that such structures were studied by Kasner [2], who gave the idea of n-ary algebras. A ternary semigroup is a non-empty set together with an associative ternary operation. It is well-known that every semigroup can be reduced to a ternary semigroup, but a ternary semigroup generally needs not necessarily reduce to a semigroup. One of generalizations of a ternary semigroup, is so-called a partially ordered ternary semigroup (Shortly: po-ternary semigroup). A partially ordered ternary semigroup sometimes was said to be an ordered ternary semigroup. The structures of partially ordered ternary semigroups have been studied by many authors, for example, [3-11]. In ring theory, an ideal of a ring is a special subset of its elements. The study of ideals in rings is one of the important fields of research in ring theory. Similarly, ideal theory in semigroups is one of the important research fields. The notion of covered one-sided ideals in semigroups was

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first introduced by Fabrici [12] in 1981. This notion was applied to study in ordered semigroups by Changphas and Summaprab [13] in 2017. Later, Khan et al. [8] introduced the notion of covered lateral ideals in partially ordered ternary semigroups. One-sided ideals in ternary semigroups consist of left ideals, right ideals and lateral ideals. Since we want to completely study in covered one-sided ideals of partially ordered ternary semigroups, we will consider to study covered left ideals and covered right ideals of partially ordered ternary semigroups. They are motivated to study this paper. In this paper, the notion of covered left and right ideals of partially ordered ternary semigroups will be introduced, and some examples are also presented. The structure of partially ordered ternary semigroups containing covered left ideals will be studied. Moreover, the structure of partially ordered ternary semigroups containing covered right ideals can be considered dually. In Section 2, we recall some definitions and results of partially ordered ternary semigroups. The main results are contained in Section 3. In the main results, we focus only in the case of covered left ideals. We investigate conditions of the union and the intersection of two covered left ideals of a partially ordered ternary semigroup to be also a covered left ideal. We characterize a partially ordered ternary semigroup containing covered left ideals. Finally, we conclude in Section 4.

2. **Preliminaries.** In this section, we recall some definitions and results used throughout the paper.

A non-empty set T is called a *ternary semigroup* if there exists a ternary operation $[\,]: T \times T \times T \to T$, written as $(a, b, c) \longmapsto [abc]$, such that

$$[[abc]de] = [a[bcd]e] = [ab[cde]]$$

for all $a, b, c, d, e \in T$. For non-empty subsets A, B and C of a ternary semigroup $(T, [\,])$, we let

$$[ABC] := \{[abc] \mid a \in A, b \in B, c \in C\}.$$

For simplicity, a ternary operation $[\,]$ will be identified with a multiplication of three elements, i.e., $[abc]$ will be identified with abc (in case a, b and c elements) and $[ABC]$ will be identified with ABC (in case A, B and C are sets). In the case $A = \{a\}$, we write $\{a\}BC$ as aBC and similarly, in the cases $B = \{b\}$ or $C = \{c\}$, we respectively write AbC and ABc.

A *partially ordered ternary semigroup* $(T, [\,], \leq)$ is a ternary semigroup $(T, [\,])$ together with a partial order relation $\leq$ on T which is compatible with the ternary operation, i.e.,

$$a \leq b \Rightarrow axy \leq bxy,\ xay \leq xby \text{ and } xya \leq xyb$$

for all $a, b, x, y \in T$. Throughout this paper, we will write T for a partially ordered ternary semigroup, unless specified otherwise.

Let T be a partially ordered ternary semigroup. For a non-empty subset A of T, we denote by $(A]$ the subset of T defined by

$$(A] := \{t \in T \mid t \leq a \text{ for some } a \in A\}.$$

If $A = \{a\}$, then we write $(\{a\}]$ as $(a]$. A non-empty subset A of T is called a *left (right) ideal* of T if (1) $TTA \subseteq A$ ($ATT \subseteq A$) and (2) $A = (A]$, that is, for any $x \in A$ and $y \in T$, $y \leq x$ implies $y \in A$. A left (right) ideal A of T is said to be *proper* if $A \subset T$. The symbol $\subset$ stands for proper inclusion for sets. A proper left (right) ideal A of T is said to be *maximal* if for any left (right) ideal B of T such that $A \subseteq B \subseteq T$, then $A = B$ or $B = T$. If T has no proper left (right) ideal, then it is *left (right) simple*. Note that the union of left (right) ideals of T is also a left (right) ideal of T, and the intersection of left (right) ideals of T is also a left (right) ideal of T, if it is non-empty. For a non-empty subset A of T, the intersection of all the left (right) ideals of T containing A is said to be the *left (right) ideal of T generated by A*, which is denoted by $L(A)$. The left (right) ideal of T generated by $\{a\}$ where $a \in T$ is denoted by $L(a)$ and called the *principal left ideal* *generated by a*. Clearly, $L(a) = (a \cup TTa]$. Similarly, $R(a) = (a \cup aTT]$ is the *principal right ideal generated by a*.

Following in [6], if A, B and C are non-empty subsets of a partially ordered ternary semigroup T, then

(1) $A \subseteq (A]$.
(2) $A \subseteq B \Rightarrow (A] \subseteq (B]$.
(3) $((A]] = (A]$.
(4) $(A](B](C] \subseteq (ABC]$.
(5) $((A](B](C]] = ((A](B]C] = (AB(C]] = (ABC]$.
(6) $(A \cup B] = (A] \cup (B]$.
(7) $(A \cap B] \subseteq (A] \cup (B]$.

In particular, if A and B are left (right) ideals of T, then $(A \cap B] = (A] \cup (B]$.

(8) $(TTA]$ is a left ideal of T.
(9) $(ATT]$ is a right ideal of T.

3. **Main Results.** Hereafter, we deal only with covered left ideals of a partial ordered ternary semigroup because the results on covered right ideals of a partial ordered ternary semigroup are left-right dual.

Definition 3.1. *A proper left ideal L of a partial ordered ternary semigroup T is called a covered left ideal (CL-ideal) of T if $L \subseteq (TT(T - L)]$.*

We now provide some examples.

Example 3.1. *Let $T = \{a, b, c, d, e\}$ with the ternary operation on T by, for all $x, y, z \in T$, $xyz = x * (y * z)$, where the binary operation $*$ is defined by*

$*$	a	b	c	d	e
a	a	a	c	d	a
b	a	b	c	d	a
c	a	a	c	d	a
d	a	a	c	d	a
e	a	a	c	d	e

and the partial order is defined as

$$\leq = \{(a, a), (a, c), (a, d), (b, a), (b, b), (b, c), (b, d), (c, c), (c, d), (d, d), (e, a), (e, c), (e, d), (e, e)\}.$$

In [8], *T is a partially ordered ternary semigroup. We have that the proper left ideals of T are $L_1 = \{a, b, e\}$ and $L_2 = \{a, b, c, e\}$. We can deduce that the proper left ideals L_1 and L_2 are CL-ideals of T.*

Lemma 3.1. *If L_1 and L_2 are any different two proper left ideals of a partially ordered ternary semigroup T such that $L_1 \cup L_2 = T$, then both L_1 and L_2 are not CL-ideals of T.*

Proof: Assume that L_1 and L_2 are any different two proper left ideals of a partially ordered ternary semigroup T such that $L_1 \cup L_2 = T$. Thus, $T - L_1 \subseteq L_2$ and $T - L_2 \subseteq L_1$. If L_1 is a CL-ideal of T, then we have that

$$L_1 \subseteq (TT(T - L_1)] \subseteq (TTL_2] \subseteq (L_2] = L_2.$$

Since $L_1 \cup L_2 = T$, this implies that $T = L_2$, which is a contradiction. Thus, L_1 is not a CL-ideal of T. Next, suppose that L_2 is a CL-ideal of T, then we have that

$$L_2 \subseteq (TT(T - L_2)] \subseteq (TTL_1] \subseteq (L_1] = L_1.$$

Since $L_1 \cup L_2 = T$, it implies that $T = L_1$, which is a contradiction. Hence, L_2 is not a CL-ideal of T. $\square$

Example 3.2. *Let $T = \{a, b, c, d, e\}$ with the ternary operation on T by, for all $x, y, z \in T$, $xyz = x * (y * z)$, where the binary operation $*$ is defined by*

$*$	a	b	c	d	e
a	a	b	a	d	a
b	a	b	a	d	a
c	a	b	a	d	a
d	a	b	a	d	a
e	a	b	a	d	a

and the partial order is defined as

$$\leq = \{(a, a), (a, b), (a, c), (a, e), (b, b), (c, b), (c, c), (c, e), (d, d), (e, b), (e, e)\}.$$

In [8]*, T is a partially ordered ternary semigroup. We have that the proper left ideals of T are $L_1 = \{a\}$, $L_2 = \{a, c\}$, $L_3 = \{a, d\}$, $L_4 = \{a, c, d\}$, $L_5 = \{a, c, e\}$, $L_6 = \{a, b, c, e\}$ and $L_7 = \{a, c, d, e\}$. We can obtain that the proper left ideals L_1, L_2 and L_5 are CL-ideals of T. However, the proper left ideals L_3, L_4, L_6 and L_7 are not CL-ideals of T. By Lemma 3.1, we can conclude that a proper left ideal of T is not a CL-ideal of T.*

Corollary 3.1. *If a partially ordered ternary semigroup T contains more than one maximal left ideal, then all maximal left ideals are not CL-ideals of T.*

Proof: Assume that a partially ordered ternary semigroup T contains two maximal different proper left ideals L_1 and L_2. Since

$$TT(L_1 \cup L_2) = TTL_1 \cup TTL_2 \subseteq L_1 \cup L_2 \text{ and } (L_1 \cup L_2] = (L_1] \cup (L_2] = L_1 \cup L_2,$$

we have that $L_1 \cup L_2$ is a left ideal of T. Since $L_1 \subset L_1 \cup L_2$ and L_1 is a maximal left ideal of T, it follows that $L_1 \cup L_2 = T$. By Lemma 3.1, we infer that both L_1 and L_2 are not *CL*-ideals of T. □

Lemma 3.2. *If L is a left ideal of a partially ordered ternary semigroup T such that $L \subseteq (TTx]$ and $L \neq (TTx]$ for some $x \in T$, then L is a CL-ideal of T.*

Proof: Assume that L is a left ideal of a partially ordered ternary semigroup T such that $L \subseteq (TTx]$ and $L \neq (TTx]$ for some $x \in T$. If $x \in L$, we have that

$$(TTx] \subseteq (TTL] \subseteq (L] = L \subseteq (TTx].$$

Thus, $L = (TTx]$, which is a contradiction. So, we obtain that $x \in T - L$, and hence $L \subseteq (TTx] \subseteq (TT(T - L)]$. This implies L is a *CL*-ideal of T. □

Corollary 3.2. *A partially ordered ternary semigroup T in which an element x does not belong to $(TTx]$ contains a CL-ideal.*

Proof: Let $L = (TTx]$. Since $TTL \subseteq (T](T](TTx] \subseteq ((TTT)Tx] \subseteq (TTx] = L$ and $(L] = ((TTx]] = (TTx] = L$, it follows that L is a left ideal of T. If $x \notin L$, we obtain that L is a proper left ideal of T, and so $L = (TTx] \subseteq (TT(T - L)]$. Hence, L is a *CL*-ideal of T. □

The next theorem gives the condition of the union of any two *CL*-ideals of partially ordered ternary semigroups to be a *CL*-ideal.

Theorem 3.1. *Let L_1 and L_2 be any two CL-ideals of a partially ordered ternary semigroup T. If $L_1 \cup L_2 \neq T$, then $L_1 \cup L_2$ is also a CL-ideal of T.*

Proof: Assume that L_1 and L_2 are any two *CL*-ideals of a partially ordered ternary semigroup T such that $L_1 \cup L_2 \neq T$. By the proof of Corollary 3.1, $L_1 \cup L_2$ is a left ideal of T. By assumption, $L_1 \cup L_2$ is a proper left ideal of T. Since L_1 and L_2 are *CL*-ideals of T, we have that $L_1 \subseteq (TT(T - L_1)]$ and $L_2 \subseteq (TT(T - L_2)]$. Now, we will show that $L_1 \cup L_2$ is a *CL*-ideal of T. Let $x \in L_1 \cup L_2$. We consider two following cases.

Case 1: If $x \in L_1$, then $x \in (TT(T-L_1)]$, we have that $x \in (TTy]$ for some $y \in T-L_1$. If $y \in (T-L_1)-L_2$, we obtain that $y \in T-(L_1 \cup L_2)$. Hence, $x \in (TTy] \subseteq (TT(T-(L_1 \cup L_2))]$, and so $L_1 \subseteq (TT(T-(L_1 \cup L_2))]$. If $y \in (T-L_1) \cap L_2$, we have that $y \in T-L_1$ and $y \in L_2$. Thus, $y \in (TTz]$ for some $z \in T-L_2$. If $z \in L_1$, then $y \in (TTz] \subseteq (TTL_1] \subseteq (L_1] = L_1$, which contradicts to $y \in T-L_1$. So, we obtain that $z \in T-L_1$ and $z \in T-L_2$, it implies that $z \in T-(L_1 \cup L_2)$. Thus,

$$x \in (TTy] \subseteq (TT(TTz]] \subseteq ((T](T](TTz]] = ((TTT)Tz] \subseteq (TTz] \subseteq (TT(T-(L_1 \cup L_2))].$$

Hence, $L_1 \subseteq (TT(T-(L_1 \cup L_2))]$.

Case 2: If $x \in L_2$, we have that $x \in (TT(T-L_2)]$, hence $x \in (TTu]$ for some $u \in T-L_2$. If $u \in (T-L_2)-L_1$, then $u \in T-(L_2 \cup L_1)$. Thus, $x \in (TTu] \subseteq (TT(T-(L_1 \cup L_2))]$, and so $L_2 \subseteq (TT(T-(L_1 \cup L_2))]$. If $u \in (T-L_2) \cap L_1$, then $u \in L_1$. So, we have that $u \in (TTv]$ for some $v \in T-L_1$. If $v \in L_2$, then $u \in (TTv] \subseteq (TTL_2] \subseteq (L_2] = L_2$, which is a contradiction. Thus, $v \in (T-L_1) \cap (T-L_2) = T-(L_1 \cup L_2)$, and so

$$x \in (TTu] \subseteq (TT(TTu]] \subseteq ((T](T](TTu]] = ((TTT)Tu] \subseteq (TTu] \subseteq (TT(T-(L_1 \cup L_2))].$$

Hence, $L_2 \subseteq (TT(T-(L_1 \cup L_2))]$.

From both two cases, we infer that $L_1 \cup L_2 \subseteq (TT(T-(L_1 \cup L_2))]$. Therefore, $L_1 \cup L_2$ is a *CL*-ideal of T. □

Lemma 3.3. *If L_1 is a CL-ideal and L_2 is a left ideal of a partially ordered ternary semigroup T such that $L_1 \cap L_2 \neq \varnothing$, then $L_1 \cap L_2$ is a CL-ideal of T.*

Proof: Assume that L_1 is a *CL*-ideal and L_2 is a left ideal of a partially ordered ternary semigroup T such that $L_1 \cap L_2 \neq \varnothing$. Since $TT(L_1 \cap L_2) = TTL_1 \cap TTL_2 \subseteq L_1 \cap L_2$ and $(L_1 \cap L_2] = (L_1] \cap (L_2] = L_1 \cap L_2$, it follows that $L_1 \cap L_2$ is a left ideal of T. Since L_1 is a *CL*-ideal of T, we obtain that $L_1 \cap L_2$ is a proper left ideal of T and $L_1 \subseteq (TT(T-L_1)]$. Hence, $L_1 \cap L_2 \subseteq L_1 \subseteq (TT(T-L_1)] \subseteq (TT(T-(L_1 \cap L_2))]$. We can conclude that $L_1 \cap L_2$ is a *CL*-ideal of T. □

We can easily prove the following fact.

Theorem 3.2. *If L_1 and L_2 are any two CL-ideals of a partially ordered ternary semigroup T such that $L_1 \cap L_2 \neq \varnothing$, then $L_1 \cap L_2$ is also a CL-ideal of T.*

The following theorem gives the example of partially ordered ternary semigroups containing a *CL*-ideal.

Theorem 3.3. *If a partially ordered ternary semigroup T is not left simple such that there are not any two proper left ideals in which their intersection is empty, then T contains a CL-ideal.*

Proof: Assume that a partially ordered ternary semigroup T is not left simple such that there are not any two proper left ideals in which their intersection is empty. Since T is not left simple, then T contains a proper left ideal L. Now, we claim that $(TT(T-L)]$ is left ideal of T. Let $L_1 = (TT(T-L)]$. Consider

$$TTL_1 = TT(TT(T-L)] \subseteq (T](T](TT(T-L)] \subseteq ((TTT)T(T-L)] \subseteq (TT(T-L)] = L_1$$

and $(L_1] = ((TT(T-L)]] = (TT(T-L)] = L_1$. Thus, L_1 is a left ideal of T. Then by assumption, $L \cap L_1 \neq \varnothing$, $L \cap L_1$ is a proper left ideal of T. Since $L \cap L_1 \subseteq L$, we have that $T-L \subseteq T-(L \cap L_1)$. Thus,

$$L \cap L_1 \subseteq L_1 = (TT(T-L)] \subseteq (TT(T-(L \cap L_1))].$$

Hence, $L \cap L_1$ is a *CL*-ideal of T. □

A proper left ideal L of a partially ordered ternary semigroup T is called the *greatest left ideal* of T if it contains every proper left ideal of T. If a partially ordered ternary semigroup T contains the greatest left ideal, we denote the left ideal by L^*. In Example

3.1, it is observed that a partially ordered ternary semigroup T contains the greatest left ideal L^*, that is $L^* = L_2$. Moreover, L^* is a *CL*-ideal and we will prove in the following lemma.

Lemma 3.4. *Let L^* be the greatest left ideal of a partially ordered ternary semigroup T. If $\left(T^3\right] = T$, then L^* is a CL-ideal of T.*

Proof: Let L^* be the greatest left ideal of a partially ordered ternary semigroup T. By the proof of Theorem 3.3, $(TT(T - L^*)]$ is a left ideal of T. Thus, $(TT(T - L^*)] = T$ or $(TT(T - L^*)] \subseteq L^*$. We consider the three following cases.
Case 1: If $(TT(T - L^*)] = T$, then $L^* \subseteq (TT(T - L^*)]$. Hence, L^* is a *CL*-ideal.
Case 2: If $(TT(T - L^*)] = L^*$, then L^* is clearly a *CL*-ideal.
Case 3: If $(TT(T - L^*)] \subset L^*$, we have that

$$\begin{aligned}\left(T^3\right] = (TTT] &= (TT((T - L^*) \cup L^*)] \\ &= (TT(T - L^*) \cup TTL^*] \\ &= (TT(T - L^*)] \cup (TTL^*] \\ &\subset L^* \cup (L^*] \\ &= L^* \cup L^* \\ &= L^* \subset T.\end{aligned}$$

Thus, $\left(T^3\right] \subset T$, which contradicts to $\left(T^3\right] = T$. □

In Example 3.1, it is observed that a partially ordered ternary semigroup T contains the greatest left ideal L^* and every proper left ideal of T is a *CL*-ideal of T. In the following theorem we characterize a partially ordered ternary semigroup in which every proper left ideal is a *CL*-ideal.

Theorem 3.4. *Let T be a partially ordered ternary semigroup such that $T = \left(T^3\right]$ which satisfies just one of the two following conditions.*
(1) T contains the greatest left ideal L^*.
(2) For any proper left ideal L and for every principal left ideal $L(a) \subseteq L$, there exists $b \in T - L$ such that $L(a) \subseteq L(b)$.
Then every proper left ideal of T is a CL-ideal of T.

Proof: Let L be a proper left ideal of a partially ordered ternary semigroup T. First, we assume that T satisfies (1). Then $L \subseteq L^*$ and so $T - L^* \subseteq T - L$. Since $T = \left(T^3\right]$, then by Lemma 3.4, L^* is a *CL*-ideal of T. Thus, $L \subseteq L^* \subseteq (TT(T - L^*)] \subseteq (TT(T - L)]$. Hence, L is a *CL*-ideal of T.

Secondary, we assume that T satisfies (2). Let L be a proper left ideal of T and $a \in L$. Then $L(a) \subseteq L$. So, there exists $b \in T - L$ such that $L(a) \subseteq L(b)$. Since $T = \left(T^3\right]$, we have that $b \in (TTx]$ for some $x \in T$. Since $b \in (TTx]$, it follows that $(b] \subseteq ((TTx]] = (TTx]$ and $(TTb] \subseteq ((T](T](TTx]] = ((TTT)Tx] \subseteq (TTx]$. Thus, $L(b) = (b \cup TTb] = (b] \cup (TTb] \subseteq (TTx]$. If $x \in L$, then $b \in (TTx] \subseteq (TTL] \subseteq (L] = L$, which contradicts to $b \in T - L$. Thus, $x \in T - L$, and so $L(b) \subseteq (TTx] \subseteq (TT(T - L)]$. Hence, $a \in L(a) \subseteq L(b) \subseteq (TT(T - L)]$. This shows that $L \subseteq (TT(T - L)]$. Therefore, L is a *CL*-ideal of T. □

We end this paper with an example by illustrating the condition (2) of Theorem 3.4.

Example 3.3. *In Example 3.1, one may easily check that $\left(T^3\right] = T$. Consider the proper subset $L_2 = \{a, b, c, e\}$ of T. We have that L_2 is a proper left ideal of T such that $L(x) \subseteq L_2$ for all $x \in L_2$. For $d \in T - L_2$, $L(x) \subseteq L(d)$ for all $x \in T$. Thus, by the hypothesis of the condition (2) of Theorem 3.4, L_2 becomes CL-ideal of T.*

4. **Conclusions.** In this paper, we focus only the results for covered left ideals of a partially ordered ternary semigroup. For covered right ideals, we can show dually. We show that a proper left ideal of a partially ordered ternary semigroup is not a covered left ideal. In Theorem 3.1, we give the condition of the union of any two CL-ideals of partially ordered ternary semigroups to be a CL-ideal. In Theorem 3.2, we give the condition of the intersection of any two CL-ideals of partially ordered ternary semigroups to be a CL-ideal. As a main result, we give the necessary conditions for every proper left ideal of a partially ordered ternary semigroup is a covered left ideal.

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