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FAULT DETECTION AND ISOLATION OBSERVER DESIGN FOR A CLASS OF HIGH-ORDER DYNAMIC SYSTEMS

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Abstract. *This paper investigates the problem of fault detection and isolation for a class of high-order dynamic systems with actuator fault. Firstly, fault detection observer and fault decision mechanism are constructed to detect the fault in time. Secondly, a series of fault isolation observers are designed to isolate the fault, and fault isolation algorithms are given. Based on Lyapunov stability theory, the stability of dynamic observation errors is analyzed. Finally, the simulation results show the effectiveness of the proposed techniques.*
Keywords: High-order dynamic systems, Actuator faults, Fault detection, Fault isolation

1. **Introduction.** Fault detection and isolation of linear systems have attracted attention from control theory field in recent years. [1] studies sensor fault estimation and accommodation for switched linear systems. In [2], an observer-based fault detection method and a fault self-restore controller are proposed for linear systems to realize fault detection, isolation, and self-restore. A new sensor fault detection and diagnosis method for nonlinear systems is presented based on a pseudo deviation separation estimation algorithm and a Bayesian classification algorithm in [4]. In [5], an unknown input observer for a class of nonlinear systems for fault diagnosis. The input powers of the above systems, whether linear or nonlinear systems, are equal to 1. However, in practical applications, some high-order dynamic systems have an input power larger than 1, and this assumption may be unreasonable in some cases.

In recent years, many scholars have made in-depth research on the control problem of high-order dynamic systems. [6] studies the finite-time stabilization problem of high-order uncertain dynamic systems. [7] deals with the problem of distributed fault detection and isolation in multi-agent systems with disturbed high-order dynamics subject to communication uncertainties and faults. However, in real applications, faults may occur in the controlled systems and impose adverse effect on system performance. Thus, many effective fault-tolerant control (FTC) approaches have been proposed to improve the reliability and safety of the faulty systems in all situations [10-14]. With the development of FTC, the models for fault detection and isolation (FDI) [8,11] are constructed to detect and isolate the faults. However, these above works just focused on the systems where the systems input powers are equal to one. For the high-order system, one of the possible difficulties of fault diagnosis is that it is difficult to deal with the problem of additional items caused by $p > 1$. So how to design the fault detection and isolation observer for high-order systems and deal with the additional items is the goal of this paper.

In this paper, we investigate the fault detection and fault isolation problems of high-order dynamic systems with known system input power, and propose an adaptive control

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method to ensure the system is stable. Then, by using Lyapunov stability theory, it is proven that detection and isolation errors converge to the origin. Compared with the existing works in literature, the contributions in our work are generalized in the following aspects.

1) Different from the results in [9] where the assumption coefficient about system uncertainty must meet the conditions which are given that it should be larger than one, the control scheme in this paper removes this known condition. For practical application, the assumptions of this paper are more reasonable, and the theoretical results of this paper have more extensive practical value.

2) Unlike some of the existing literature where the methods of fault detection and isolation are only suitable for the system with system input powers equal to one, the method proposed in this paper is more reasonable, considering some of the system input power of high-order system is greater than one.

3) To solve the problem that the additional items caused by $p > 1$ are difficult to deal with, this paper introduces an auxiliary function to reduce the computational complexity.

The rest of this paper is organized as follows. In Section 2, the system formulation is introduced and the description of FLS is presented. In Section 3, main results are presented, which includes fault detection and fault isolation. Section 4 gives simulation results. These simulation results prove the effectiveness of the technique proposed in this paper. Finally, Section 5 draws the conclusion.

2. Problem Statement and Description of Fuzzy Logic System.

2.1. Problem statement.

Let us consider the following high-order dynamic systems:

$$\begin{cases} \dot{x} = Ax^p + Bu^p + B_d d(t) + f(x) \\ y = Cx \\ k = Cx^p \end{cases} \tag{1}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, y and $u^p = [u_1^p, u_2^p, \ldots, u_m^p]^T \in R^m$ respectively denote system's state, output and input; $p \in R > 0$ is the input power of the system, which is known bounded positive odd number; the unknown nonlinear term $d(t)$ is dynamic disturbances; $f(x) = [f_1, f_2, \ldots, f_n]^T$ are unknown smooth nonlinear functions; k is an auxiliary function to be designed later; the time-independent matrices A, B, B_d and C are known matrices with appropriate dimension.

Remark 2.1. *In (1), the positivity of k is used as an auxiliary function. The purpose of the auxiliary function is to ensure that the subtracted term containing the parameter L can be combined like terms with the formula in (8).*

In practical applications, control system components such as actuators may fail. In this paper, actuator fault has the following form,

$$u_i^f(t) = \rho_i(t)u_i(t), \quad i = 1, 2, \ldots, m \tag{2}$$

where unknown function ρ_i denotes the retained control rate, and $t \geq t_f$ is unknown fault occurrence time.

Control objective is to design an active FTC scheme so that the system (1) will be stable in all cases. Under normal conditions, the control law $u(t)$ is designed to ensure that the system (1) is stable. In the case of faulty, fault detection and isolation observer is designed to ensure system (1) is stable.

To design the observer, the following assumptions are given.

Assumption 2.1. *The known matrix B is full rank and $[A, C]$ is observable.*

Assumption 2.2. *The function $d(t)$ is unknown nonlinear function, but bounded, such that $\|d(t)\| \leq \bar{d}$, where $\bar{d}$ is known normal constant.*

Assumption 2.3. *There exist known constants* $\bar{\rho}_i$ *(*$i = 1, 2, \ldots, m$*) satisfying that* $|\rho_i(t)| \leq \bar{\rho}_1$*, where known real constants* $\bar{\rho}_1 > 0$.

Lemma 2.1. *For* $\alpha \in R^{n_a}$, $\beta \in R^{n_b}$, $M \in R^{n_a \times n_b}$ *and arbitrary matrices* $X \in R^{n_a \times n_a}$, $Y \in R^{n_a \times n_b}$, $Z \in R^{n_b \times n_b}$*, if there exists* $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$*, then we have*

$$-2\alpha^T M\beta \leq \begin{bmatrix} \alpha \\ \beta \end{bmatrix}^T \begin{bmatrix} X & Y - M \\ Y^T - M^T & Z \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$$

2.2. **Description of fuzzy logic system.** Fuzzy logic system includes four parts: the knowledge base, the fuzzifier, the fuzzy inference engine working on fuzzy rules and defuzzifier. Its knowledge base contains a series of fuzzy if-then rules that can be expressed in the following form:

$$R^l\text{: if } x_1 \text{ is } A_1^l \text{ and } x_2 \text{ is } A_2^l \ldots \text{ and } x_n \text{ is } A_n^l, \text{ then } y \text{ is } B^l,\ l = 1, 2, \ldots, M \tag{3}$$

where $\underline{x} = [x_1, x_2, \ldots, x_n]^T \in U \subset R^n$ and y are respectively the input and output of the fuzzy logic system; Fuzzy sets A_i^l and B^l are related to fuzzy function $\mu_{A_i^l} = \exp\left(-\left(\frac{x_i - a_i^l}{b_i^l}\right)^2\right)$ and $\mu_{B^l}\left(y^l\right) = 1$; M is the number of fuzzy rules. Applying singleton function, center average defuzzification and product inference, define $\bar{y}^l = \max_{y \in R} \mu_{B^l}$, the fuzzy logic system can be expressed as

$$y(x) = \sum_{l=1}^{M} \bar{y}^l \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right) \Bigg/ \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)$$

Define the fuzzy basis functions:

$$\xi^l(x) = \prod_{i=1}^{n} \mu_{A_i^l}(x_i) \Bigg/ \sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{A_i^l}(x_i)\right)$$

and define $\theta^T = \left[\bar{y}^1, \bar{y}^2, \ldots, \bar{y}^M\right] = [\theta_1, \theta_2, \ldots, \theta_M]$ and $\xi = \left[\xi^1, \xi^2, \ldots, \xi^M\right]^T$, then the fuzzy logic system can be expressed as

$$y(x) = \theta^T \xi(x) \tag{4}$$

Lemma 2.2. $f(x)$ *is a continuous function defined on a closed set* Ω*. Then, for any real constant* $\varepsilon > 0$*, there exists a fuzzy logic system such as* $\sup_{x \in \Omega} \left|f(x) - \theta^T \xi(x)\right| \leq \varepsilon$.

By Lemma 2.2, the FLS (4) can approximate any smooth function on a compact set to any degree of accuracy. In this paper, fuzzy logic system is used to approximate unknown continuous function $f_i(x)$, $i = 1, 2, \ldots, n$ as $\hat{f}_i\left(\hat{x}_i, \hat{\theta}_{f_i}\right) = \hat{\theta}_{f_i}^T \xi_{f_i}(\hat{x}_i)$, where $\hat{x}$, $\hat{\theta}_{f_i}$ are the estimates of x, $\theta_{f_i}^*$, respectively. Let us define the optimal parameter vectors $\theta_{f_i}^* = \arg\min_{\theta_{f_i} \in \Omega_{f_i}} \left[\sup_{x \in U, \hat{x} \in \hat{U}} \left|f_i(x_i) - \hat{f}_i(\hat{x}_i)\right|\right]$, where Ω_{f_i}, U and $\hat{U}$ are closed sets with respect to $\hat{\theta}_{f_i}$, x and $\hat{x}$. The minimum approximation errors can be defined as

$$\begin{aligned} \varepsilon_{f_i} &= f(x_i) - \theta_{f_i}^{*T} \xi_{f_i}(\hat{x}_i) \\ \delta_{f_i} &= f(x_i) - \hat{\theta}_{f_i}^T \xi_{f_i}(\hat{x}_i) \end{aligned} \tag{5}$$

Assumption 2.4. *There exist unknown constants* $\varepsilon_{f_i}^* > 0$ *and* $\delta_{f_i}^* > 0$*, known constants* $M_{\varepsilon_{f_i}} > 0$, $M_{\delta_{f_i}} > 0$ *and* $M_{\theta_{f_i}} > 0$, $|\varepsilon_{f_i}| \leq \varepsilon_{f_i}^*$, $\left|\varepsilon_{f_i}^*\right| \leq M_{\varepsilon_{f_i}}$, $|\delta_{f_i}| \leq \delta_{f_i}^*$, $\left|\delta_{f_i}^*\right| \leq M_{\delta_{f_i}}$ *and* $\left\|\theta_{f_i}^*\right\| \leq M_{\theta_{f_i}}$.

3. Main Results.

3.1. Fault detection.

To detect the fault occurring in the system, the following fault detection observer is designed as

$$\begin{cases} \dot{\hat{x}} = A\hat{x}^p + Bu^p + \hat{f}(\hat{x}) + L\left(k - \hat{k}\right) + \operatorname{sgn}\left(2e_D^T P\right)\hat{M}_{\varepsilon_f} \\ \hat{y} = C\hat{x} \\ \hat{k} = C\hat{x}^p \end{cases} \tag{6}$$

where $\hat{f} = \left[\hat{f}_1, \hat{f}_2, \ldots, \hat{f}_n\right]^T$, $\hat{M}_{\varepsilon_f} = \left[\hat{M}_{\varepsilon_{f_1}}, \hat{M}_{\varepsilon_{f_2}}, \ldots, \hat{M}_{\varepsilon_{f_n}}\right]$, $\hat{f}_i = \hat{\theta}_{f_i}^T \xi_{f_i}(\hat{x}_i)$ and $\hat{M}_{\varepsilon_{f_i}}$ are the estimates of ${\theta_{f_i}^*}^T \xi_{f_i}(\hat{x}_i)$ and $M_{\varepsilon_{f_i}}$, respectively; $e_D = [e_{D_1}, e_{D_2}, \ldots, e_{D_n}]^T$ is the fault detection observer error; L is the observer gain matrix with appropriate dimensions for the sth observer which is chosen to ensure that $A-LC$ is Hurwitz; P^i $(i = 1, 2, \ldots, n)$ represents the ith column of matrix P, $\operatorname{sgn}\left(2e_D^T P\right) = diag\left\{\operatorname{sgn}\left(2e_D^T P^1\right), \ldots, \operatorname{sgn}\left(2e_D^T P^n\right)\right\}$.

Define

$$\begin{aligned} e_D &= x - \hat{x} \\ \tilde{f} &= \left[\tilde{f}_1, \ldots, \tilde{f}_n\right]^T \end{aligned} \tag{7}$$

where $\tilde{f}_i = f_i - \hat{f}_i$, $i = 1, \ldots, n$. From (1) and (6), the error dynamic is obtained

$$\begin{aligned} \dot{e}_D &= (A - LC)e_{Dp} + B_d d + \tilde{f} - \operatorname{sgn}\left(2e_D^T P\right)\hat{M}_{\varepsilon_f} \\ &= (A - LC)e_{Dp} + B_d d + \tilde{\theta}^T \xi_f(\hat{x}) + \varepsilon_f - \operatorname{sgn}\left(2e_D^T P\right)\hat{M}_{\varepsilon_f} \end{aligned} \tag{8}$$

where $e_{D_p} = \left[e_{D_{p_1}}, e_{D_{p_2}}, \ldots, e_{D_{p_n}}\right]^T$, $e_{D_{p_i}} = x_i^p - \hat{x}_i^p$ $(i = 1, \ldots, n)$, ${\theta_f^*}^T = diag\{{\theta_{f_1}^*}^T, {\theta_{f_2}^*}^T, \ldots, {\theta_{f_n}^*}^T\}$, $\xi_f(\hat{x}) = [\xi_{f_1}(\hat{x}_1), \xi_{f_2}(\hat{x}_2), \ldots, \xi_{f_n}(\hat{x}_n)]^T$, $\varepsilon_f = [\varepsilon_{f_1}, \varepsilon_{f_2}, \ldots, \varepsilon_{f_n}]^T$, $\tilde{\theta}_f = \left[\tilde{\theta}_{f_1}, \tilde{\theta}_{f_2}, \ldots, \tilde{\theta}_{f_n}\right]^T$, $\tilde{\theta}_{f_l} = \theta_{f_l}^* - \hat{\theta}_{f_l}$, $l = 1, \ldots, n$.

Define the following Lyapunov function $V_1 = e_D^T P e_D$, differentiating V_1 with respect to time t, we have

$$\begin{aligned} \dot{V}_1 = {} & e_D^T\left[P(A - LC) + (A - LC)^T P\right]e_{Dp} + 2e_D^T P B_d d + 2e_D^T P\left(\tilde{\theta}^T \xi_f(\hat{x}) + \varepsilon_f\right) \\ & - 2e_D^T P \operatorname{sgn}\left(2e_D^T P\right)\hat{M}_{\varepsilon_f} \end{aligned} \tag{9}$$

From Assumption 2.4, we have

$$2e_D^T P\left(\tilde{\theta}^T \xi_f(\hat{x}) + \varepsilon_f\right) \le 2e_D^T P\left(\tilde{\theta}^T \xi_f(\hat{x}) + M_{\varepsilon_f}\right) \tag{10}$$

where $M_{\varepsilon_f} = \left[M_{\varepsilon_{f_1}}, M_{\varepsilon_{f_2}}, \ldots, M_{\varepsilon_{f_n}}\right]^T$. Substituting (10) into (9), we further have

$$\begin{aligned} \dot{V}_1 \le {} & e_D^T\left[P(A - LC) + (A - LC)^T P\right]e_{Dp} + 2e_D^T P B_d d + 2e_D^T P\tilde{\theta}^T \xi_f(\hat{x}) \\ & + \left|2e_D^T P\right| M_{\varepsilon_f} - 2e_D^T P \operatorname{sgn}\left(2e_D^T P\right)\hat{M}_{\varepsilon_f} \\ \le {} & e_D^T\left[P(A - LC) + (A - LC)^T P\right]e_{Dp} + 2e_D^T P B_d d + 2e_D^T P\tilde{\theta}^T \xi_f(\hat{x}) \\ & + 2e_D^T P \operatorname{sgn}\left(2e_D^T P\right)\tilde{M}_{\varepsilon_f} \end{aligned} \tag{11}$$

where $\tilde{M}_{\varepsilon_f} = \left[\tilde{M}_{\varepsilon_{f_1}}, \tilde{M}_{\varepsilon_{f_2}}, \ldots, \tilde{M}_{\varepsilon_{f_n}}\right]$, $\tilde{M}_{\varepsilon_{f_i}} = M_{\varepsilon_{f_i}} - \hat{M}_{\varepsilon_{f_i}}$, $i = 1, \ldots, n$.

Since Young's inequality, it follows that for any positive constant $r > 0$, one has

$$2e_D^T P B_d d \le r e_D^T P B_d B_d^T P e_D + \frac{1}{r}\|d\|^2 \le r e_D^T P B_d B_d^T P e_D + \frac{1}{r}\left(\bar{d}\right)^2 \tag{12}$$

Substituting (12) into (11) and from Lemma 2.1, one has

$$\dot{V}_1 \leq \left[\begin{array}{c} e_D \\ e_{Dp} \end{array}\right]^T \left[\begin{array}{cc} X + rPB_dB_d^TP & Y + P(A-LC) \\ Y^T + (A-LC)^TP & Z \end{array}\right] \left[\begin{array}{c} e_D \\ e_{Dp} \end{array}\right] + \frac{1}{r}\left(\bar{d}\right)^2 + 2e_D^TP\tilde{\theta}_f^T\xi_f + 2e_D^TP\mathrm{sgn}\left(2e_D^TP\right)\tilde{M}_{\varepsilon_f} \tag{13}$$

Obviously, if matrix $X, Y, Z, Q > 0$ and $P = P^T > 0$ are choosing such that $\left[\begin{array}{cc} X & Y \\ Y^T & Z \end{array}\right] > 0$ and

$$\left[\begin{array}{cc} X + rPB_dB_d^TP & Y + P(A-LC) \\ Y^T + (A-LC)^TP & Z \end{array}\right] < -Q \tag{14}$$

then we have

$$\dot{V}_1 \leq -\left[\begin{array}{c} e_D \\ e_{Dp} \end{array}\right]^T Q \left[\begin{array}{c} e_D \\ e_{Dp} \end{array}\right] + \frac{1}{r}\left(\bar{d}\right)^2 + 2e_D^TP\tilde{\theta}_f^T\xi_f + 2e_D^TP\mathrm{sgn}\left(2e_D^TP\right)\tilde{M}_{\varepsilon_f} \tag{15}$$

Choose adaptive laws as follows:

$$\dot{\hat{\theta}}_f = 2\eta_1 e_D^TP\xi_f - \eta_\theta\hat{\theta}_f \tag{16}$$

$$\dot{\hat{\varepsilon}}_f = 2\eta_2 e_D^TP\mathrm{sgn}\left(2e_D^TP\right) - \eta_\varepsilon\hat{M}_{\varepsilon_f} \tag{17}$$

where $\eta_1 > 0$, $\eta_\theta > 0$, $\eta_2 > 0$, $\eta_\varepsilon > 0$ are design parameters.

Define

$$V_D = V_1 + \frac{1}{2\eta_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{1}{2\eta_2}\tilde{M}_{\varepsilon_f}^2 \tag{18}$$

Differentiating V_D with respect to time t leads to

$$\dot{V}_D \leq -\left[\begin{array}{c} e_D \\ e_{D_p} \end{array}\right]^T Q \left[\begin{array}{c} e_D \\ e_{D_p} \end{array}\right] + \frac{1}{r}\left(\bar{d}\right)^2 + \tilde{\theta}_f^T\left(2e_D^TP\xi_f - \frac{1}{\eta_1}\dot{\hat{\theta}}_f\right) + \tilde{M}_{\varepsilon_f}\left(2e_D^TP\mathrm{sgn}\left(2e_D^TP\right) - \frac{1}{\eta_2}\dot{\hat{M}}_{\varepsilon_f}\right) \tag{19}$$

Substituting (16) and (17) into (19), we have

$$\dot{V}_D \leq -\left[\begin{array}{c} e_D \\ e_{D_p} \end{array}\right]^T Q \left[\begin{array}{c} e_D \\ e_{D_p} \end{array}\right] + \frac{1}{r}\left(\bar{d}\right)^2 + \frac{\eta_\theta}{\eta_1}\tilde{\theta}_f^T\hat{\theta}_f + \frac{\eta_\varepsilon}{\eta_2}\tilde{M}_{\varepsilon_f}\hat{M}_{\varepsilon_f} \tag{20}$$

Since

$$\begin{aligned} &\frac{\eta_\theta}{\eta_1}\tilde{\theta}_f^T\hat{\theta}_f \leq \frac{\eta_\theta}{\eta_1}\tilde{\theta}_f^T\left(\theta_f^* - \tilde{\theta}_f\right) \leq -\frac{\eta_\theta}{2\eta_1}\tilde{\theta}_f^T\tilde{\theta}_f + \frac{\eta_\theta}{2\eta_1}M_{\theta_f}^2 \\ &\frac{\eta_\varepsilon}{\eta_2}\tilde{M}_{\varepsilon_f}\hat{M}_{\varepsilon_f} \leq \frac{\eta_\varepsilon}{\eta_2}\tilde{M}_{\varepsilon_f}\left(M_{\varepsilon_f} - \tilde{M}_{\varepsilon_f}\right) \leq -\frac{\eta_\varepsilon}{2\eta_2}\tilde{M}_{\varepsilon_f}^2 + \frac{\eta_\varepsilon}{2\eta_2}M_{\varepsilon_f}^2 \end{aligned} \tag{21}$$

Then (20) can be rewritten as

$$\begin{aligned} \dot{V}_D &\leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}e_D^TPe_D - \frac{\eta_\theta}{2\eta_1}\tilde{\theta}_f^T\tilde{\theta}_f - \frac{\eta_\varepsilon}{2\eta_2}\tilde{M}_{\varepsilon_f}^2 + \frac{\eta_\theta}{2\eta_1}M_{\theta_f}^2 + \frac{\eta_\varepsilon}{2\eta_2}M_{\varepsilon_f}^2 + \frac{1}{r}\left(\bar{d}\right)^2 \\ &\leq -c_1V_D + \mu_1 \end{aligned} \tag{22}$$

where $c_1 = \min\left\{\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}, \frac{\eta_\theta}{2\eta_1}, \frac{\eta_\varepsilon}{2\eta_2}\right\}$, $\mu_1 = \frac{\eta_\theta}{2\eta_1}M_{\theta_f}^2 + \frac{\eta_\varepsilon}{2\eta_2}\left(M_{\varepsilon_f}\right)^2 + \frac{1}{r}\left(\bar{d}\right)^2$.

Theorem 3.1. *Considering system (1) and observer (6) under Assumptions 2.1-2.4, and the adaptive laws (16) and (17), if there exist matrices $X, Y, Z > 0$, $P = P^T > 0$, $Q > 0$ and positive constants r satisfying $\left[\begin{array}{cc} X & Y \\ Y^T & Z \end{array}\right] > 0$ and*

$$\left[\begin{array}{cc} X + rPB_dB_d^TP & Y + P(A-LC) \\ Y^T + (A-LC)^TP & Z \end{array}\right] < -Q \tag{23}$$

then the observer error is asymptotically stable and all signals in the closed-loop systems are bounded.

Proof: Since $\frac{d}{dt}(V(t){e_D}^{c_1 t}) \leq {e_D}^{c_1 t}\mu_1$, we have

$$0 \leq V_D(t) \leq \frac{\mu_1}{c_1} + \left[V_D(0) - \frac{\mu_1}{c_1}\right] {e_D}^{-c_1 t} \leq \frac{\mu_1}{c_1} + V_D(0) = \alpha_D$$

Hence, it can be known that the observer error is asymptotically stable. In addition, according to the definition of $V_D(t)$ defined in (18), $\hat{\theta}_f$, $\hat{M}_{\varepsilon_f}$ are bounded. Therefore, it means that all the signals in the closed-loop system are bounded and we have $\|e_D\| \leq \sqrt{\alpha_D/\lambda_{\min}(P)}$, $\left\|\tilde{\theta}_f\right\| \leq \sqrt{2\eta_1\alpha_D}$, $\left\|\tilde{M}_{\varepsilon_f}\right\| \leq \sqrt{2\eta_2\alpha_D}$.

The proof is completed.

According to Theorem 3.1, the following fault detection residuals are defined as

$$J(t) = \|y(t) - \hat{y}(t)\|$$

In the case of no fault, we have

$$J(t) \leq \|Ce_D\| \leq \|C\|\sqrt{\alpha_D/\lambda_{\min}(P)}$$

Further, the following mechanism is used for fault detection

$$\begin{cases} J(t) \leq T_D & \text{no fault occurred;} \\ J(t) > T_D & \text{fault has occurred} \end{cases} \tag{24}$$

where threshold value $T_D = \|C\|\sqrt{\alpha_D/\lambda_{\min}(P)}$.

3.2. **Fault isolation.** Because the system has m actuators and it is assumed that only one actuator becomes faulty at one time, we have m possible faulty cases in total. When the sth ($1 \leq s \leq m$) actuator is faulty, the faulty system can be described as

$$\begin{cases} \dot{x}_s = Ax_s^p + Bu^p - b_s\rho_s^p u_s^p + B_d d(t) + f_s(x_s) \\ y_s = Cx_s \\ k_s = Cx_s^p \end{cases} \tag{25}$$

where $x_s = [x_{s_1}, x_{s_2}, \ldots, x_{s_n}]^T$, $B = [b_1, b_2, \ldots, b_m]$, $b_s \in R^{n\times 1}$, $f_s(x_s) = [f_{s_1}, f_{s_2}, \ldots, f_{s_n}]^T$, ρ_s represents the failure of the sth actuator, u_s is the control input under normal conditions, $s = 1, 2, \ldots, m$.

When a fault has been detected, the fault isolation algorithm is activated. Now, design the following m fault isolation observers:

$$\begin{cases} \dot{\hat{x}}_s = A\hat{x}_s^p + L\left(k_s - \hat{k}_s\right) + Bu^p - b_l\mu_l\bar{\rho}_l^p|u_l|^p + \hat{f}_s\left(\hat{x}_s\right) + \operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}} \\ \hat{y}_s = C\hat{x}_s \\ \hat{k}_s = C\hat{x}_s^p \end{cases} \tag{26}$$

where $\hat{x}_s$ and $\hat{y}_s$ are the state and output of the sth observer, respectively; $\hat{f}_s\left(\hat{x}_s\right) = \left[\hat{f}_{s_1}, \hat{f}_{s_2}, \ldots, \hat{f}_{s_n}\right]^T$, $\hat{M}_{\varepsilon_{f_s}} = \left[\hat{M}_{\varepsilon_{f_{s_1}}}, \hat{M}_{\varepsilon_{f_{s_2}}}, \ldots, \hat{M}_{\varepsilon_{f_{s_n}}}\right]^T$ and $\hat{M}_{\varepsilon_{f_{s_i}}}$ is the estimate of $M_{\varepsilon_{f_{s_i}}}$, $i = 1, \ldots, n$, $\mu_s = -e_I^T P b_s$, $\bar{\rho}_s = \rho_1$ represents the upper bound on the gain failure of the lth actuator, $l = 1, \ldots, m$; L is the observer gain matrix and the appropriate dimension is chosen such that $A - LC$ is Hurwitz; $e_I = [e_{I_1}, e_{I_2}, \ldots, e_{I_n}]$, $e_{I_i} = x_{s_i} - \hat{x}_{s_i}$, $e_{I_p} = \left[e_{I_{p_1}}, e_{I_{p_2}}, \ldots, e_{I_{p_n}}\right]$, $e_{I_{p_i}} = x_{s_i}^p - \hat{x}_{s_i}^p$, $e_{y_I} = y_s - \hat{y}_s$.

In the following, l is used to denote the practical faulty case, namely, the faulty actuator is actuator l.

For $s = l$, the error dynamics between (25) and (26) is

$$\dot{e}_I = (A - LC)e_{Ip} - b_s\left(\rho_s^p u_s^p - \mu_s\bar{\rho}_s^p|u_s|^p\right) + B_d d(t) + \tilde{f}_I - \operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}} \tag{27}$$

and for $s \neq l$, we have

$$\dot{e}_I = (A - LC)e_{Ip} - (b_s\rho_s^p u_s^p - b_l\mu_l\bar{\rho}_l^p|u_l|^p) + B_d d(t) + \tilde{f}_I - \operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}} \tag{28}$$

where $\tilde{f}_I = \left[\tilde{f}_{I_1}, \ldots, \tilde{f}_{I_n}\right]^T$, $\tilde{f}_{I_i} = f_{I_i} - \hat{f}_{I_i}$, $i = 1, \ldots, n$.

1) For $s = l$, from (27), we have

$$\dot{e}_I = (A - LC)e_{Ip} - b_s\left(\rho_s^p u_s^p - \mu_s\bar{\rho}_s^p|u_s|^p\right) + B_d d(t) + \tilde{f}_I - \operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}}$$

Similar to the previous subsection, differentiating $V_2 = e_I^T P e_I$ with respect to time t, one has

$$\begin{aligned}\dot{V}_2 = {} & e_I^T\left[(A - LC)^T P + P(A - LC)\right]e_{Ip} + 2e_I^T P\left(B_d d + \tilde{f}\right) \\ & + 2e_I^T P b_s\left(-\rho_s^p u_s^p + \mu_s\bar{\rho}_s^p|u_s|^p\right) - 2e_I^T P\operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_f}\end{aligned} \tag{29}$$

From the definition of μ_s and Assumption 2.3, it yields

$$2e_I^T P b_s\left(-\rho_s^p u_s^p + \mu_s\bar{\rho}_s^p|u_s|^p\right) \leq 2e_I^T P b_s\rho_s^p u_s^p - \operatorname{sgn}\left(2e_I^T P b_s\right)\bar{\rho}_s^p|u_s|^p \leq 0 \tag{30}$$

Substituting (30) into (29), we have

$$\begin{aligned}\dot{V}_2 = {} & e_I^T\left[(A - LC)^T P + P(A - LC)\right]e_{Ip} + 2e_I^T P\left(B_d d + \tilde{f}_I\right) \\ & - 2e_I^T P\operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}}\end{aligned} \tag{31}$$

Similar to (10) and (12) in the previous subsection, we have

$$\begin{aligned}2e_I^T P\left(f - \hat{f}_s\right) & \leq 2e_I^T P\left[\theta_{f_s}^{*\,T}\xi_{f_s}(\hat{x}_s) - \hat{\theta}_{f_s}\xi_{f_s}(\hat{x}_s) + \varepsilon_{f_s}\right] \\ & \leq 2e_I^T P\left(\tilde{\theta}_{f_s}^T\xi_{f_s}(\hat{x}_s) + M_{\varepsilon_{f_s}}\right) \\ 2e_I^T P B_d d & \leq r e_I^T P B_d B_d^T P e_I + \frac{1}{r}\left(\bar{d}\right)^2\end{aligned}$$

where $M_{\varepsilon_{f_s}} = \left[M_{\varepsilon_{f_{s_1}}}, M_{\varepsilon_{f_{s_2}}}, \ldots, M_{\varepsilon_{f_{s_i}}}\right]^T$, $\tilde{\theta}_{f_s} = \left[\tilde{\theta}_{f_{s_1}}, \ldots, \tilde{\theta}_{f_{sn}}\right]^T$, $\tilde{\theta}_{f_{s_l}} = \theta_{f_{s_l}}^* - \hat{\theta}_{f_{s_l}}$, $\hat{\theta}_{f_{s_l}}$ is the estimate of $\theta_{f_{s_l}}^*$, $l = 1, \ldots, n$.

Further, we have

$$\begin{aligned}\dot{V}_2 \leq {} & \begin{bmatrix} e_I \\ e_{Ip}\end{bmatrix}^T \begin{bmatrix} X + rPB_d B_d^T P & Y + P(A - LC) \\ Y^T + (A - LC)^T P & Z\end{bmatrix}\begin{bmatrix} e_I \\ e_{Ip}\end{bmatrix} \\ & + \frac{1}{r}\left(\bar{d}\right)^2 + 2e_I^T P\tilde{\theta}_{f_s}^T\xi_{f_s} + 2e_I^T P\operatorname{sgn}\left(2e_I^T P\right)\tilde{M}_{\varepsilon_{f_s}}\end{aligned} \tag{32}$$

where $\tilde{M}_{\varepsilon_{f_s}} = \left[\tilde{M}_{\varepsilon_{f_{s_1}}}, \tilde{M}_{\varepsilon_{f_{s_2}}}, \ldots, \tilde{M}_{\varepsilon_{f_{sn}}}\right]$, $\tilde{M}_{\varepsilon_{f_{s_i}}} = M_{\varepsilon_{f_{s_i}}} - \hat{M}_{\varepsilon_{f_{s_i}}}$, $i = 1, \ldots, n$.

Similar to (15), it yields

$$\dot{V}_2 \leq -\begin{bmatrix} e_I \\ e_{Ip}\end{bmatrix}^T Q\begin{bmatrix} e_I \\ e_{Ip}\end{bmatrix} + \frac{1}{r}\left(\bar{d}\right)^2 + 2e_I^T P\tilde{\theta}_{f_s}^T\xi_{f_s} + 2e_I^T P\operatorname{sgn}\left(2e_I^T P\right)\tilde{M}_{\varepsilon_{f_s}} \tag{33}$$

Choose adaptive laws as follows:

$$\dot{\hat{\theta}}_{f_s} = 2\eta_3 e_D^T P\xi_{f_s} - \eta_{\theta_s}\hat{\theta}_{f_s} \tag{34}$$

$$\dot{\hat{\varepsilon}}_{f_s} = 2\eta_4 e_D^T P\operatorname{sgn}\left(2e_D^T P\right) - \eta_{\varepsilon_s}\hat{M}_{\varepsilon_{f_s}} \tag{35}$$

where $\eta_3 > 0$, $\eta_{\theta_s} > 0$, $\eta_4 > 0$, $\eta_{\varepsilon_s} > 0$ are design parameters.

Define $V_I = V_2 + \frac{1}{2\eta_3}\tilde{\theta}_{f_s}^T\tilde{\theta}_{f_s} + \frac{1}{2\eta_4}\tilde{M}_{\varepsilon_{f_s}}^2$, differentiating V_I with respect to time t and from (33)-(35) lead to

$$\dot{V}_I \leq -\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}e_I^T P e_I - \frac{\eta_{\theta_s}}{2\eta_3}\tilde{\theta}_{f_s}^T\tilde{\theta}_{f_s} - \frac{\eta_{\varepsilon_s}}{2\eta_4}\tilde{M}_{\varepsilon_{f_s}}^2 + \frac{\eta_{\theta_s}}{2\eta_3}M_{\theta_{f_s}}^2 + \frac{\eta_{\varepsilon s}}{2\eta_4}M_{\varepsilon_{f_s}}^2 + \frac{1}{r}\left(\bar{d}\right)^2$$

$$\leq -c_2 V_I + \mu_2 \tag{36}$$

where $c_2 = \min\left\{\frac{\lambda_{\min}(Q)}{2\lambda_{\max}(P)}, \frac{\eta_{\theta_s}}{2\eta_3}, \frac{\eta_{\varepsilon_s}}{2\eta_4}\right\}$, $\mu_2 = \frac{\eta_{\theta_s}}{2\eta_3}M_{\theta_{f_s}}^2 + \frac{\eta_{\varepsilon_s}}{2\eta_4}\left(M_{\varepsilon_{f_s}}\right)^2 + \frac{1}{r}\left(\bar{d}\right)^2$.

Then, one has $\frac{d}{dt}(V(t){e_I}^{c_2 t}) \leq {e_I}^{c_2 t}\mu_2$. Further

$$0 \leq V_I(t) \leq \frac{\mu_2}{c_2} + \left[V_I(0) - \frac{\mu_2}{c_2}\right]{e_I}^{-c_2 t} \leq \frac{\mu_2}{c_2} + V_I(0) = \alpha_I \tag{37}$$

Therefore, if the appropriate matrices X, Y, Z, Q and positive definite symmetric matrix P are chosen such that $\begin{bmatrix} X & Y^T \\ Y & Z \end{bmatrix} > 0$ holds and (38) holds, the adaptive law can guarantee that $V_I(t)$ is bounded, that is, the closed-loop system is semi-globally uniformly asymptotically bounded. That is, all signals of the closed-loop system remain within the compact set Ω_1 defined below:

$$\Omega_{e_I} := \left\{ \begin{array}{l} \left(e_I, \tilde{\theta}_{f_s}, \tilde{M}_{\varepsilon_{f_s}}\right) \Big| \ \|e_I\| \leq \sqrt{\alpha_I / \lambda_{\min}(P)}, \\ \left\|\tilde{\theta}_{f_s}\right\| \leq \sqrt{2\eta_3\alpha_I}, \quad \left\|\tilde{M}_{\varepsilon_{f_s}}\right\| \leq \sqrt{2\eta_4\alpha_I} \end{array} \right\}$$

2) For $s \neq l$, from the faulty (25) and the observer (26), one has

$$\begin{aligned} \dot{e}_I &= (A - LC)e_{Ip} - (b_l\rho_l^p u_l^p - b_s\mu_s\bar{\rho}_s^p|u_s|^p) + B_d d(t) + \tilde{f}_I - \operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}} \\ \dot{V}_I &= e_I^T\left[(A - LC)^T P + P(A - LC)\right]e_{Ip} + 2e_I^T P\left(-b_l\rho_l^p u_l^p + b_s\mu_s\bar{\rho}_s^p|u_s|^p\right) \\ &\quad - 2e_I^T P \operatorname{sgn}\left(2e_I^T P\right)\hat{M}_{\varepsilon_{f_s}} \end{aligned}$$

since $s \neq l$, $u_l \neq u_s$, $\rho_l \neq \rho_s$, it is found that $2e_I^T P\left(-b_l\rho_l^p u_l^p + b_s\mu_s\bar{\rho}_s^p|u_s|^p\right)$ varies infinitely, which further causes that all signals involved in the closed-loop systems do not converge to the above-obtained results.

From 1) and 2), the conclusion is easily obtained.

Theorem 3.2. *Consider system (1) and observer (26) under Assumptions 2.1-2.4, and the adaptive laws (34) and (35), if there exist matrices* $X, Y, Z > 0$, $P = P^T > 0$, $Q > 0$ *and positive constants* r *satisfying* $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} > 0$ *and*

$$\begin{bmatrix} X + rPB_dB_d^TP & Y + P(A - LC) \\ Y^T + (A - LC)^T P & Z \end{bmatrix} < -Q \tag{38}$$

then, if the faulty actuator is actuator l*, 1) for* $s = l$*, the closed-loop system is asymptotically stable, all closed-loop system signals converge to a small neighborhood of the origin* Ω_1*; 2) for* $s \neq l$*, then all closed-loop system signals do not converge to a small neighborhood of the origin* Ω_1*.*

Now, we denote the residuals between the real system and isolation estimators as follows:

$$J_s(t) = \|\hat{y}_s(t) - y(t)\| = \|Ce_I(t)\|, \quad 1 \leq s \leq m \tag{39}$$

From Theorem 3.2, we know, if the faulty actuator is the lth one, namely, $s = l$, $J_s(t)$ must converge to Ω_1; if $s \neq l$, $J_s(t)$ does not basically converge to Ω_1. Therefore, the isolation law for actuator fault can be designed as

$$\begin{cases} J_s(t) \leq T_I, & s = l \Rightarrow \text{the } l\text{th actuator is faulty} \\ J_s(t) > T_I, & s \neq l \end{cases} \tag{40}$$

where $T_I = \|C\|\sqrt{\alpha_I/\lambda_{\min}(P)}$ is a threshold.

4. **Simulation.** To verify the effectiveness of the proposed method, the following example is given

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_d = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} -4 & 1 \\ -1 & -3 \end{bmatrix} \tag{41}$$

In this simulation, $d(t) = 0.01 \sin t$, $f(x_1) = x_1^2 + 0.5\cos(x_2)$, $f(x_2) = 0.5x_1 - x_1x_2^2$. According to the empirical values of references, the initial values and parameter values in the simulation are selected as $[x_1(0), x_2(0)]^T = [0.1, -0.1]^T$.

Assume that only one actuator fails at a time. Consider the following condition:

$$u_1^f(t) = u_1(t), \quad u_2^f(t) = \begin{cases} u_2(t), & t < 10 \\ (1 - 0.04\cos(t)), & t \geq 10 \end{cases}$$

Firstly, the matrix inequality (23) and (38) is transformed into linear matrix inequality, and then the matrix X, Y, Z, P, Q is solved by Matlab. The simulation results are shown in Figures 1-4. In the case of no fault, the error convergence of the observers is shown in

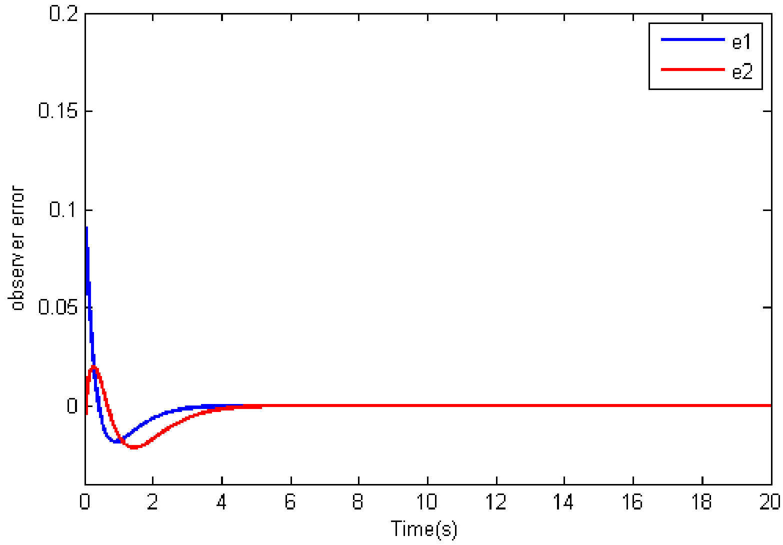

FIGURE 1. The observer error $x_i - \hat{x}_i$ in the fault-free case

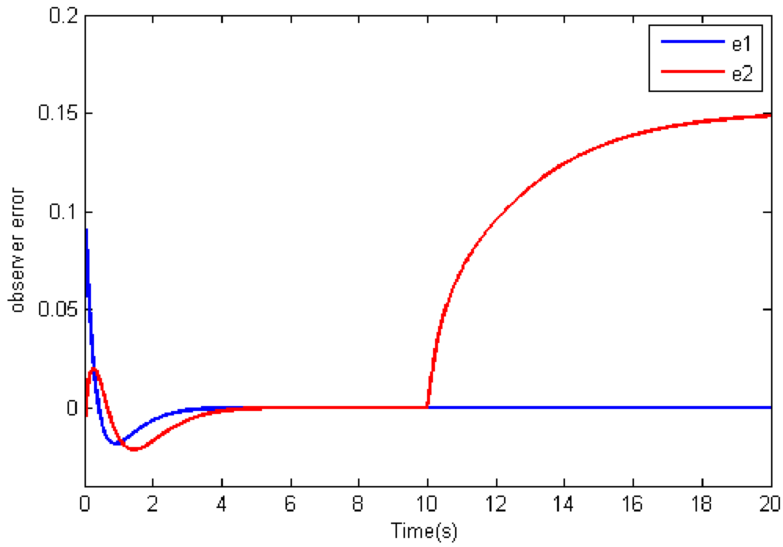

FIGURE 2. The observer error $x_i - \hat{x}_i$ in the faulty case

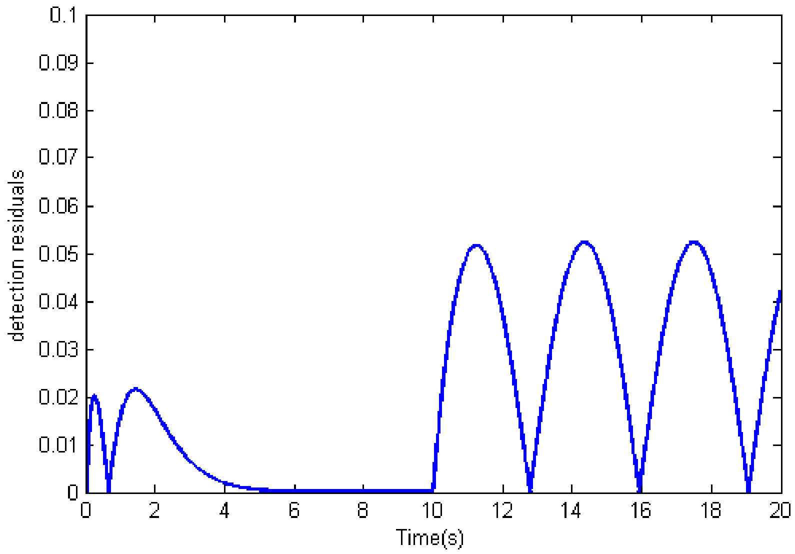

FIGURE 3. Fault detection residual

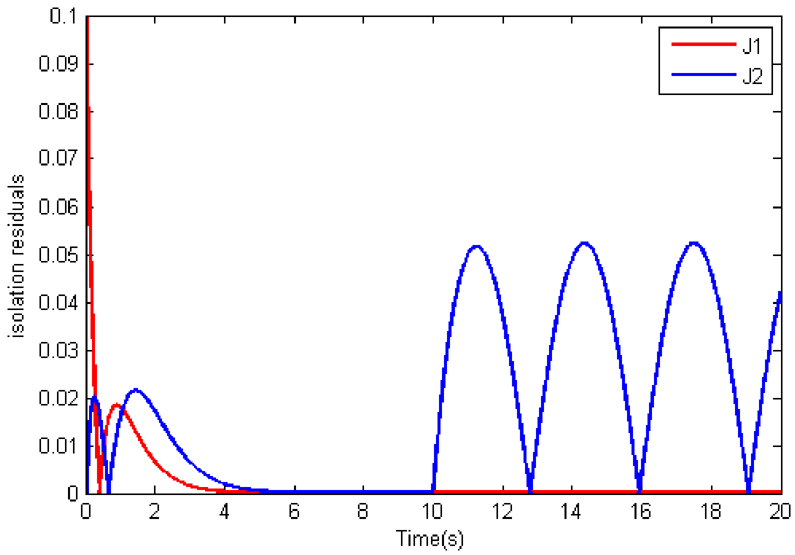

FIGURE 4. Fault isolation residuals

Figure 1, indicating that the designed observer has a good performance. At $t = 10$ s, the actuator of the second subsystem fails, as shown in Figures 2 and 3, the observer error and detection residual deviates from the origin obviously after the failure occurs, while that the isolation residual signals shows in Figure 4.

5. **Conclusions.** This paper studies the problem of fault detection and isolation for a class of high-order nonlinear dynamic systems. We design a bank of observers to detect and isolate the faults. Simulations show that the designed fault detection and isolation algorithms have better dynamic performances in the presence of actuator faults. The faults studies in this paper are mainly single gain faults, and other types of faults such as bias faults are not considered. Therefore, for high-order nonlinear system, how to isolate multiple types of faults is important and also is worthy of further research.

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